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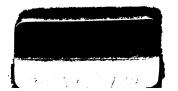
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# NATIONAL ARITHMETIC,

ON THE

# INDUCTIVE SYSTEM,

COMBINING THE

# ANALYTIC AND SYNTHETIC METHODS;

FORMING A COMPLETE

# COURSE OF HIGHER ARITHMETIC.

BY BENJAMIN GREENLEAF, A. M. AUTHOR OF THE "COMMON SCHOOL ARITHMETIC," "ALGEBRA," RTC.

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1860.

OFFICE OF THE CONTROLLERS OF PUBLIC ECHOOLS
FIRST SCHOOL DISTRICT OF PRINSTLYANIA.
PHILADELPHIA, December 14, 1859.

At a Meeting of the Controllers of Public Schools, First District of Pennsylvania, held at the Controllers' Chamber, on Tuesday, December 18th, 1859, the following Resolution was adopted:—

Resolved: That GREENLEAF'S COMMON SCHOOL AND NATIONAL ARITHMETICS be introduced to be used in the Public Schools of this District.

ROBERT J. HEMPHILL, Secretary.

# 748/8

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# PREFACE.

The National Arithmetic was first presented to the American public in 1835. The generous favor with which it was received assured the author that he had not misunderstood the wants of the public in the department of arithmetical instruction, and that his labors had, to

a considerable extent, supplied those wants.

During the ten years following, increased attention was given to the subject of popular education, and great improvements were made in methods of imparting knowledge. Accomplished teachers soon began to demand a work on Arithmetic, which should embody the numerous improvements which had enriched that science. In response to a demand so reasonable, the author was induced, in 1847, to prepare a revised and enlarged edition of the National Arithmetic. Aided by important suggestions from eminent teachers, and directly assisted by gentlemen intimately acquainted with arithmetical science, he was enabled to produce a work which, up to the present time, has been steadily increasing in public favor.

The last ten years have formed a period of unprecedented activity in all that relates to the interests of education. The numerous Arithmetics which, within this period, have become candidates for popular patronage, afford ample evidence that the department of knowledge to which they relate has meanwhile received its share of attention. Vigorous emulation among authors and publishers has produced thor-

ough investigation, careful preparation, and valuable results.

The author of this work, wishing, if possible, to keep pace with the rapid march of improvement, has again thoroughly revised, rewritten, and considerably enlarged it. The results of a long experience as a mathematical instructor, and the suggestions of many distinguished

teachers of the present day, are embodied in this volume.

In preparing this as well as the former editions of his National Arithmetic, the author has regarded the end to be sought in the study of Arithmetic as twofold,—a practical knowledge of numbers, and the discipline of the mind. With reference to the former, he has endeavored to present methods which are brief, accurate, and especially adapted to the wants of business life; with reference to the latter, he has aimed to give a clear and logical analysis of every operation, from the simplest to the most involved.

The author adheres to his opinion long since advanced, in relation

to the value of rules in an arithmetical treatise. It is not an easy thing for the experienced teacher to express in the most concise and accurate language the method of solving a problem. Much less can such an expression be given by the untrained scholar. Now, as precision in thought is essentially aided by precision in language, it is deemed expedient to furnish the scholar with rules which shall state in the fewest and clearest words the results of previous logical inductions. Moreover, when an intricate reasoning process may have been forgotten and cannot readily be recalled, the brief form of words impressed upon the memory in one's youth may oftentimes enable him in after life to perform an important mathematical operation in which he must otherwise have failed.

It will be observed, that, while the author has expressed in rules his modes of operating, he has in every case first given the analysis upon which each rule is based.

The author flatters himself that the present edition of the National Arithmetic embraces many improvements on former editions. He has endeavored to present clearer definitions, more rigid analyses, and briefer and more accurate rules. While almost every topic included in earlier editions has been treated in a more elaborate and comprehensive manner, this volume comprises a large amount of new matter, which it is believed will be found useful in business.

On comparing this with preceding editions, teachers will find extensive additions and improvements under the heads of Numeration, Addition, and the other fundamental rules, Properties of Numbers, Fractions, Ratio, Percentage, Notes and Banking, Roots, etc. Among the new material will be discovered methods of finding the greatest common divisor and the least common multiple of fractions, of reducing fractions to a common numerator, of contracting the operations in the multiplication and division of decimal fractions, of reducing continued fractions, of averaging accounts, of alligating, of extracting roots to any degree, and of reducing numbers from one system of notation to another.

Especial attention is invited to the section on averaging accounts,—
a subject rarely taught in schools, though of great importance in the
counting-room,— to the manner of treating the roots, and to the many
new problems which will be found in all parts of the book.

In closing these remarks, the author desires to tender his hearty thanks to many teachers who have favored him with valuable suggestions; and to acknowledge in an especial manner his indebtedness to Mr. H. B. Maglathlin, who has been constantly associated with him in making this revision, and to whose accurate scholarship and sound judgment the value of the work is largely due.

BRADFORD, MASS., May 30, 1857.

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## INTRODUCTION.

#### HISTORY OF A-RITHMETIC.

It is difficult to determine who was the inventor of Arithmetic, or in what age or among what people it originated. In ordinary history, we find the origin of the science attributed by some to the Greeks, by some to the Chaldeans, by some to the Phænicians, by Josephus to Abraham, and by many to the Egyptians. The opinion, however, which modern investigations have rendered most probable, is, that Arithmetic, properly so called, is of Indian origin, — that is, that the science received its first definite form, and became the germ of modern Arithmetic, in the regions of the East.

It is evident, from the nature of the case, that some knowledge of numbers and of the art of calculation was necessary to men in the earliest periods of society, since without this they could not have performed the simplest business transactions, even such as are incidental to an almost savage state. The question, therefore, as to the invention of Arithmetic, deserves to be considered only as it respects the origin of the science as we now have it, and which, as all scholars admit, has reached a surprising degree of perfection. In this sense the honor of the invention must be awarded to the Hindoos.

The history of the various methods of Notation, or the different means by which numbers have been expressed by signs or characters, is one of much interest to the advanced and curious scholar; but the brevity of this sketch allows us barely to touch upon it here. Among the ancient nations which possessed the art of writing, it was a natural and common device to employ letters to denote what we express by our numeral figures. Accordingly we find, that, with the Hebrews and Greeks, the first letter of their respective alphabets was used for 1, the second for 2, and so on to the number 10,—the latter, however, inserting one new character to denote the number 6, and evidently in order that their notation might coincide with that of the Hebrews, the sixth letter of the Hebrew alphabet having no corresponding one in the Greek.

The Romans, as is well known, employed the letters of their alphabet as numerals. Thus I denotes 1; V, 5; X, 10; L, 50; C, 100; D, 500; and M, 1,000. The intermediate numbers were expressed by a repetition of these letters in various combinations; as, II for 2; VI for 6; XV for 15; IV for 4; IX for 9, &c. They frequently expressed any number of thousands by the letter or letters denoting so many units, with a line drawn above; thus,  $\overline{V}$ , 5,000;  $\overline{VI}$ , 6,000;  $\overline{X}$ , 10,000;  $\overline{L}$ , 50,000;  $\overline{C}$ , 100,000;  $\overline{M}$ , 1,000,000.

In the classification of numbers, as well as in the manner of expressing them, there has been a great diversity of practice. we adopt the decimal scale and reckon by tens, the aborigines of Mexico, according to Humboldt, and some of the early nations of Europe, adopted the vicenary, reckoning by twenties; some of the Indian tribes, and several of the African tribes, use the quinary. reckoning by fives; and the Chinese for more than 4,000 years have used the binary, reckoning by twos. The adoption of one or another of these scales has been so general, that they have been regarded as natural, and accounted for by referring them to a common and natural cause. The reason for assuming the binary scale probably lay in the use of the two hands, which were employed as counters in computing; that for employing the quinary, in a similar use of the five fingers on either hand; while the decimal and vicenary scales had respect, the former to the ten fingers on the two hands, and the latter to the ten fingers combined with the ten toes on the naked feet, which were as familiar to the sight of a rude, uncivilized people as their fingers. — It is an interesting circumstance, that in the common name of our numeral figures, digits (digiti) or fingers, we preserve a memento of the reason why ten characters and our present decimal scale of numeration were originally adopted to express all numbers, even of the highest order.

It is now almost universally admitted, that our present numeral characters, and the method of estimating their value in a tenfold ratio from right to left, have decided advantages over all other systems, both of notation and numeration, that have ever been adopted. There have been those, as Leibnitz and De Lagni, who have advocated the binary scale; a few, with Claudius Ptolemy, have claimed advantages for the sexagenary scale, or that by sixty; and there are those who think that a duodecimal scale, and the use of twelve numeral figures instead of ten, would afford increased facility for rapid and extensive calculations; but most mathematicians are satisfied with the present number of numerals and the scale of numeration, which have attained an adoption all but universal.

It was long supposed, that for our modern Arithmetic the world is indebted to the Arabians. But this, as we have seen, is not the

case. The Hindoos at least communicated a knowledge of it to the Arabians, and, as we are not able to trace it beyond the former people, they must have the honor of its invention. They do not, however, claim this honor; but refer it to the Divinity, declaring that the invention of nine figures, with device of place, is to be ascribed to the beneficent Creator of the universe.

But though the invention of modern Arithmetic is to be ascribed to the Hindoos, the honor of introducing it into Europe belongs unquestionably to the Arabians. It was they who took the torch from the East and passed it along to the West. The precise period, however, at which this was done, it is not easy to determine. It is evident, that our numeral characters and our method of computing by them were in use among the Arabians about the beginning of the eighth century, when they invaded Spain, and it is probable that a knowledge of them was soon afterwards communicated to the inhabitants of Spain, and gradually to those of the other European countries.

It is said, that the celebrated Gerbert, afterward Pope Sylvester II., returning to France from Spain, where he had been to acquire a knowledge of the Arabic or Indian notation, about the year 970, introduced it among the French.

About the middle of the eleventh century it is supposed to have been introduced into England by John of Basingstoke, Archdeacon of Leicester.

The Arabic characters, having been first used by astronomers, became circulated over Europe in their almanacs; but do not seem to have secured general adoption in Europe earlier than the twelfth or thirteenth century.

The science of Arithmetic, like all other sciences, was very limited and imperfect at the beginning, and the successive steps by which it has reached its present extension and perfection have been taken at long intervals and among different nations. It has been developed by the necessities of business, by the strong love of certain minds for mathematical science and numerical calculation, and by the call for its higher offices by other sciences, especially that of Astronomy. In its progress, we find that the Arabians discovered the method of proof by casting out the 9's, and that the Italians early adopted the practice of separating numbers into periods of six figures, for the purpose of enumeration. To facilitate the process of multiplication, this latter people also introduced, probably from the writings of Boethius, the Multiplication Table of Pythagoras.

The invention of the Decimal Fraction was a great step in the advancement of arithmetical science, and the honor of it has generally been given to John Muller, commonly called Regiomontanus, about the year 1464. It appears, however, that Stevinus, in 1582, wrote

the first express treatise on the subject. The credit of first using the decimal point, by which the invention became permanently available, is given by Dr. Peacock to Napier, the inventor of Logarithms; but De Morgan says, that it was used by Richard Witt as early as 1613, while it is not shown that Napier used it before 1617. Circulating Decimals received but little attention till the time of Dr. Wallis, the author of the Arithmetic of Infinites. Dr. Wallis died at Oxford, in 1703.

The greatest improvement which the art of computation ever received was the invention of Logarithms, the honor of which is unquestionably due to Baron Napier, of Scotland, about the end of the sixteenth or the commencement of the seventeenth century.

The oldest treatises on Arithmetic now known are the 7th, 8th, 9th, and 10th books of Euclid's Elements, in which he treats of proportion and of prime and composite numbers. These books are not contained in the common editions of the great geometer, but are found in the edition by Dr. Barrow, the predecessor of Sir Isaac Newton in the mathematical chair at Cambridge. Euclid flourished about 300 B. C.

A century later, Eratosthenes invented a method, which is known as his "sieve," for separating prime numbers from others.

The next writer on Arithmetic mentioned in history is Nicomachus, the Pythagorean, who wrote a treatise relating chiefly to the distinctions and divisions of numbers into classes, as plain, solid, triangular, &c. He is supposed to have lived near the Christian era.

About the middle of the fourth century lived Diophantus, a celebrated mathematician, who, besides being the first known author on the subject of Algebra, composed thirteen books on Arithmetic, six of which are still extant.

The next writer of note is Boethius, the Roman, who, however, copied most of his work from Nicomachus. He lived at the beginning of the sixth century, and is the author of the well-known work on the Consolation of Philosophy.

The next writer of eminence on the subject is Jordanus, of Namur, who wrote a treatise about the year 1200, which was published by Joannes Faber Stapulensis in the fifteenth century, soon after the invention of printing.

The author of the first printed treatise on Arithmetic was Pacioli, or, as he is more frequently called, Lucas de Burgo, an Italian monk, who in 1484 published his great work entitled Summa de Arithmetica, &c., in which our present numerals appear under very nearly their modern form.

In 1522, Bishop Tonstall published a work on the Art of Computatien, in the Dedication of which he says, that he was induced to study Arithmetic to protect himself from the frauds of money-changers and stewards, who took advantage of the ignorance of their employers. In his preparation for this work, he professes to have read all the books which had been published on this subject, adding, also, that there was hardly any nation which did not possess such books.

About the year 1540, Robert Record, Doctor in Physic, printed the first edition of his famous Arithmetic, which was afterward augmented by John Dee, and subsequently by John Mellis, and which did much to advance the science and practice of Arithmetic in England in its early stages. This work, which is now quite a curiosity, effectually destroys the claim to originality in some things of which authors much more modern have obtained the credit. In it we find the celebrated case of a will, which we have in the Miscellaneous Questions of Webber and Kinne, and which, altered in language and the time of making the testament, is the 2nd Miscellaneous Question in the present work. This question is, by his own confession, older than Record, and is said to have been famous since the days of Lucas de Burgo. In Record it occurs under the "Rule of Fellowship." Record was the author of the first treatise on Algebra in the English language.

In 1556, a complete work on Practical Arithmetic was published by Nicolas Tartaglia, an Italian, and one of the most eminent mathema-

ticians of his time.

From the time of Record and Tartaglia, works on Arithmetic have been too numerous to mention in an ordinary history of the science. De Morgan, in his recent work (Arithmetical Books), has given the names of a large number, with brief observations upon them, and to this the inquisitive student is referred for further information in regard both to writers and books on this subject since the invention of Printing. It is remarkable that De Morgan knew next to nothing of any American works on Arithmetic. He mentions the "American Accountant" by William Milns, New York, 1797, and gives the name of Pike (probably Nicholas Pike) among the names of which he had heard in connection with the subject. Of the compilation of Webber and the original work of Walsh, he seems to have been entirely ignorant.

The various signs or symbols, which are now so generally used to abridge arithmetical as well as algebraical operations, were introduced gradually, as necessity or convenience taught their importance. The earliest writer on Algebra after the invention of printing was Lucas de Burgo, above mentioned, and he uses p for plus and m for minus, and indicates the powers by the first two letters, in which he is followed by several of his successors. After this, Steifel, a German, who in 1544 published a work entitled Arithmetica Integra, added

considerably to the use of signs, and, according to Dr. Hutton, was the first who employed + and — to denote addition and subtraction. To denote the root of a quantity he also used our present sign  $\checkmark$ , originally r, the initial of the word radix, root. The sign =, to denote equality, was introduced by Record, the above-named English mathematician, and for this reason, as he says, that "noe 2 thynges can be moar equalle," namely, than two parallel lines. It is a curious circumstance that this same symbol was first used to denote subtraction. It was also employed in this sense by Albert Girarde, who lived a little later than Record. Girarde dispensed with the vinculum employed by Steifel, as in  $\overline{3+4}$ , and substituted the parenthesis (3+4), now so generally adopted. The first use of the St. Andrew's cross,  $\times$ , to signify multiplication, is attributed to William Oughtred, an Englishman, who in 1631 published a work entitled Clavis Mathematica, or Key of Mathematics.

It was intended to notice several other works, ancient and modern, but the length to which this sketch has already extended forbids it.

We had thought of alluding to the ancient philosophic Arithmetic. and the elevated ideas which many of the early philosophers had of the science and properties of numbers. But a word must here suffice. Arithmetic, according to the followers of Plato, was not to be studied. "with gross and vulgar views, but in such a manner as might enable men to attain to the contemplation of numbers; not for the purpose of dealing with merchants and tavern-keepers, but for the improvement of the mind, considering it as the path which leads to the knowledge of truth and reality." These transcendentalists considered perfect numbers, compared with those which are deficient or superabundant, as the images of the virtues, which, they allege, are equally remote from excess and defect, constituting a mean between them; as in the case of true courage, which, they say, lies midway between audacity and cowardice, and of liberality, which is a mean between profusion and avarice. In other respects, also, they regard this analogy as remarkable; perfect numbers, like the virtues, are "few in number and generated in a constant order; while superabundant and deficient numbers are, like vices, infinite in number, disposable in no regular series, and generated according to no certain and invariable law."

We conclude this brief sketch in the earnest hope that the noble science of numbers may erelong find some devoted friend, who shall collect, arrange, and bring within the reach of ordinary students, much more fully than we have done, the scattered details of its long-neglected history.

# ARITHMETIC.

#### DEFINITIONS.

ARTICLE 1. QUANTITY is anything that can be increased, diminished, or measured; as time, weight, lines, surfaces, and solids.

- 2. A unit is a single thing or quantity regarded as a whole.
- 3. An abstract unit is one that has no reference to any particular thing or quantity.
- 4. A concrete unit is one that has reference to some particular thing or quantity.
- 5. A number is an expression of quantity, representing either a unit or a collection of units.
- 6. An abstract number is a number whose unit is abstract; as, one, six, nine.
- 7. A concrete or denominate number is a number whose unit is concrete; as, one dollar, six pounds, nine men.
- 8. A simple number is a unit, or a collection of units, either abstract, or concrete of a single kind or denomination; as, 1, 15, 1 book, 13 dollars.
- **9.** The *unit of measure* of any quantity is one of the same kind with that by which the quantity is measured or compared; as, in the abstract number, six, the abstract unit is that of measure or comparison; and in six pounds, the concrete unit, one pound, is that of measure or comparison.
- 10. ARITHMETIC is the science of numbers and the art of computing by them. It treats of the properties and relations

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of numbers, and teaches the methods of applying the principles of the science to practical purposes.

11. An axiom is a self-evident truth.

12. A problem is a question proposed for solution, or something to be done.

13. An operation is the process of finding, from given

quantities, others that are required.

14. A sign is a symbol employed to indicate the relations of quantities, or operations to be performed upon them.

15. A rule is a direction for performing an operation.

- 16. An example is a particular application of a general principle or rule.
- 17. The principal or fundamental processes of arithmetic are Notation and Numeration, Addition, Subtraction, Multiplication, and Division.

# SIGNS.

- 18. The sign of equality, two short horizontal lines, =, is read equal, or equal to, and denotes that the quantities between which it is placed are equal to each other. Thus, 12 inches = 1 foot, signifies that 12 inches are equal to 1 foot.
- 19. The sign of addition, an erect cross, +, is read plus, and, or added to, and denotes that the quantities between which it is placed are to be added together. Thus, 8+6 signifies that 6 is to be added to 8.
- 20. The sign of subtraction, a short horizontal line, —, is read minus, or less, and denotes that the quantity on the right of it is to be subtracted from the quantity on the left. Thus, 8 6 signifies that 6 is to be subtracted from 8.
- 21. The sign of multiplication, an inclined cross,  $\times$ , is read times, or multiplied by, and denotes that the quantities between which it is placed are to be multiplied together. Thus,  $7 \times 6$  signifies that 7 is to be multiplied by 6.
- 22. The sign of division, a horizontal line between two dots,  $\div$ , is read divided by, and denotes that the quantity on the left of it is to be divided by that on the right. Thus,  $42 \div 6$  signifies that 42 is to be divided by 6.
  - 23. The sign of aggregation, a parenthesis, (), includ-

ing several numbers, or a vinculum, —, drawn over them, indicates that the value of the expression is to be used as a single number. Thus,  $(17+3) \times 5$ , indicates that the sum of 17 and 3, or 20, is to be multiplied by 5; and  $12+(9-3)\div 2$ , indicates that the difference between 9 and 3 divided by 2, or 3, is to be added to 12.

# AXIOMS.

- 24. Arithmetic, in common with other branches of the mathematics, is based upon axioms, few in number, and universally admitted to be so clearly true, that no process of reasoning can make them plainer; as,
- 1. If the same quantity, or equal quantities, be added to equal quantities, the sums will be equal.
- 2. If the same quantity, or equal quantities, be subtracted from equal quantities, the remainders will be equal.
- 3. If the same quantity, or equal quantities, be added to unequal quantities, the sums will be unequal.
- 4. If the same quantity, or equal quantities, be subtracted from unequal quantities, the remainders will be unequal.
- 5. If equal quantities be multiplied by the same quantity, or equal quantities, the products will be equal.
- 6. If equal quantities be divided by the same quantity, or equal quantities, the quotients will be equal.
- 7. If the same quantity be both added to and subtracted from another, the value of the latter will not be changed.
- 8. If a quantity be both multiplied and divided by the same quantity, its value will not be changed.
- 9. If two quantities be equally increased or diminished, their difference will not be changed.
- 10. Quantities which are equal to the same quantity are equal to each other.
- 11. Quantities which are like parts of equal quantities are equal to each other.
- 12. The whole of a quantity is greater than any of its
- 13. The whole of a quantity is equal to the sum of all its parts.

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#### NOTATION AND NUMERATION.

#### NOTATION.

25. NOTATION is the process of representing numbers by letters, figures, or other symbols.

The common methods of expressing numbers are three: by words, written or spoken; by letters, called the Roman method; and by figures, called the Arabic method.

26. In common language, we express numbers by the terms one, two, three, four, five, six, seven, eight, nine, ten, eleven, twelve, thirteen, fourteen, fifteen, sixteen, seventeen, eighteen, nineteen, twenty, twenty-one, etc., giving a distinct name to each unit as far as ten, when we begin a second ten, and pass on to twenty; a third ten, and pass on to thirty; and so on to forty, fifty, sixty, seventy, eighty, and ninety. Proceeding thus we reach ten tens, which we call one hundred, when we begin a second hundred, and pass to two hundred; a third hundred, and pass to three hundred; and so on as far as ten hundred, which we call one thousand. A thousand thousand we call one million; a thousand million, one billion; a thousand billion, one trillion; and so on with numbers still higher.

Note 1. — The term eleven is a contraction of one left after ten; and twelve, of two left after ten. Thirteen, fourteen, fifteen, sixteen, seventeen, eighteen, nineteen, are derived from three and ten, four and ten, five and ten, etc. Twenty, thirty, forty, fifty, sixty, seventy, eighty, and ninety are contractions of two tens, three tens, four tens, etc.

Note 2.— Billion is a contraction of the Latin bis, twice, and million; and trillion, of the Latin tres, three, and million. In like manner from the Latin numerals, quatuor, four; quinque, five; sex, six; septem, seven; octo, eight; novem, nine; decem, ten; undecim, eleven; duodecim, twelve; tredecim, thirteen; quatuordecim, fourteen; quindecim, fifteen; sexdecim, sixteen; septendecim, seventeen; octodecim, eighteen; novemdecim, nineteen; viginti, twenty,—are formed quadrillions, quintillions, sextillions, septillions, octillions, nonillions, decillions, undecillions, duodecillions, etc.

#### ROMAN NOTATION.

27. THE ROMAN NOTATION, so called from its having originated with the ancient Romans, employs in expressing numbers seven capital letters, viz.:

I, V, X, L, C, D, M. one, five, ten, fifty, one hundred, five hundred, one thousand.

All intervening and succeeding numbers are expressed by use of these letters, either in repetitions or combinations. By a letter being written after another denoting equal or less value, the sum of their values is represented; as, II represents two; VI, six. By writing a letter denoting a less value before a letter denoting a greater, their difference of value is represented; as, IV represents four; XL, forty.

A dash (—) placed over a letter increases the value denoted by the letter a thousand times; as,  $\overline{V}$  represents five thousand;  $\overline{IV}$ , four thousand.

#### TABLE.

Ī	der	otes	one.	XXX de	notes	thirty.
II.		"	two.	$\mathbf{XL}$	"	forty.
$\mathbf{III}$		"	three.	${f L}$	"	fifty.
IV		"	four.	LX	"	sixty.
V.		"	five.	LXX	"	seventy.
$\mathbf{v}\mathbf{I}$		".	six.	LXXX	"	eighty.
VII		"	seven.	$\mathbf{XC}$	"	ninety.
VII	Ι	u	eight.	C	"	one hundred.
$\mathbf{I}\mathbf{X}$		"	nine.	CI	"	one hundred and one.
$\mathbf{X}$		"	ten.	$\mathbf{CC}$	"	two hundred.
$\mathbf{XI}$		"	eleven.	CCC	"	three hundred.
XII	[	66	twelve.	CCCC -	"	four hundred.
$\mathbf{X}\Pi$	ΙΙ	"	thirteen.	D	"	five hundred.
XI	V	"	fourteen.	DC	"	six hundred.
$\mathbf{x}\mathbf{v}$	•	"	fifteen.	DCC	"	seven hundred.
XV	Ι	к	sixteen.	DCCC	"	eight hundred.
XV	Π	"	seventeen.	DCCCC	"	nine hundred.
$\mathbf{X}\mathbf{V}$	Ш	"	eighteen.	M	"	one thousand.
$\mathbf{X}\Gamma$	X	"	nineteen.	MD	"	fifteen hundred.
$\mathbf{X}\mathbf{X}$		"	twenty.	MM	"	two thousand.
XX	$\mathbf{I}$	"	twenty-one.	$\overline{XIX}$	"	nineteen thousand.
$\mathbf{X}\mathbf{X}$	П	"	twenty-two.	$\overline{\mathbf{M}}$	66	one million.
$\mathbf{X}\mathbf{X}$	Ш	"	twenty-three.	$\overline{\mathbf{M}}$	"	two million.

Note 1.— The Roman method of Notation is now but little used, except in numbering sections, chapters, and other divisions of books; and for indicating the hours on the face of clocks, watches, or dials.

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NOTE 2. — Formerly CIO was used to represent one thousand, and the pre-fixing of a C and the annexing of a O increased the number denoted ten times; thus, CCIOO represented ten thousand, and CCCIOOO, one hundred thousand.

#### EXERCISES.

Represent the following numbers by letters:-

1. Forty-nine.

Ans. XLIX.

- 2. Ninety-seven.
- 3. One hundred and eighty-eight.
- 4. Two hundred and nineteen.
- 5. Six hundred and sixty-three.
- 6. One thousand five hundred and six.
- 7. One thousand eight hundred and fifty-seven.
- 8. Four thousand four hundred and forty-four.
- 9. Eleven thousand nine hundred and eleven.
- 10. One hundred fifty thousand and fifty.
- 11. One million twenty thousand and twenty.
- 12. Three million one hundred thousand.

#### ARABIC NOTATION.

- 28. ARABIC NOTATION, so called from its having been made known through the Arabs, employs in expressing numbers ten characters or figures, viz.:
- 1, 2, 3, 4, 5, 6, 7, 8, 9, 0. one, two, three, four, five, six, seven, eight, nine, cipher. The first nine are sometimes called digits, and the cipher, naught or zero.
- 29. The place of a figure is its particular position with regard to other figures; as in 61 (sixty-one) counting from the right, the 1 occupies the first place and the 6 the second place, and so on for any other like arrangement of figures.
- 30. The digits have been denominated significant figures, because each of itself expresses a positive value, always representing so many units, or ones, as its name indicates. But the size or value of the units represented by a figure differs with the place occupied by the figure. Thus in 235 (two hundred and thirty-five), each of the figures, without regard to its place, expresses units, or ones; but these units or ones differ in value. The 5 occupying the first place represents

5 single units; the 3 occupying the second place represents 3 tens, or 3 units each ten times the size or value of a unit of the first place; and the 2 occupying the third place represents 2 hundreds, or 2 units each one hundred times the size or value of a unit of the first place; the value expressed by any figure being always made tenfold by each removal of it one place to the left.

- 31. The cipher becomes significant when connected with other figures, by filling a place that otherwise would be vacant; as in 10 (ten) where it occupies the vacant place of units, and in 102 (one hundred and two) where it fills the vacant place of tens.
- 32. The simple value of a unit is the value expressed by a figure standing alone; or, in a collection, when standing in the right-hand place. Thus 2 alone, or in 32 (thirty-two), expresses a simple value of two single units or ones.
- 33. The local value of a unit is the value expressed by a figure when it is used in combination with another figure or figures, and depends upon the place the figure occupies. Thus, in 44 (forty-four), the 4 in the first place expresses the local value of 4 units, and the 4 in the second place, the local value of 4 tens, or forty.
- 34. The successive places occupied by figures are often called orders. Thus a figure in the first or units' place is called a figure of the first order, or of the order of units; a figure in the second place is a figure of the second order, or of the order of tens; in the third place, of the third order, or of the order of hundreds; and so on, each figure next to the left belonging to a distinct order, the unit of which is tenfold the size or value of a unit of the order at the right.

#### EXERCISES.

- 1. Write three units of the first order.
- 2. Write five units of the first order.
- 3. Write eight units of the second order, with seven of the first.
- 4. Write two units of the third order, with none of the second, and one of the first.
- 5. Write seven units of the fourth order, with two of the third, none of the second, and none of the first.

- 6. Write one unit of the fifth order, with none of the four lower orders.
- 7. Write six units of the sixth order, five of the fifth, four of the fourth, three of the third, one of the second, and two of the first.
- 8. Write one unit of the eighth order, with none of the seven lower orders.
- 9. Write nine units of the ninth order, with six of each of the eight lower orders.
- 10. Write two units of the twelfth order, with none of the eleventh, none of the tenth, one of the ninth, five of the eighth, nine of the seventh, none of the sixth, none of the fifth, none of the fourth, three of the third, none of the second, and three of the first.
- 11. Write three units of the fifteenth order, with none of the fourteenth, none of the thirteenth, none of the twelfth, one of the eleventh, seven of the tenth, five of the ninth, one of the eighth, none of the seventh, none of the sixth, five of the fifth, three of the fourth, two of the third, two of the second, and seven of the first.
- 12. Write four units of the twenty-fifth order, with three of the twenty-fourth, two of the twenty-third, none of the twenty-second, none of the twenty-first, none of the twentieth, none of the nineteenth, none of the eighteenth, none of the seventeenth, five of the sixteenth, and none of the fifteen lower orders.

## NUMERATION.

- 35. Numeration is the process of reading numbers when expressed by figures.
- 36. There are two methods of numeration; the French and the English.

## FRENCH NUMERATION.

37. The French method of numeration is that in general use on the continent of Europe and in the United States. Beginning at the right, figures occupying more than three places being separated into as many groups as possible of three figures each, called *periods*, it gives a distinct name to each period.

#### FRENCH NUMERATION TABLE.

La Hundreds of Sextillions.  Tens of Sextillions.  Sextillions.	Thundreds of Quintillions.  The Countillions.  Countillions.	Fundreds of Quadrillions.	La Hundreds of Trillions.  The control of Trillions.  The Trillions.	L Hundreds of Billions.  Tens of Billions.  By Billions.	Hundreds of Millions.  c. Tens of Millions.	> Hundreds of Thousands. ∞ Tens of Thousands. ∴ Thousands.	Thundreds.  Tens.  Units.
	$\sim$	$\sim$	$\sim$	$\sim$	$\sim$	$\sim$	$\sim$
8th Period. Sextil- lions.	7th Period. Quintil- lions.	6th Period. Quadril- lions.	5th Period. Trillions.	4th Period. Billions.	8d Period. Millions.	2d Period. Thousands.	1st Period. Units.

The value of the number represented in the table is, seven hundred eighty-nine sextillions, one hundred twenty-three quintillions, four hundred fifty-six quadrillions, seven hundred eighty-nine trillions, one hundred twenty-three billions, four hundred fifty-six millions, seven hundred eighty-nine thousands, one hundred twenty-three.

38. The unit of the first period, or right-hand group, is 1; of the second, 1 thousand; of the third, 1 million; of the fourth, 1 billion; of the fifth, 1 trillion; of the sixth, 1 quadrillion; of the seventh, 1 quintillion; of the eighth, 1 sextillion, etc.

The periods above sextillions, in their order, are, Septillions, Octillions, Nonillions, Decillions, Undecillions, Duodecillions, Tredecillions, Quatuordecillions, Quindecillions, Sexdecillions, Septendecillions, Octodecillions, Novemdecillions, Vigintillions, etc.

Note.—The idea of number is the latest and most difficult to form. Before the mind can arrive at such an abstract conception, it must be familiar with that process of classification by which we successively ascend from individuals to species, from species to genera, from genera to orders. The savage is lost in his attempts at numeration, and significantly expresses his inability to proceed by holding up his expanded fingers or pointing to the hair of his head. It is, indeed, difficult for any mind to form an adequate idea of the larger numbers. To count a million, at the rate of one in a second, would require upward of twenty-three days of twelve hours each. A billion is equal to a million a thousand times repeated, or a number so great, as to exceed all the seconds of time that would elapse in thirty-two years.

To read numbers represented by figures according to the French method; -

Begin at the right hand, and point off the figures into as many periods as possible of three places each.

Then, commencing at the left hand, read the figures of each period, giving the name of each period excepting that of units.

# EXERCISES.

Read orally, or write in words, the numbers represented by the following figures, according to the French method: -

1.	31620	7.	21628	3417	13.	325617189	4
2.	55216	8.	101313	5125	14.	516225191	2
3.	29156	9.	876223	3399	15.	1314142121	1
4.	179213	10.	12343	625	16.	45786349763	9
<b>5.</b>	918512	11.	21177	7792	17.	43437878343	4
6.	1219615	12.	6665551	161	18.	785246376744	5
19.	4070000107	03801	21.	4	781278	1501666606070	7
20.	2000700078	01000	22.	8008	008008	0080080080080	0
	. 93	19708	1061071	0810	100090	07007	

24. 407144140070060700007101800808

40. To write numbers in figures according to the French method:-

Begin at the left hand, and write in each successive order the figure belonging to it.

If any intervening order would otherwise be vacant, fill the place by

a cipher.

#### Exercises.

Represent by figures, and read, the following numbers, according to the French method: -

- Twenty-nine.
- Four hundred and seven.
- 3. Twenty-three thousand and seven.
- 4. Five millions and twenty-seven.
- 5. Seven millions, two hundred five thousand and five.
- 6. Two billions, two hundred seven millions, six hundred four thousand and nine.
- 7. One hundred five billions, nine hundred nine millions, three hundred eight thousand two hundred and one.

- 8. Nine quintillions, eight billions and forty-six.
- 9. Fifteen quintillions, thirty-one millions and seventeen.
- 10. Five hundred seven septillions, two hundred three trillions, fifty-seven millions and eighteen.
- 11. Nine nonillions, forty-seven trillions, seven billions, two millions, three hundred ninety-two.
- 12. Fifteen duodecillions, ten trillions, one hundred twenty-seven billions, twenty-six millions, three hundred twenty thousand four hundred twenty-six.

#### ENGLISH NUMERATION.

41. The English method of numeration is that generally used in Great Britain, and in the British Provinces. It divides numbers into periods of six figures, and gives a distinct name to each.

#### ENGLISH NUMERATION TABLE.

c Hund of Thousands of Trillions. c Tens of Thousands of Trillions. c Thousands of Trillions. c Hundreds of Trillions. c Tens of Trillions.	c. Hund. of Thousands of Billions.  Thousands of Billions.  Thousands of Billions.  Thurdreds of Billions.  Hundreds of Billions.  The State of Billions.	C. Hund. of Thousands of Millions. Thousands of Millions. Thousands of Millions. Hundreds of Millions. Than of Millions. The Relations. The Millions.	Thursday of Thousands.  The Tens of Thousands.  Thousands.  Thousands.  Thursday.  Thursday.  Thursday.  Thursday.  Thursday.  Thursday.
4th Period.	3d Period.	2d Period.	1st Period.
Trillions.	Billions.	Millions.	Units.

The value of the figures in the above table, expressed in words according to the English method, is, Three hundred ninety-eight thousand, eight hundred thirty-two trillions; five hundred sixty-three thousand, eight hundred seventy-one billions; three hundred fifty-one thousand, six hundred fifteen millions; one hundred twenty-three thousand five hundred sixty-one.

42. To read numbers represented by figures according to the English method;—

Begin at the right hand, and point off the figures into periods of six

places each.

Then, commencing at the left hand, read the figures of each period, giving the name of each period except that of units.

#### EXERCISES.

Read orally, or write in words, the numbers represented by the following figures, according to the English method:—

1.	23457896	4.	98765421910311
2.	325487691	5.	5632411132321300012
3.	1678912161	6.	6961771889133201443345567

43. To write numbers in figures according to the English method; —

Begin at the left hand, and write in each successive order the figure belonging to it.

If any intervening order would otherwise be vacant, fill the place by

a cipher.

#### EXERCISES.

Represent by figures, and read, the following numbers, according to the English method:—

- 1. Thirty-two million three hundred.
- 2. Seven billion seventeen thousand.
- 3. Five hundred sixty thousand one hundred two million, nine hundred twenty-nine thousand four hundred eleven.
- 4. One trillion, seven hundred forty-eight thousand nine hundred fifty-five billion.

### ADDITION.

44. Addition is the process of finding the sum of two or more numbers. The result obtained is called the amount.

Numbers can be added together only when their units are of the same kind. When the numbers added are simple, the process is termed Addition of Simple Numbers. 45. To add simple numbers.

Ex. 1. A man has three farms; the first contains 378 acres, the second 586 acres, and the third 168 acres. How many acres are there in the three farms.

Ans. 1132.

Acres. Having arranged the numbers so that all the units of the same order shall stand in the same column, we first add the column of units; thus, 8 and 6 are 14, and 8 are 22 units, = 2 tens and 2 units. We write the two units under the column of units, and

Ans. 1132 carry or add the 2 tens to the column of tens; thus,

Ans. 1132 added to 6 make 8, and 8 are 16, and 7 are 23

tens, = 2 hundred and 3 tens. We write the 3 tens under the column of tens, and add the 2 hundred to the column of hundreds; thus, 2 added to one make 3, and 5 are 8, and 3 are 11 hundred, = 1 thousand and 1 hundred. We write the 1 hundred under the column of hundreds; and there being no other column to be added, we set down the 1 thousand in the thousands place, and find the amount of the several numbers to be 1132.

In practice, it is better not to name each figure added, but only

the results, thus, 8, 14, 22 units, = 2 tens and 2 units, etc.

Rule. — Write the numbers so that all the figures of the same order shall stand in the same column.

Add, upward, all the figures in the column of units, and, if the amount be less than ten, write it underneath. But if the amount be ten or more, write down the unit figure only, and add in the figure denoting the ten or tens with the next column.

Proceed in this way with each column, until all are added, observing

to write under the last column its whole amount.

46. First Method of Proof.—Begin at the top and add the columns downward in the same manner as they were before added upward; and if the two sums agree, the work is presumed to be right.

The reason of this proof is, that, by adding downward, the order of the figures is inverted; and, therefore, any error made in the first addition would probably be detected in the second.

NOTE. — This method of proof is generally used in business.

47. Second Method of Proof. — Separate the numbers to be added into two parts, by drawing a horizontal line between them. Add the numbers below the line, and set down their sum. Then add this sum and the number or numbers above the line together; and if their sum is equal to the first amount, the work is presumed to be right.

The reason of this proof depends on the principle, that the sum of all the parts into which any number is separated is equal to the whole. (Art. 24, Ax. 13.)

## EXAMPLES.

5	2.	2.		3.		3.
OPERATI	on. Operati <b>Iges</b> .	ON AND PROOF. Dollars.	. 01	Tons.	OPERATION	AND PROOF. Tons.
	6 5	765		126		126
•	81	38.1		384		384
_	7 <b>2</b>	872		876		876
_	15	315		243		243
	3 3 First a		Ane		First am	
200. 20	oo rusta		Zus.	1025	rnst am	
		.1568				$\frac{1503}{}$
	A	ns. 2333			Ans	. 1629
4.	<b>5.</b>	6.	<b>7.</b> :	8.	9.	10.
Barrels.	Pounds.		ents.	Engles.	Rods.	Poles.
123	678	456 7	89	456	781	889
456	901	789 9	87	781	175	77.6
789	278	127 1	23	197	564	432
341	633	815 8	21	715	337	876
1709	$\overline{2490}$	187 22	20	2149	1857	2973
11.	12.	13.	·1	4.	15.	16.
Ounces.	Inches.	Rods.	Furl	ongs.	Cords.	Feet.
7891	3256	6789	12	34	4567	4561
3245	7890	1234	<b>5</b> 6	<b>78</b>	8912	7890
6789	1234	5678	90	1 2	3456	7658
1234	5678	6543	34	56	7891 .	8888
5678	7801	1234	<b>7</b> 8	91	4567	9199
24887	$\overline{25859}$	21478	272	71 2	9393	38196
17.	18.	19.	20	).	21.	22.
Hogsheads.	Furlongs.	Miles.	Dolls	rs.	Casks.	Pence.
1789	6781	7890	17	85	7895	4371
6543	1371	1070	56	78	5678	1699
2177	8715	4437	9 1		7186	1098
8915	6371	6789	81		5176	8816
6781	1234	5378	18		4321	6171
4325	7171	1234	19		4127	7185

23.	24.	25.	26.	27.
Shillings.	Tons.	Miles.	Trees.	Loads.
78956	12345	34567	76717	56789
32167	87655	78901	77776	12345
41328	34517	$3\ 2\ 1\ 9\ 9$	67890	67819
45678	6548.3	17188	71444	34567
13853	79061	88888	47474	71888
71667	20939	12345	16175	83197
*		************		

28.	29.	30.	31.
Acres.	Roods.	Poles.	Yards.
789.516	451237	1234567	789123
377895	813715	8901234	456789
378567	679919	5678901	987654
832156	787651	3456789	357913
789567	637171	5432115	245678
813138	813785	7177444	999999

- 32. What is the sum of 15 + 26 + 18 + 91? Ans. 150.
- 33. What is the sum of 6789 + 5832 + 4671 + 8907?
  Ans. 26199.
- 34. Required the sum of 76 + 48 + 59 + 81. Ans. 264.
- 35. Required the sum of 123456 + 789012 + 345678 + 901234 + 567890 + 987654 + 321032 + 765437.
- 36. Required the sum of 876543 + 789112 + 345678 + 965887 + 445566 + 788743 + 399378 + 456789.
- 37. Required the sum of 789012 + 345678 + 901234 + 789037 + 891133 + 477666 + 557788 + 888878.
- 38. Required the sum of 987654 + 456112 + 222333 + 456789 + 987654 + 321178 + 123456 + 789561.
- 39. Required the sum of 678953 + 467631 + 117777 + 888888 + 444444 + 667679 + 998889 + 671236.
- 40. Required the sum of 783256 + 7128 + 39 + 432815 + 99 + 67851 + 125 + 641236 + 801 + 4328.
- 41. Required the sum of 12004 + 32 + 1 + 7836 + 100 + 46 + 3 + 6176 + 32 + 91876.
  - 42. Add together 763, 4663, 37, 49763, 6178, and 671. Ans. 62075.
  - 43. Add together 15, 7896, 1, 13, 106, 113, 156, 100, 2201.



- 44. A butcher sold to A 369 lbs. of beef, to B 169 lbs., to C 861 lbs., to D 901 lbs., to E 71 lbs., and to F 8716 lbs.; what did they all receive?

  Ans. 11087 lbs.
- 45. A owes to one creditor 596 dollars, to another 3961, to another 581, to another 6116, to another 469, to another 506, to another 69381, and to another 1261. What does he owe them all?

  Ans. \$82871.
- 46. If a boy earn 17 cents a day, how much will he earn in 7 days?

  Ans. 119 cts.
- 47. If a man's wages be 19 dollars per month, what are they per year?

  Ans. \$ 228.
- 48. If a boy receive a present every New Year's day of 1783 dollars, how much money will be possess when he is 21 years old?

  Ans. \$37443.
  - 48. Method of adding two or more columns at one operation.
- Ex. 1. A merchant paid for cloth 219 dollars, for flour 416 dollars, for hardware 711 dollars, and for rent 93 dollars. How much did he pay in all?

  Ans. \$1439.

OPERATION. Beginning with the number last written down, Dollars. we add the units and tens; thus, 93 and 1 = 94, 219 and 10 = 104, and 6 = 110, and 10 = 120, and 416 9 = 129, and 10 = 139. Of this amount we write 711 the 9 units and 3 tens under the columns added; and add in the 1 hundred with the column of 93 hundreds; thus, 1 (carried) and 7 = 8, and 4 = Ans. 1439 12, and 2 = 14 hundred, of which we write the 4 hundred under the column of hundreds and the

1 thousand in the thousands' place; and find the whole amount to be 1439. In like manner may be added more than two columns at one operation.

Note. — The examples that follow can be performed as the above, or by the common method, or by both.

2. How many were the members of Congress in 1856, there being 2 Senators from each State, and Maine sending 6 Representatives, New Hampshire 3, Massachusetts 11, Rhode Island 2, Connecticut 4, Vermont 3, New York 33, New Jersey 5, Pennsylvania 25, Delaware 1, Maryland 6, Virginia 13, North Carolina 8, South Carolina 6, Georgia 8, Alabama 7, Mississippi 5, Louisiana 4, Tennessee 10, Kentucky 10, Ohio 21, Indiana 11, Illinois 9, Wisconsin 3, Iowa 2, Missouri 7, Arkansas 2, Michigan 4, Florida 1, Texas 2, California 2. Ans. 296.

- 3. Water-mills were invented in the year 555 after Christ; windmills 744 years after water-mills; pumps 126 years after windmills; printing 15 years after pumps; watches 37 years after printing; the spinning-wheel 53 years after watches; the steam-engine 119 years after the spinning-wheel; the fire-engine 14 years after the steam-engine; the spinning-frame 98 years after the fire-engine; and the electro-magnetic telegraph 71 years after the spinning-frame. What year was the electro-magnetic telegraph invented?

  Ans. 1832.
- 4. During the American Revolutionary war, the British lost at the battle of Lexington 273 men, at Bunker Hill 1054 men, at Long Island 400 men, at White Plains 300 men, at Fort Washington 1000 men, at Trenton 1020 men, at Princeton 400 men, at Hubbardton 200 men, at Bennington 800 men, at Brandywine 500 men, at Stillwater 600 men, at Germantown 500 men, at Saratoga 400 men, by Burgoyne's surrender 5791 men, at Fort Mercer 500 men, at Monmouth 400 men, at Rhode Island 260 men, at Brier Creek 23 men, at Stony Point 600 men, at Savannah 130 men, at Camden 325 men, at King's Mountain 1150 men, at Cowpens 800 men, at Guilford Court-House 600 men, at Hobkirk's Hill 250 men, at Eutaw Springs 700 men, at Yorktown 7000 men. What was the entire loss?
- 5. Required the number of square miles in the following States, there being in Maine 35,000, in New Hampshire 8,030, in Vermont 8,000, in Massachusetts 7,250, in Rhode Island 1,200, in Connecticut 4,750, in New York 46,000, in New Jersey 6,851, in Pennsylvania 47,000, in Ohio 39,964, in Michigan 56,243, in Indiana 33,809, in Illinois 55,405, in Wisconsin 53,924, in Iowa 50,914, and in California 188,982.

  Ans. 643,322.
- 6. Required the number of square miles in the following States, there being in Delaware 2,120, in Maryland 11,000, in Virginia 61,352, in North Carolina 45,500, in South Carolina 28,000, in Georgia 58,000, in Alabama 50,722, in Florida 59,268, in Mississippi 47,147, in Tennessee 44,000, in Kentucky 37,680, in Missouri 67,380, in Arkansas 52,198, in Louisiana 46,431, and in Texas 325,520. Ans. 936,318.
  - 7. According to the census of 1850, Maine had 583,169

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inhabitants, New Hampshire 317,976, Massachusetts 994,514, Rhode Island 147,545, Connecticut 370,792, Vermont 314,120, New York 3,097,394, New Jersey 489,555, Pennsylvania 2,311,786, Delaware 91,532, Maryland 583,034, District of Columbia 51,687, Virginia 1,421,661, North Carolina 869,039, South Carolina 668,507, Georgia 906,185, Florida 87,445, Alabama 771,623, Mississippi 606,526, Lousiana 517,762, Texas 212,592, Arkansas 209,897, Tennessee 1,002,717, Missouri 682,044, Kentucky 982,405, Ohio 1,980,329, Indiana 988,416, Illinois 851,470, Michigan 397,654, Wisconsin 805,391, Iowa 192,214, California 92,597, and the Territories 92,298. What was the whole number of inhabitants?

Ans. 23,191,876.

## SUBTRACTION.

49. Subtraction is the process of taking one number from another to find the difference.

When the two numbers are unequal, the larger is called the *Minuend* and the less number the *Subtrahend*; and when the numbers are equal, either is the *Minuend*, and the other is the *Subtrahend*. The result, or the number found by the subtraction, is called the *Difference*, or *Remainder*.

One number can be subtracted from another only when the units of both are of the same kind; and when the numbers are simple, the process is termed Subtraction of Simple Numbers.

50. To subtract simple numbers.

Ex. 1. From 935 take 673.

Ans. 262.

Minuend 9 3 5 units, and find the difference to be 2 units, which we write under the figure subtracted.

Remainder 2 6 2

Remainder 2 6 2

Remainder 2 6 2

difficulty, since the 7 is greater than the 3, and cannot be subtracted from it. We therefore add 10 tens to the

3 tens, which makes 13 tens, and then subtract the 7 from 13, and 6 tens remain, which we write below. Then, to compensate for the 10 tens, equal to 1 hundred, added to the 3 tens in the minuend, we add 1 hundred to the 6 hundred of the subtrahend, which makes

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7 hundreds, and subtract the 7 hundreds from 9 hundreds, and 2 hundreds remain. By adding the 10 tens to the minuend and the 1 hundred to the subtrahend, the two numbers being equally increased, their difference is not changed. (Art. 24, Ax. 9.) The remainder is 262.

Note. — The addition of 10 to the minuend is sometimes called borrowing 10, and the addition of 1 to the subtrahend is called carrying 1.

RULE. — Place the less number under the greater, so that units of the same order shall stand in the same column.

Commencing at the right hand, subtract each figure of the subtrakend from the figure above it.

If any figure of the subtrahend is larger than the figure above it in the minuend, add 10 to that figure of the minuend before subtracting, and then add 1 to the next figure of the subtrahend.

51. First Method of Proof. — Add the remainder and the subtrahend together, and their sum will be equal to the minuend, if the work is right.

This method of proof depends on the principle, that the greater of any two numbers is equal to the less added to the difference between them.

**52.** Second Method of Proof. — Subtract the remainder or difference from the minuend, and the result will be like the subtrahend, if the work is right.

This method of proof depends on the principle, that the smaller of any two numbers is equal to the remainder obtained by subtracting their difference from the greater.

#### EXAMPLES.

	2.	2. •	3.	3.
	OPERATION.	OPERATION AND PROOF.	OPERATION.	OPERATION AND PROOF.
Minueno	1 469	469	788	788
Subtrah	end 183	183	369	369
Remain	der 286	286	419	419
		Min. 469		Sub. 369
	4.	5.	6.	7.
	Miles.	Gallons.	Minut	es. Pecks.
From	7654	7116	617	8 4567
Take	1978	1997	. 176	1978

	8.	9.			10.		11.	•
	Barrels.	Dogr			rlengs.		Tons.	
From	765116	567	89	5 (	3781	. '	7167	8
Take	716669	100	91	8 9	109		1881	9
•	12.	13.			4.		15.	
_	Hogsheads.	Bushel			rds.		Pounds.	
	611000	6178			1111		9900	-
Take	199999	1999	09	90	9009	1	9991	9
	16.	17			18.		19.	
_	Boods.	Acre		_	oles.	_	Cords.	
	100200	5117			0000	-	8911	
Take	98761	4191	0 9	16	6666	1	7167	0
	20.			01			22.	
	ZU. Dollars.			21. Eagles.			ZZ. Guineas.	
From	1000000	0		9999	9		8888	
Take	909901			0091	-		9999	-
Tare	303301	<del>-</del>		0031		_	0000	_
	28	-				24.		
-	Seco				• • • •	Hours		
	1002003				6007			
Take	9.08070	60504	039		1918	189	1.7 1 8	5
	25.		2	6.		:	27.	•
	Months.		D	ays.	•	W	eeks.	
From	6156710	1	100	0000		100	0000	0
Take	9167	8		1		99	9999	9
90	7777	_	05.05.0	. P 4	018123			
	What is the							
	What is the							
30.	What is the v	alue of	31671	.675	· <b>3</b> 6178	4 ?		

- 31. What is the value of 16781321 100716?
  - 32. What is the value of 1002007000 5971621?
- 33. Sir Isaac Newton was born in the year 1642, and he died in 1727; how old was he at the time of his decease? Ans. 85 years.
- 34. Gunpowder was invented in the year 1330; how long was this before the invention of printing, which was in 1440? Ans. 110 years.

- 35. The mariner's compass was invented in Europe in the year 1302; how long was this before the discovery of America by Columbus, which happened in 1492?

  Ans. 190 years.
- 36. What number is that, to which if 6956 be added, the sum will be one million?

  Ans. 993044.
- 37. A man bought an estate for seventeen thousand five hundred and sixty-five dollars, and sold it for twenty-nine thousand three hundred and seventy-five dollars. Did he gain or lose, and how much?

   Ans. Gained \$11810.
- 38. Bonaparte was declared emperor in 1804; how many years since?
- 39. The union of the government of England and Scotland was in the year 1603; how long was it from this period to 1776, the time of the declaration of the independence of the United States?

  Ans. 173 years.
- 40. Jerusalem was taken and destroyed by Titus in the year 70; how long was it from this period to the time of the first Crusade, which was in the year 1096? Ans. 1026 years.
- 41. From the Creation to the Deluge was 1656 years; thence to the founding of Rome 1595 years; thence to the death of Charlemagne, which took place 814 years after Christ, 1567 years. In what year of the world was Christ born?

  Ans. 4004.
- 42. A gentleman 83 years old has two sons; the age of the older son added to his makes 128 years, and the age of the younger son is equal to the difference between the age of the father and that of the older son. How old is each of his sons?

  Ans. The older, 45 years the younger, 38 years.
- 43. During the year 1810, there were manufactured in the United States one hundred and forty-six thousand nine hundred and seventy-four yards of cotton cloth; and during the year 1855, five hundred and twenty million yards. What was the increase?

  Ans. 519,853,026 yards.
- 53. Method of subtracting, when there are two or more subtrahends.
- Ex. 1. From a pile of wheat containing 657 bushels, A is to have 141 bushels, B 244 bushels, C 134 bushels, and D the remainder. How many bushels is D to have? Ans. 138.



Minuend	Bushels.	Minuend	Bushels.	tion, the first opera- tion, the several sub- trahends are added for
	$ \begin{array}{r} 141 \\ 244 \\ 134 \end{array} $		$   \left\{     \begin{array}{l}       \hline       141 \\       244 \\       134     \end{array}   \right. $	a single subtrahend to be taken from the minuend. In the sec- ond, the subtrahends
Subtrahen Remainde		Remainde	er 138	are subtracted as they are added, at one oper- ation, thus: beginning with units. 4 and 4 and

1 = 9, which from 17 units leaves 8 units; passing to tens, 1 (carried) and 3 and 4 and 4 = 12 tens; reserving the left-hand figure to add in with the figures of the subtrahends in the next column, the right-hand figure, 2 tens, which we subtract from the 5 tens of the minuend, and have left 3 tens; and, passing to hundreds, we add in the left-hand figure 1, reserved from the 12 tens, which with the other figures 1 and 2 and 1 = 5 hundreds, which, taken from 6 hundreds, leaves 1 hundred; and 138 is the answer sought.

- 2. John Drew has a yearly income of 2,500 dollars; his family expenses are 1,300 dollars, his expenditures in improving his estate 450 dollars, and his contributions to several worthy objects 225 dollars. What remains to lay up or invest?
- 3. A speculator bought four village lots; for the first he paid 620 dollars; for the second, 416 dollars; for the third, 350 dollars; for the fourth, 225 dollars; and sold the whole for 2,000 dollars. What did he gain?
- 4. Daniel White, dying, left property to the amount of 27,563 dollars, of which his wife received 9,188 dollars, each of his two daughters, 4,594 dollars, and his only son the balance. What did his son receive?

  Ans. 9,187 dollars.
- 5. The United States contain 2,983,153 square miles, of which the Atlantic slope includes 967,576, the Pacific slope 778,266, and the Mississippi Valley the remainder. How many square miles does the Mississippi Valley contain?

Ans. 1,237,311.

6. The British North American Provinces contain 3,125,401 square miles; of which 147,832 square miles belong to Canada West; 201,989 to Canada East; 27,700 to New Brunswick; 18,746 to Nova Scotia; 2,134 to Prince Edward's Island; 57,000 to Newfoundland; 170,000 to Labrador; and the re-

mainder to the Hudson's Bay Territory. What number of square miles belong to the Hudson's Bay Territory?

Ans. 2,500,000.

- 7. James Howe has property to the amount of 63,450 dollars, and owes in all three debts; one of 1000 dollars, another of 350 dollars, and another of 12,468 dollars. How much has he after paying his debts?
- 8. The entire coinage of the mint of the United States, including the coinage of its branches, from 1792 to 1856, amounted in value to \$498,197,382, of which \$396,895,574 was gold, \$100,729,602 silver, and the remainder of the amount copper. What was the value of the copper coinage?

Ans. \$ 572,206.

#### MULTIPLICATION.

54. MULTIPLICATION is the process of taking one number as many times as there are units in another number.

In multiplication three terms are employed, called the *Multiplicand*, the *Multiplier*, and the *Product*.

The multiplicand is the number to be multiplied or taken.

The multiplier is the number by which we multiply, and denotes the number of times the multiplicand is to be taken.

The product is the result, or number produced by the multiplication.

The multiplicand and multiplier together are called factors, from the product being made or produced by them.

When the multiplicand consists of a simple number, the process is termed Multiplication of Simple Numbers.

In the following table, the invention of Pythagoras, may be found all the elementary products necessary in performing any operation in multiplication, since the multiplication of numbers, however large, depends upon the product of one digit by another. The products, therefore, of each digit by any other, should be thoroughly committed to memory. Considerable more of the table, even, may be memorized with fully compensating results.

### MULTIPLICATION TABLE.

	•							•																
25	2	75	96 100	125	150	175	200	225	250	275	300	325	350	375	400	425	450	475	200	525	550	575	909	625
24	48	72	ı	95 1100 1105 1110 1115 120 1125	96 1102 1108 1114 120 126 132 138 144 150	98 105 112 119 126 133 140 147 154 161 168 175	192	216	240	264	288	312	336	360	384	408	432	456 475	480	420 441 462 483 504 525	440 462 484 506 528 550	460 483 506 529 552 575	576	600
23	46	69	92	115	138	191	184	207	230	253	276	533	322	345	368	391	414	437	160	483	506	529	552	575
22	44	99	88	011	132	154	176	198	220	242	264	286	308	330	352	374	396	418	440	462	484	506	528	550
21	42	63	8	105	126	147	168	189	210	231	252	273	294	315	336	357	378	399	420	441	462	483	504	525
20	40	09	8	8	120	140	160	88	200	220	077	260	280	300	320	340	360	380	90	420	440	160	180	200
18 19	38	57	16		114	133	152	171	190	209	828	247	266	285	304	323	342	361	380	399		437	456	475
	36	54	72	90	108	126	144	162	180	198	216	234	252	270	288	306	\$24	342	360	378	396	414	432	120
17	34	119	89	85	102	119	136	153	170	187	504	221	238	255	272	588	306	323	340	357	374	391	108	125
16	32	48	64	80	96	112	128	144	160	176	192	808	224	240	256	272	288	<u>8</u>	380	336	352	368	384	8
15	30	45	9	75	8	105	120	135	150	165	180	195	210	225	240	255	270	285	300	315	330	345	360	375
7	28	42	26	70	\$	86	88 96 104 112 120 128 136 144 152 160 168 176 184 192 200	99 108 117 126 135 144 153 162 171 180 189 198 207 216 225	140	154	168	182	196	210	224	238	252	566	280	294	308	322	336	င္ဆ
13	97	39	25	65	78	16	5	117	130	143	156	169	182	195	208	221	234	247	560	273	286	299	312	325
12	24	36	48	09	72	84	96	108	120	132	144	156	168	081	192	204	516	228	240	252	264	276	288	8
9 10 11 12 13 14	22	33	4	55	99	77			80  90 100 110 120 130 140 150 160 170 180 190 200 210 220 230 240 250	99 1110 121 132 143 154 165 176 187 198 209 220 231 242 253 264 275	96 108 120 132 144 156 168 180 192 204 216 228 240 252 264 276 288 300	91   104   117   130   143   156   169   182   195   208   221   234   247   260   273   286   299   312   325	98   112   126   140   154   168   182   196   210   224   238   252   266   280   294   308   322   336   350	90   105   120   135   150   165   180   195   210   225   240   255   270   285   300   315   330   345   360   375	96   112   128   144   160   176   192   208   224   240   256   272   288   304   320   336   352   368   384   400	85   102   119   136   153   170   187   204   221   238   255   272   289   306   323   340   357   374   391   408   425	108 126 144 162 180 198 216 234 252 270 288 306 524 342 360 378 396 414 432 450	114 133 152 171 190 209 228 247 266 285 304 323 342 361 380 399 418 437	80   100   120   140   160   180   200   220   240   260   280   300   320   340   360   380   400   420   440   460   480   500	84 105 126 147 168 189 210 231 252 273 294 315 336 357 378 399	88 1110 132 154 176 198 220 242 264 286 308 330 352 374 396 418	92 115 138 161 184 207 230 253 276 299 322 345 368 391 414 437	96   120   144   168   192   216   240   264   288   312   336   360   384   408   432   456   480   504   528   552   576   600	275
101	20	30	40	20	09	70	80	90	8	110	120	130	140	120	160	170	180	190	200	210	220	230	240	200
6	18	27	36	45	54	63	72	81	90	66	801	117	126	135	144	153	162	2	081	189	861	202	516	225
8	16	54	32	40	48	26	64	72	80	88	96	2	112	120	128	136	144	152	99	89	176	184	192	8
7	14	2	28	35	42	49	99	63	70	177	78	16		105	112	119	126	133	140	147	154	191	168	135
9	12	18	77	30	36	42	48	54	09	99	60 - 72	18	<del>18</del>	06	96	102	801	114	20	126	132	138	144	20
2	2	15	20	25	30	35	9	45	20	55	9	65	70	75	80	85	6	95	8	105	011	115	120	22
4	80	12	16	50	24	28	35	36	40	44	48	25	26	09	4	89	72	92	80	8	88	95	96	8
3	9	6	12	12	18	21	24	27	30	33	36	39	42	45	48	21	54	22	9	63	99	69	72	50 75 100 125 150 175 20 122 25 25 25 25 25 25 25 25 25 25 25 25 2
2	4	9	8	10	12	14	16	18	20	22	24	56	88	30	32	34	36	38	9	45	44	46	48	2
-	2	က	4	2	9	~	80	6	9	=	12	13	14	12	16	12	18		20	21	22	23		22
_		_				<u> </u>						-												

For example, suppose we wish to find the product of 7 by 5; we look for 7 at the top of the table, and for 5 at the left hand, and where the lines intersect is 35, the number sought; or, we may look for the 7 at the left hand, and the 5 at the top, and find, where the lines intersect, the same result.

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- 55. The repeated addition of a number to itself is equivalent to a multiplication of that number. Thus, 7+7+7+7 is equivalent to  $7 \times 4$ , the sum of the former and the product of the latter being the same. Hence multiplication has sometimes been called a *concise method of addition*.
- 56. The product must be of the same kind or denomination as the multiplicand, since the taking of a quantity any number of times does not alter its nature. Thus: 5, an abstract number,  $\times 3 = 15$ , an abstract number; and 9 yards  $\times 7 = 63$  yards.
- 57. The multiplier must always be considered as an abstract number. Thus, in finding the cost of 4 books at 9 dollars each, we cannot multiply books and dollars together, which would be absurd, but we can, by regarding the 4 as an abstract number, take the 9 dollars, or cost of 1 book 4 times, and the product, 36 dollars, will be the result required.
- 58. The product of two factors will be the same, whichever is taken as the multiplier. Thus,  $8 \times 6 = 6 \times 8 = 48$ ; and the cost of 5 hats at 2 dollars each gives the same product as 2 hats at 5 dollars each. Also, the product of any number of factors is the same, in whatever order they are multiplied. Thus,  $2 \times 3 \times 5 = 3 \times 5 \times 2 = 5 \times 2 \times 3 = 30$ .
- 59. A Composite number is a number produced by multiplying together two or more numbers greater than 1. Thus, 10 is a composite number, since it is the product of  $2 \times 5$ ; and 18 is a composite number, since it is the product of  $2 \times 3 \times 3$ .
  - 60. To multiply simple numbers.

Ex. 1. Let it be required to multiply 1538 by 9.

Ans. 13842.

Multiplier 9
Product 13842

Having written the multiplier, 9, under the unit figure of the multiplicand, we multiply the 8 units by the 9, obtaining 72 units = 7 tens and 2 units. We write down the 2 units in the units' place, and reserve the 7 tens to add to

the product of the tens. We then multiply the 3 tens by 9, obtaining 27 tens, and, adding the 7 tens which were reserved, we have 34 tens = 3 hundreds and 4 tens. We write down the 4 tens in the tens' place, and reserve the 3 hundreds to add to the product of the hundreds. We next multiply the 5 hundreds by 9, obtaining 45 hundreds, and, adding the 3 hundreds which were reserved, we have 48 hundreds.

dreds = 4 thousands and 8 hundreds. We write down the 8 hundreds in the hundreds' place, and reserve the 4 thousands to add to the product of the thousands. By multiplying the 1 thousand by 9 we obtain 9 thousands, and, adding the 4 thousands reserved, we have 13 thousands, which we write down in full;—and the product is 18842.

2. Let it be required to multiply 2156 by 423.

Ans. 911988.

Multiplicar Multiplier	ıd	•		1	5 2	6
Partial Products	8	<b>4</b> 6	$\frac{\bar{6}}{3}$	4 1 4	6 2	8
Product	9	1	1	9	8	8

In this example the multiplicand is to be taken 423 times = 3 units times + 2 tens times + 4 hundreds times. 3 units times 2156 = 6468 units; 2 tens times 2156 = 4312 tens; and 4 hundreds times 2156 = 8624 hundreds; the sum of which partial products = 911988, or the total product required. In the operation the right-hand figure of each partial product is written di-

rectly under its multiplier, that units of the same order may stand in the same column, for convenience in adding.

RULE. - Write the multiplier under the multiplicand, arranging

units under units, tens under lens, &c.

Multiply each figure of the multiplicand by each figure of the multiplier, beginning with the right-hand figure, writing the right-hand figure of each product underneath, and adding the left-hand figure or figures, if any, to the next succeeding product.

If the multiplier consists of more than one figure, the right-hand figure of each partial product must be placed directly under the figure of the multiplier that produces it. The sum of the partial products will be the whole product required.

NOTE. — When there are ciphers between the significant figures of the multiplier, pass over them in the operation, and multiply by the significant figures only, remembering to set the first figure of the product directly under the figure of the multiplier that produces it.

- 61. First Method of Proof. Multiply the multiplier by the multiplicand, and, if the result is like the first product, the work is supposed to be right. (Art. 58.)
- 62. Second Method of Proof. Divide the product by the multiplier, and, if the work is right, the quotient will be like the multiplicand.

NOTE. — This is the common mode of proof in business; but, as it anticipates the principles of division, it cannot be employed without a previous knowledge of that process.

63. Third Method of Proof. — Begin at the left hand of the

multiplicand, and add together its successive figures toward the right, till the sum obtained equals or exceeds the number nine. If it equals it, drop the nine, and begin to add again at this point, and proceed till you obtain a sum equal to, or greater than, nine. If it exceeds nine, drop the nine as before, and carry the excess to the next figure, and then continue the addition as before. Proceed in this way, till you have added all the figures in the multiplicand and rejected all the nines contained in it, and write the final excess at the right hand of the multiplicand.

Proceed in the same manner with the multiplier, and write the final excess under that of the multiplicand. Multiply these excesses together, and place the excess of nines in their product at the right.

Then proceed to find the excess of nines in the product obtained by the original operation; and, if the work is right, the excess thus found will be equal to the excess contained in the product of the above excesses of the multiplicand and multiplier.

NOTE. — This method of proof, though perhaps sufficiently sure for common purposes, is not always a test of the correctness of an operation. If two or more figures in the work should be transposed, or the value of one figure be just as much too great as another is too small, or if a nine be set down in the place of a cipher, or the contrary, the excess of nines will be the same, and still the work may not be correct. Such a balance of errors will not, however, be likely to occur.

#### EXAMPLES.

3. Multipl	y 7325 by 3612.	Ans. 26457900.
36 14 11	OPERATION.	PROOF BY MULTIPLICATION.
Multiplica	nd 7325	Multiplicand 3612
Multiplier	3612	Multiplier 7 3 2 5
	$\overline{14650}$	18060
	7325	7224
	43950	10836
	21975	25284
Product	26457900	Product 26457900

4. Required the product of 82967 by 652.

Ans. 54094484.

Multiplicand Multiplier	8 2 9 6 7 6 5 2	proof by the rings. 5 excess. 4 excess.				
	165934	20	2 excess.			
	414835					
	497802		}			
Product	54094484	•	2 excess.			
<b>5.</b> ·	6.	7.	8.			
789123	1234567	989898	3789588			
4	5	2	8			
9.	10.	11.	12.			
678954	616783	789563	789567			
24	36.	57	98			
13.	14.	15.	16.			
892001	230442	425016	5061029			
329	701	645	3408			

17. What will 365 acres of land cost at 73 dollars per acre?

Ans. \$26645.

18. What will 97 tons of iron cost at 57 dollars a ton?

Ans. \$5529.

19. What will 397 yards of cloth cost at 7 dollars per yard?

Ans. \$2779.

20. What will 569 hogsheads of molasses cost at 37 dollars per hogshead?

Ans. \$21053.

21. If a man travel 37 miles in one day, how far will he travel in 365 days?

Ans. 13505 miles.

22. If a vessel sails 169 miles in one day, how far will she sail in 144 days?

23. What will 698 barrels of flour cost at 7 dollars a barrel?

24. What will 376 lbs. of sugar cost at 13 cents a pound?

Ans. 4888 cts.

25. What will 97 lbs. of tea cost at 93 cents a pound?

Ans. 9021 cts.

26. If a regiment of soldiers consists of 1128 men, how many men are there in an army of 53 regiments?

Ans. 59784.

27. What is the product of  $75432 \times 47$ . Ans. 3545304.

28. What is the product of  $76785316 \times 7615$ .

Ans. 584720181340.

29. What is the product of  $67853 \times 8765$ .

Ans. 594731545.

30. What is the product of  $3812345 \times 31243$ .

Ans. 119109094835.

31. What is the product of  $40670007 \times 10002$ .

Ans. 406781410014.

32. What is the product of  $31235678 \times 10203$ .

Ans. 318697622634.

33. What is the product of  $76786321 \times 3007$ .

Ans. 230896467247.

34. What is the product of  $6176777 \times 22222$ .

Ans. 137260338494.

35. What is the product of  $7060504 \times 30204$ .

Ans. 213255462816.

36. Multiply 88888 by 4444.

Ans. 395018272.

37. Multiply 7008005 by 10008. Ans. 70136114040.

38. Multiply 987648 by 481007. Ans. 475065601536.

39. Multiply 101010101 by 202020202.

Ans. 20406081008060402.

40. Multiply 304050607 by 3011101.

Ans. 915527086788307.

41. Multiply 908007004 by 500123.

Ans. 454115186861492.

42. Multiply 2003007001 by 6007023.

Ans. 12032109124168023.

43. Multiply 9000006 by 9000006.

Ans. 81000108000036.

44. A full-grown elm will, it is computed, yearly, on an average, produce three hundred twenty-nine thousand three hundred and seventy-five seeds. How many seeds will three such trees produce in fifty-three years. Ans. 52370625.

45. John Alden can plant 3 plats of corn, containing 11 rows of 67 hills each, in 1 day, and Loring Blanchard can

plant twice as much in the same time. How many hills can Blanchard plant in a month of 26 working days? Ans. 114972.

46. If the multiplicand be three hundred and seventy-five millions two hundred and ninety-six thousand three hundred and twenty-one, and the multiplier seventy-nine thousand and twenty-four, what will be the product?

Ans. 29657416470704.

- 64. When the multiplier is a composite number.
- Ex. 1. What cost 35 acres of land at 316 dollars an acre?

  Ans. 11060 dollars.

 $\begin{array}{c} \textbf{316} \\ \textbf{316} \\ \hline \textbf{2212} \\ \textbf{dollars, cost of 7 acres.} \end{array}$ 

7 and 5. Now, if we multiply the price of one acre by 7, we get the cost of 7 acres; and, then, by multiplying the cost of 7 acres by the factor 5, it is evident, we obtain the ace, when the multiplier is a

The factors of 35 are

1 1 0 6 0 dollars, cost of 35 acres. acres by the factor 5, it is evident, we obtain the cost of 5 times 7 acres, or 35 acres. Hence, when the multiplier is a composite number, we may

Multiply the multiplicand by one of the factors of the multiplier, and the product thus obtained multiply by another, and so on until each of the factors has been used as a multiplier; and the last product will be the one sought.

## EXAMPLES.

- 2. Multiply 3121 by 81, using its factors.
- 3. What will 63 horses cost at 175 dollars each?
- 4. A certain house contains 87 windows, and each window has 32 panes of glass. How many panes in the whole house?

  Ans. 2784.
  - 5. What is the product of 47134987 by 56?

Ans. 2639559272.

6. If a garrison consume 6231 pounds of bread in 1 day, how many pounds will the same consume in 144 days?

Ans. 897264 pounds.

65. When there are ciphers on the right in the multiplier or multiplicand, or both.

Ex. 1. In 1 yard there are 36 inches; how many inches in 10 yards? In 100 yards? Ans. 360, 3600.

We annex one cipher to the Multiplicand 36 36 multiplicand to multiply it by Multiplier 10 100 10, and two ciphers to multiply it by 100; since annexing one 3600 360 Product cipher removes each figure of the multiplicand one place to Or thus: 360, 3600. the left, and thus increases it 10 times; annexing two ciphers removes each figure two places to the

10 times; annexing two ciphers removes each figure two places to the left, and increases it 100 times; and so on, each additional cipher having the effect to increase its value 10 times (Art. 30).

2. What will 700 bales of cotton cost at 40 dollars per bale?

Ans. 28000 dollars.

Multiplicand Multiplier	7 0 0 4 0	The multiplicand we resolve into the factors 7 and 100, and the multiplier into the factors 4 and 10. Now, it is
Product	28000	evident (Art. 58), that, if these several factors be multiplied together, they will
	3 . 4 . 5777	produce the same product as the origi-

nal factors, 700 and 40. Thus  $7 \times 4 = 28$ , and  $28 \times 100 = 2800$ , and  $2800 \times 10 = 28000$ , the same result as in the operation.

Hence, when there are ciphers, one or more, on the right of the multiplier, or multiplicand, or both, we may, for the required product.

Multiply the significant figures together, and to their product annex as many ciphers as there are on the right in both multiplicand and multiplier.

### EXAMPLES.

- 3. Multiply 1819 by 10.
- 4. Multiply 4106 by 100.

5. Multiply 10000 by 7000. Ans. 70000000.

6. Multiply 123000 by 78000. Ans. 9594000000.

7. Multiply 70000 by 10000. Ans. 700000000.

8. Multiply 900900 by 70070. Ans. 63126063000.

- 9. What must be the distance sailed by a steamship, whose average rate is 310 miles a day, in making a voyage from New York to Liverpool, in 12 days?
- 10. The annual salary of a member of Congress being 3,000 dollars, how much do 296 members receive?

Ans. 888,000 dollars.

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11. The salary of the President of the United States is 25,000 dollars a year; how much will it amount to in 82 years?

Ans. 2,050,000 dollars.

12. The earth is 95,000,000 of miles from the sun, and the planet Neptune is 30 times as far. How far is Neptune from the Sun?

Ans. 2,850,000,000 miles.

### DIVISION.

66. Division is the process of finding how many times one number is contained in another; or the process of separating a number into a proposed number of equal parts.

In division there are three principal terms: the Dividend, the Divisor, and the Quotient.

The dividend is the number to be divided.

The divisor is the number by which we divide.

The quotient is the result, or number produced by the division, and denotes the number of times the divisor is contained in the dividend, or one of the equal parts into which the dividend is divided.

When the dividend does not contain the divisor an exact number of times, the excess is called a remainder, and may be regarded as a fourth term in the division.

When the dividend consists of a simple number, the process is termed Division of Simple Numbers.

67. Division is frequently indicated by writing the dividend above a short horizontal line and the divisor below; thus,  $\frac{6}{2}$ . The expression  $\frac{6}{2} = 3$  is read, 6 divided by 2 is equal to 3.

Another method of indicating division, is by a curved line placed between the divisor and dividend. Thus, the expression 6) 12 shows that 12 is to be divided by 6.

68. When a number is divided into two equal parts, one of the parts is called one half; when divided into three equal parts, one of the parts is called one third, two of the parts two thirds; when divided into four equal parts, one of the

parts is called one fourth, two of the parts two fourths, three of the parts three fourths; etc.

Such equal parts are called fractions, since they are fractured or broken numbers. They are expressed by figures, in a form of division; thus, one half is written  $\frac{1}{2}$ ; one third,  $\frac{1}{3}$ ; two thirds,  $\frac{2}{3}$ ; one fourth,  $\frac{1}{4}$ ; two fourths,  $\frac{2}{4}$ ; three fourths,  $\frac{2}{4}$ ; and may also be read, one divided by two, one divided by three, and so on. In any fraction, expressed in the manner now explained, the number above the line is called the *numerator*, and that below the line its *denominator*. Thus, in  $\frac{1}{2}$ , 1 is the numerator, and 2 the denominator.

- 69. When the divisor and dividend are of the same kind or denomination, the quotient will denote the number of times the divisor is contained in the dividend, and will be an abstract number. Thus, to find how many pencils at 6 cents each can be bought for 24 cents, we inquire how many times 6 cents are contained in 24 cents, which are 4 times. Hence, 4 pencils, at 6 cents each, can be bought for 24 cents.
- 70. When the divisor and dividend are not of the same kind or denomination, the divisor must be regarded as an abstract number, and will denote the number of equal parts into which the dividend is divided, and the quotient will denote the number of units in each part, and will be of the same kind or denomination as the dividend. Thus, to find the cost of 1 pencil, when 4 pencils cost 24 cents, we separate or divide the 24 cents into 4 equal parts, of which one part is 6 cents. Hence, 1 pencil costs 6 cents, when 4 pencils cost 24 cents.
- 71. The remainder will always be of the same kind or denomination as the dividend, since it is a part of the dividend.
- 72. Division is the reverse of multiplication. The dividend answers to the product, and the divisor and quotient to the factors, of multiplication. In multiplication, two factors are given, to find their product; and in division, one of two factors and their product are given, to find the other factor.
  - 73. To divide simple numbers.
- Ex. 1. How many yards of cloth, at 4 dollars a yard, can be bought for 948 dollars?

  Ans. 237 yards.

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OPERATION. Divisor 4) 948 Dividend. 237 Quotient.

We first inquire how many times 4, the divisor, is contained in 9, the first left-hand figure of the dividend, which is hundreds, and find it contained 2 times,

and 1 hundred remaining. We write the 2 directly under 9, its dividend, for the hundreds' figure of the quotient. To 4, the next figure of the dividend, which is tens, we regard as prefixed the 1 hundred that was remaining, which equals 10 tens, and thus form 14 tens, in which we find the divisor 4 to be contained 3 times. and 2 tens remaining. We write the 3 for the tens' figure in the quotient, and the 2 tens that were remaining, equal 20 units, we regard as prefixed to 8, the last figure of the dividend, which is units, in which the divisor 4 is contained 7 times. Writing the 7 for the units' figure of the quotient, we have 237 as the entire quotient. equal the number of yards of cloth at 4 dollars a yard that can be bought for 948 dollars.

# 2. How many times does 3979 contain 17?

Ans. 23417 times.

OPERATION. Dividend. Divisor 17) 3979 (234 17 Quotient. 57 51 69 68 Remainder.

We say, 17 in 39, times. The 2 we 2 times. write in the quotient.  $17 \times 2 =$ 34, which we write under the 39. 39 -34 = 5, to which bringing down the next figure of the dividend, we form 57. 17 in 57, 3 The 3 we times. write in the quo-

tient.  $17 \times 3 = 51$ , which we write under the 57. 57 - 51 = 6, to which bringing down the next figure of the dividend, we form 69. 17 in 69, 4 times. The 4 we write in the quotient.  $17 \times 4 = 68$ , which we write under the 69. 69 - 68 = 1, a remainder, or a part of the dividend left undivided. 1 divided by  $17 = \frac{1}{17}$  (Art. 68). The 17 we write in the quotient, and obtain as the answer required 2341.

In this illustration, to render the explanation the more concise, the

naming of the denominations of the figures has been omitted.

When, as in the operation preceding the last, results only are written down, the method is called short division; and when, as in the last operation, the work is written out at length, it is called long division. The principle is the same in both cases. Hence the general

Rule. - Beginning at the left, find how many times the divisor is contained in the fewest figures of the dividend that will contain it, for the first quotient figure.

Multiply the divisor by this quotient figure, and subtract the product from the figures of the dividend used. With the remainder, if any, unite the next figure of the dividend.

Find how many times the divisor is contained in the number thus formed, and write the figure denoting the result at the right of the

former quotient figure.

Thus proceed until all the figures of the dividend are divided.

- NOTE 1. The proper remainder is in all cases less than the divisor. If, in the course of the operation, it is at any time found to be as large as, or larger than, the divisor, it will show that there is an error in the work, and that the quotient figure should be increased.
- Note 2. If at any time the divisor, multiplied by the quotient figure, produces a product *larger* than the part of the dividend used, it shows that the quotient figure is too *large*, and must be diminished.
- Note 3.—It will often happen that, when a figure of the dividend is taken, the number will not contain the divisor; and, in that case, a cipher must be placed in the quotient, and another figure of the dividend taken, and so on, until the number is large enough to contain the divisor.
- NOTE 4. If there be a remainder after dividing the last figure of the dividend, write it with the divisor underneath, with a line between them, at the right of the quotient.
- 74. First Method of Proof. Multiply the divisor by the quotient, and to the product add the remainder, if any, and if the work be right, the sum thus obtained will be equal to the dividend.
- Note. This method follows from division being the reverse of multiplication. (Art. 72.)
- 75. Second Method of Proof. Find the excess of nines in the divisor, quotient, and remainder. Multiply the excess of nines in the divisor and quotient together, and to the product add the excess of nines in the remainder. If the excess of nines in this sum equal the excess of nines in the dividend, the work may be supposed to be right.
- 76. Third Method of Proof. Add together the remainder, if any, and all the products that have been produced by multiplying the divisor by the several quotient figures, and the result will be like the dividend, if the work be right.
- 77. Fourth Method of Proof. Subtract the remainder, if any, from the dividend, and divide the difference by the quotient. The result will be like the original divisor, if the work be right.
- Note. The first method of proof (Art. 74) is usually most convenient, and is most commonly employed.

# EXAMPLES.

3. Divide 9184 by 7.	Ans. 1312.
Divisor 7) 9184 Dividend.	1312 Quotient.
· 1312 Quotient.	7 Divisor.
	9184 Dividend.
4. Divide 18988 by 759.	Ans. 25,13.
OPERATION.	PROOF BY THE NINES.
Dividend.	Divisor = 3 excess.
Divisor 759) 18988 (25 Quotie	
1518	Remainder $= 4$ excess.
3808	-
<u>3795</u>	$(3 \times 7) + 4 = 7 \text{ excess.}$
13 Remainder.	Dividend $= 7$ excess.
5. Divide 147856 by 97.	Ans. 1524 <del>3</del> \$.
openation. Dividend.	
Divisor 97) 147856(1524 Que	otiont
+97	owen.
	PROOF BY ADDITION.
508	97
+485	485 Products
235	$\begin{array}{c} 400 \\ 194 \end{array}$ Products.
<b>+194</b>	388
416	28 Remainder.
+388	147856 Dividend.
+28 Remainder	r <b>.</b>
6. Divide 84645 by 285.	Ans. 297.
OPERATION.	PROOF BY DIVISION.
Dividend.	Dividend.
Divisor 285) 84645 (297 Quotient.	
570	594
2764	2524
$2\ 5\ 6\ 5$	2376
1995	1485
1995	1485

$ \begin{array}{c} 7.\\ 8) \underline{67856336}\\ 226187782\\ 3 \end{array} $	8. 7) <u>178985</u>		9. 67789
10. 5) <u>334652</u>	11. 8) <u>9672357</u> 8		12. 3 6 7 3 1
13. 17)678916(	14. 35)9106013(	15 91)6210	
16. Divide 67167	2052 by 6	Quotients. 111946492	Rem. 1.
17. Divide 16633		23762387	2.
18. Divide 16133		20166474	1.
19. Divide 16167		17964186	4.
20. Divide 36389		33081425	3.
21. Divide 16437		13698246	4.
22. Divide 78950		10000210	1.
23. Divide 67895			3.
24. Divide 66778			1.
25. Divide 77777		-	3.
26. Divide 88888			6.
27. Divide 78963			٠.
28. Divide 79678		153227	44.
29. Divide 16785		275175	
30. Divide 67575		2.01.0	
31. Divide 56789			1.
32. Divide 67163		71451	
33. Divide 11678		8650	111.
34. Divide 78617		90365	28.
35. Divide 16785		4598	297.
36. Divide 87635		226447	174.
37. Divide 34567		5091	5091.
	alue of 213255467	$083 \div 30204$ ?	
	alue of 395020613		2341.
	alue of 720727663		4567.
· 41. What is the va			3 ? 8765.

<sup>42.</sup> How many barrels of flour, at 9 dollars a barrel, can be bought for 18621 dollars?

- 43. How much sugar at 15 dollars a hundred may be bought for 405 dollars?
- 44. A tailor has 938 yards of broadcloth; how many cloaks can be made of the cloth, if it require 7 yards to make one cloak?
  - 45. What number multiplied by 1728 will produce 1705536?

    Ans. 987.
- 46. A. Hartmann has sold his wagon to J. Herr for 85 dollars. He is to receive his pay in wood at 5 dollars a cord. How many cords will it require to pay for the wagon?

Ans. 17 cords.

- 47. The Bible contains 31,173 verses; how many must be read each day, that the book may be read through in a year of 365 days?

  Ans. 85\frac{1}{6}\frac{3}{6}\frac{3}{6}\text{ verses.}
- 48. A train on the Liverpool Railroad runs at the rate of 65 miles an hour; how long would it take at that velocity to pass round the earth, the distance being about 25,000 miles?

Ans. 384<sub>13</sub> hours.

49. A gentleman possessing an estate of 66,144 dollars, bequeathed one fourth to his wife, and the remainder was divided among his 4 children; what was the share of each?

Ans. 12,402 dollars.

- 50. If the dividend is 6756785 and the quotient 193051, what is the divisor?

  Ans. 35.
- 51. A's age multiplied by 17, or B's age multiplied by 19, is equal to 1292 years, and the sum of their ages is equal to C's age multiplied by 3. What is the age of each?

Ans. A's 76 years; B's 68 years; C's 48 years.

- 78. When the divisor is a composite number.
- Ex. 1. A farmer bought 21 horses for 2625 dollars; how many dollars did each cost?

  Ans. 125 dollars.

OPERATION.

3) 2625 dolls., cost of 21 horses.

7) 875 dolls., cost of 7 horses. 125 dolls., cost of 1 horse. The factors of 21 are 3 and 7. Now, if we divide the 2625 dollars, the cost of 21 horses, by 3, we obtain 875 dollars, the cost of 7 horses, since

there are 7 times 3 in 21. Then, dividing the 875 dollars, the cost of 7 horses, by 7, we obtain 125 dollars, the cost of 1 horses.

2. Divide 3515 by 42.

Ans. 8343.

OPERATION.

FINDING THE TRUE REMAINDER.

2)3515  $4 \times 3 \times 2 = 24$ , 1st Product.

3) 1757, 1, 1st Rem.

 $2 \times 2 = 4$ , 2d Product.

7) 585, 2, 2d Rem.

1, 1st Remainder.

83, 4, 3d Rem.

29, true Remainder.

Or,  $(4 \times 3 \times 2) + (2 \times 2) + 1 = 29$ , true Rem.

Using as divisors 2, 3, and 7, the factors of 42, we obtain for

remainders, 1, 2, and 4.

The first remainder, 1, is a unit of the given dividend, since it is a part of it (Art. 71). The second remainder, 2, is units of the quotient 1757, whose units are 2 times as great as those of the given dividend. The third remainder, 4, is units of the quotient 585, whose units are 3 times as great as those of the quotient 1757, of which the units are 2 times as great as those of the given dividend. Now, these remainders must be all of the same units as the given dividend, to constitute the whole or true remainder. We therefore multiply the third remainder by 3 and 2, the divisors used in producing the quotient of which it is a part; and the second remainder by 2, the divisor used in producing the quotient of which it is a part; and the products with the first remainder added together give 29, the whole or true remainder sought. Hence, as shown by these illustrations, when the divisor is a composite number, we may

Divide the dividend by one of the factors, and the quotient thus found by another, and thus proceed till each of the factors has been made a The last quotient will be the quotient required.

If there be remainders, multiply each remainder, except the first, by all the divisors preceding the one which produced it; and the first remainder being added to the sum of the products, the amount will be the true remainder.

Note, - There will be but one product to add to the first remainder when there are only two divisors and two remainders.

## EXAMPLES.

3. Divide 7704 by  $24 = 4 \times 6$ .

Ans. 321.

4. Divide 8317 by  $27 = 3 \times 9$ .

5. Divide 3116 by  $81 = 9 \times 9$ .

6. Divide 61387 by  $121 = 11 \times 11$ . Ans. 507-49.

7. Divide 19917 by  $144 = 12 \times 12$ . Ans. 13844.

8. Divide 91746 by  $336 = 6 \times 7 \times 8$ . Ans.  $273\frac{18}{336}$ .

9. At 45 dollars an acre, a farm of how many acres can be Ans. 12119 acres. bought for 5464 dollars?

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79. When the divisor contains one or more ciphers at the right hand.

Ex. 1. If 10 men receive 792 dollars for a job of work, what will be each man's share of it?

Ans. 79<sub>7</sub> dollars.

1 | 0 ) 7 9 | 2 Quotient 7 9, 2 Rem.

Or thus: 79 | 2.

To multiply by 10, we annex one cipher, which removes the figures one place to the left, and thus increases their value 10 times (Art. 65). Now, it is obvious, that, if we reverse the process, and cut off the right-hand

figure by a line, we remove the remaining figures one place to the right, and consequently diminish the value of each 10 times, and thus, divide the whole number by 10. The figures on the left of the line are the quotient, and the one on the right is the remainder, which may be written over the divisor and annexed to the quotient. Hence each man's share is 79 fb.

2. How many years will it take a man at a yearly salary of 700 dollars to earn 3664 dollars?

Ans. 5 + 5 + 5 years.

7 ) 3 6, 6 4, 1st Rem.
5, 1, 2d Rem.

Or thus: 7|00|36|645, 164. The divisor, 700, may be resolved into the factors 7 and 100. We first divide by the factor 100, by cutting off two figures at the right, and get 36 for the quotient, and 64 for a remainder. We then divide the quotient, 36, by the other factor, 7, and obtain

5 for a quotient and 1 for a remainder. The last remainder, 1, being multiplied by the divisor, 100, and 64, the first remainder, added, we obtain 164 for the true remainder (Art. 77); and for the answer required, 5 165 years. Hence, when the divisor contains one or more ciphers at the right, we may, to perform the division,

Cut off the ciphers from the right of the divisor, and the same number of figures from the right of the dividend; and then divide the remaining figures of the dividend by the remaining figures of the divisor.

Note. — When, by the operation, there is a remainder, to it must be annexed the figures cut off from the dividend to form the true remainder. Should there be no last remainder, then the significant figures, if any, cut off from the dividend, will form the true remainder.

## EXAMPLES.

3. Divide 123456789 by 10. 12345678 4. Divide 987654300 by 100.

		-	
• •	•	Quotients.	Rem.
5.	Divide 32100 by 6000.	5	2100.
6.	Divide 3678953 by 326100.	11	91853.
	Divide 1637851 by 500000.	3	137851.
8.	Divide 41111111 by 1100000.	37	411111.

9. Divide 89765432156 by 1000000.

10. The entire annual loss to the United States in consequence of intemperance has been estimated to be about 98,400,000 dollars. How many schools at a yearly expense of 600 dollars would that sum support?

Ans. 164,000 schools.

- 11. The late war with Russia was carried on at a cost of 600,000,000 dollars to Great Britain. Allowing that country to have a population of 28,000,000, what was the cost to each individual?
- 12. If light moves at the rate of 192,000 miles in a second, how long is it in passing from the sun to the earth, a distance of 95,000,000 of miles.

  Ans. 494182 seconds.

## GENERAL PRINCIPLES AND APPLICATIONS.

- 80. In division, the value of the quotient depends upon the divisor and dividend.
- 81. If the dividend be multiplied, or the divisor divided, by any number, the quotient is multiplied by the same number. Thus, if the dividend be 20 and the divisor 4, the quotient will be 5; but if the dividend be multiplied by any number, as 2, and the divisor remain unchanged, the quotient will be 2 times as large as before, or 10; as  $(20 \times 2) \div 4 = 10$ ; and if the divisor be divided by the 2, and the dividend remain unchanged, the quotient will be, likewise, 2 times as large, or 10; as  $20 \div (4 \div 2) = 10$ .
- 82. If the dividend be divided, or the divisor multiplied, by any number, the quotient is divided by the same number. Thus, if the dividend be 32 and the divisor 8, the quotient will be 4. But if the dividend be divided by any number, as 2, and the

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divisor remain unchanged, the quotient will be only half as large as before, or 2; as  $(32 \div 2) \div 8 = 2$ ; and if the divisor be multiplied by 2, and the dividend remain unchanged, the quotient will be, likewise, only half as large, or 2; as  $32 \div (8 \times 2) = 2$ .

- 83. If the dividend and divisor be both multiplied, or both divided, by the same number, the quotient will not be changed. Thus, if the dividend be 16 and the divisor 4, the quotient will be 4. Now, if we multiply the dividend and divisor by some number, as 2, their relative values are not changed, and we obtain 32 and 8 respectively, and  $32 \div 8 = 4$ , the same as the original quotient. Also, if we divide the dividend and quotient by some number, as 2, their relative values are not changed, and we obtain 16 and 2 respectively, and  $16 \div 2 = 8$ , the same quotient as before.
- 84. If a factor in any number is rejected or cancelled, the number is divided by that factor. Thus, if 24 is the dividend and 6 the divisor, the quotient will be 4. Now, since the divisor and quotient are the two factors which, being multiplied together, produce the dividend (Art. 72), it follows, if we reject or cancel the factor 6, the remaining 4 is the quotient; and, by the operation, the dividend 24 has been divided by 4.

#### CANCELLATION.

- 85. CANCELLATION is the method of abbreviating arithmetical operations by rejecting any factor or factors common to the divisor and dividend.
- Ex. 1. Sold 19 thousand shingles at 4 dollars a thousand, and received pay in wood at 4 dollars a cord; how many cords of wood was received?

  Ans. 19 cords.

Dividend 
$$\frac{4 \times 19}{4} = 19$$
 Quotient.

Having indicated by signs the multiplication and division required by the question, then, since dividing both dividend and divisor

by the same number will not change the quotient (Art. 83), we divide them by the common factor 4, by cancelling it in both, and obtain 25 for the quotient. 2. Divide the product of 15, 3, 28, and 13, by the product of 7, 30, and 4.

Dividend 
$$\frac{\cancel{15} \times \cancel{3} \times \cancel{25} \times \cancel{13}}{\cancel{7} \times \cancel{30} \times \cancel{4}} = \frac{39}{2} = 19\frac{1}{2}$$
 Quotient.

The product of the 7 and 4 in the divisor equals the 28 in the dividend; we therefore cancel all these numbers. Finding 15 in the dividend to be a factor of 30 in the divisor, we cancel both of the numbers, and use the remaining factor 2 in place of the 30. There now being no factor common to both dividend and divisor uncancelled, we multiply together the remaining factors in the dividend, and divide the product by the remaining factor in the divisor, and obtain the quotient 19½.

RULE. — Cancel the factor or factors common to the dividend and divisor, and then divide the product of the factors remaining in the dividend by the product of those remaining in the divisor.

Note. —1. In arranging the numbers for cancellation, the dividend may be written above the divisor, with a horizontal line between them, as in division (Art. 67); or, as some prefer, the dividend may be written on the right of the divisor, with a vertical line between them.

Note. — 2. Cancelling a factor does not leave 0, but the quotient 1, to take its place, since rejecting a factor is the same as dividing by that factor (Art. 84). Therefore, for every factor cancelled, either in the dividend or divisor, the factor 1 remains.

### EXAMPLES.

3. Multiply 24 by 16, and divide the product by 12.

Ans. 32.

4. Divide 48 by 16, and multiply the quotient by 8.

Ans. 24.

- 5. Divide the product of 7, 10, 12, and 5, by the product of 14, 18, and 6.
- 6. If 15 be multiplied by 7, 27, and 40, and the product divided by 54 multiplied by 14, 10, and 2, what will be the result?

  Ans. 74.
- 7. Divide the product of 13, 15, 20, and 5, by the product of 26, 10, 2, and 3.

  Ans. 12½.
- 8. Divide the product of 28, 27, 21, 15, and 18, by the product of 7, 54, 7, 3, and 9.
- 9. How many pounds of butter at 28 cents a pound will be required to pay for 56 pounds of sugar at 11 cents a pound?

  Ans. 22 pounds.

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- 10. A. Holmes sold 14 boxes of soap, each containing 24 pounds, at 9 cents a pound, and received for pay 63 barrels of ashes, each containing 3 bushels. What was allowed a bushel for the ashes?

  Ans. 16 cents.
- 11. M. Gardner sold 5 piles of brick, each containing 12 thousand, at 7 dollars a thousand, and was paid in wood, 3 ranges, at 4 dollars a cord. How many cords in each range?

  Ans. 35 cords.
- 12. A merchant exchanged 8 cases of shoes, each containing 60 pairs, at 75 cents a pair, for a certain number of casks of molasses, each containing 90 gallons, at 40 cents a gallon. How many casks did he get?

  Ans. 10 casks.

#### CONTRACTIONS IN MULTIPLICATION.

- 86. A CONTRACTION is the process of shortening any operation.
- 87. When the multiplier is 13, 14, etc., or 1 with a significant figure annexed.

Ex. 1. Multiply 3126 by 14.

Ans. 43764.

FIRST OPERATION.	SECOND OPERATION.
$3126 \times 14$	3 1 2 6
12504	14
43764 Ans.	43764 Ans.

In the first operation, we multiply the multiplicand by the 4 units of the multiplier, and write the product under the multiplicand one

place to the right. To this partial product we add the multiplicand, since, as it stands, it represents the product of the multiplicand by the 1 ten of the multiplier; and obtain 43764, the answer required. In the second operation, we add in the multiplicand taken as the product by the 1 ten of the multiplier, as we multiply by the 4 units; thus,  $6 \times 4 = 24$ , of which we write down the 4 and carry the 2.  $2 \times 4 = 8$ , + 2 (carried) + 6 (from the multiplicand) = 16, of which we write down the 6 and carry the 1.  $1 \times 4 = 4$ , + 1 + 2 = 7, which we write down.  $3 \times 4 = 12$ , + 1 = 13 of which we write down the 3 and carry the 1. 3 + 1 = 4, which we write down; and have as the entire result 43764 as before.

RULE. — Write the product by the units' figure of the multiplier under the multiplicand, one place to the right, and add them together. Or.

Multiply each figure of the multiplicand by the units' figure of the multiplier, and, after the units' place, add in the preceding figure of the multiplicand.

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## EXAMPLES.

2. Multiply 63013 by 17.

Ans. 1071221.

3. Multiply 79245 by 19.

4. Multiply 32067812 by 16.

Ans. 513084992.

88. When the multiplier is 101, 102, etc., or 1 with one or more ciphers and a significant figure annexed.

Ex. 1. Multiply 8107 by 103.

Ans. 835021.

 We multiply by 3, the units' figure of the multiplicand, and write the product under the multiplicand, two places to the right, so that the multiplicand, as it stands over this partial product, will represent the product of the multiplicand by the 1 hundred

of the multiplier; and, adding these, we obtain 835021, the result required. For the reason given, if the multiplier had been such as to have contained one more intervening cipher, we should have written the product by the units' figure three places to the right, and so on, one place farther to the right for every additional intervening cipher.

Rule. — Write the product by the units figure of the multiplier under the multiplicand, as many places to the right as there are in the multiplier intervening ciphers plus 1; and add them together.

### EXAMPLES.

2. Multiply 6651 by 108.

Ans. 718308.

8. Multiply 111223 by 104.

Ans. 11567192.

4. Multiply 2042 by 1009.

89. When the multiplier is 21, 31, etc., or 1 with a significant figure prefixed.

Ex. 1. Multiply 3113 by 41.

Ans. 127633.

 $3113 \times 41$  12452 127633 Ans.

We multiply by 4, the tens' figure of the multiplier, and write the product under the multiplicand, one place to the left, so that the multiplicand, as it stands over this partial product, will represent the product of the multiplicand by the 1 unit of the multi-

plier; and, adding these, obtain the result required.

RULE. — Write the product by the tens' figure of the multiplier under the multiplicand, one place to the left, and add them together.

#### EXAMPLES.

Ans. 679167. 2. Multiply 13317 by 51.

3. Multiply 71389 by 21.

4. Multiply 12062 by 91. Ans. 1097642.

When the multiplier is 201, 301, etc., or 1 with one or more ciphers and a significant figure prefixed.

Ex. 1. Multiply 14118 by 601.

Ans. 8484918.

OPERATION.  $14118 \times 601$ . 84708 8484918 Ans.

We multiply by 6, the hundreds' figure of the multiplier, and write the product under the multiplicand, two places to the left, so that the multiplicand, as it stands over this partial product, will represent the product of the

multiplicand by the 1 unit of the multiplier; and adding these we have the answer required.

RULE. — Write the product by the hundreds' figure of the multiplier under the multiplicand, as many places to the left as there are in the multiplier intervening ciphers plus 1; and add them together.

#### EXAMPLES.

Ans. 58528360. 2. Multiply 8360 by 7001. 3. Multiply 10613 by 801.

4. Multiply 91603 by 2001.

Ans. 8501013.

91. When the multiplier or multiplicand has a fraction annexed.

Ex. 1. Multiply 426 by 7\frac{1}{3}. By 8\frac{2}{3}. Ans. 3124; 3692.

OPERATION. OPERATION. 426 426 71 .84 2982 = Product by 7.3408 =Product by 8.  $142 = Product by \frac{1}{4}$ . 284 =Product by  $\frac{2}{3}$ . 3124 =Product by  $7\frac{1}{3}$ . 3692 = Product by 83.

In multiplying 426 by 73, we first obtain the product of 426 by 7, and then the product of 426 by 1, and, adding these two partial products together, have 3124, the product by 73. In multiplying by 1, we take one third of the multiplicand, by dividing it by 3; thus,  $426 \times \frac{1}{2} = 426 + 3 = 142$ . In multiplying 426 by 83, we proceed as in the other case, except in obtaining the product by the fraction; we take two thirds of the multiplicand, by taking one third of it 2

times; thus,  $426 \times \frac{2}{3} = (426 \div 3) \times 2 = 284$ .

If the fraction had been annexed to the multiplicand instead of the multiplier, we then would have found the product of the fraction and multiplier, for a partial product, in like manner as above.

RULE. — Multiply the fractional part and the whole number separately, and add the products.

## EXAMPLES.

2. Multiply 915 by 223.

Ans. 20496.

3. Multiply 12244 by 18.

Ans. 220344.

- 4. If 69<sup>1</sup> miles make 1 degree, how many miles are 180 degrees?

  Ans. 12450 miles.
- **92.** When the multiplier is a convenient part of a number of tens, hundreds, or thousands.

Note. — The following are some of the convenient parts often occurring as multipliers;  $2\frac{1}{2} = \frac{1}{4}$  of 10;  $3\frac{1}{3} = \frac{1}{5}$  of 10;  $12\frac{1}{2} = \frac{1}{5}$  of 100;  $16\frac{2}{3} = \frac{1}{5}$  of 100;  $25 = \frac{1}{4}$  of 100;  $33\frac{1}{3} = \frac{1}{5}$  of 100;  $125 = \frac{1}{5}$  of 1000;  $166\frac{2}{3} = \frac{1}{5}$  of 1000;  $250 = \frac{1}{4}$  of 1000;  $333\frac{1}{3} = \frac{1}{5}$  of 1000.

Ex. 1. Multiply 785643 by 25.

Ans. 19641075.

operation.
4)78564300

19641075 Product.

We multiply by 100, by annexing two ciphers to the multiplicand (Art. 65), and obtain a product 4 times as large as it should be, since 25, the multiplier, is only

one fourth part of 100; we therefore take one fourth part of that

product, by dividing by 4, for the true product.

Upon the same principle, if the multiplier had been 31, we should have annexed one cipher and divided by 3, or, if the multiplier had been 125, we should have annexed three ciphers and divided by 8. Hence the following

RULE. — Multiply by the number of tens, hundreds, or thousands, of which the multiplier is a part, and, of the product thus found, take the same part.

# EXAMPLES.

2. Multiply 68056 by  $12\frac{1}{2}$ .

Ans. 850700.

3. Multiply 17924 by 2½.

4. Multiply 192378 by 163.

Ans. 3206300. Ans. 1543209750.

5. Multiply 12345678 by 125. Ans. 1543209750.
 6. How much can be earned in one year of 313 working days, at 34 dollars a day?

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- 7. What will a final ontaining 534 acres cost, at 33\frac{1}{2} dollars an acre?
- 8. In a large field there are 771 rows of corn, each contain the large field there are 771 rows of corn, each contain the large field there are 771 rows of corn, each contain the large field there are 771 rows of corn, each contain the large field there are 771 rows of corn, each contain the large field there are 771 rows of corn, each contain the large field there are 771 rows of corn, each contain the large field there are 771 rows of corn, each contain the large field there are 771 rows of corn, each contain the large field there are 771 rows of corn, each contain the large field there are 771 rows of corn, each contain the large field there are 771 rows of corn, each contain the large field there are 771 rows of corn, each contain the large field there are 771 rows of corn, each contain the large field there are 771 rows of corn, each contain the large field there are 771 rows of corn, each contain the large field the large

Ans. 192750 hills.

From a port in Louisiana there were exported, in a given time, 9168 boxes of sugar, averaging 1663 pounds to a box; required the whole number of pounds.

Ans. 1528000 pounds.

10. An agent has bought for the army, at different times, in the aggregate, 1993 horses, at an average price of 125 dollars each. How much did he pay for them in all?

Ans. 249125 dollars.

- 11. How many are  $333\frac{1}{3}$  times 28044? Ans. 9348000.
- 93. When a part of the multiplier is a factor of another part.
  - Ex. 1. Multiply 3263 by 568.

Ans. 1853384.

3 2 6 3 Multiplicand. 5 6 8 Multiplier.

26104 =Product by 8 units. 182728 =Product by 56 tens.

1853384 =Product by 568.

We regard the multiplier as separated into two parts, 56 tens and 8 units, or 560 + 8; of which the smaller part is evidently a factor of the larger, since the 56 tens, or 560, is equal to 7 tens × 8. We next multiply by the 8 units, obtaining the product

for that part of the multiplier. Now, as this product is the same as that by the factor 8 of the other part of the multiplier, we multiply it by 7 tens, obtaining the product of the multiplicand by  $8 \times 7$  tens or 56 tens. These products of the parts, 560 and 8, added together, give the true product by 568.

Rule. — Multiply first by the smaller part of the multiplier; and then that partial product by a factor, or factors, of a larger part; and so on with all the parts. The sum of the several partial products will be the product required.

Note. — Care must be taken in writing down the partial products to have the units of the different orders stand in their proper places for adding.

#### EXAMPLES.

2. Multiply 112345678 by 288144486.

Ans. 32371787641631508.

OPPRATION.

112345678 Multiplicand. 288144486 Multiplier.

674074068 =Product by 6 units.

5392592544 = 1st product x 8 tens for product by 48 tens;

16177777632

2d product × 3 thousands for product by 144 thousands.

32355555264

3d product × 2 millions for product by 288

32371787641631508 =Product by 288144486.

- 3. Multiply 61370913 by 96488. Ans. 5921556653544.
- 4. Multiply 8649347864 by 1325769612.

Ans. 11467042561708308768.

94. When the multiplier is any number of nines.

Multiply 87654 by 999.

Ans. 87566346.

OPERATION.

 $87654000 = 87654 \times 1000$  $87654 = 87654 \times$ 1

 $87566346 = 87654 \times$ 999

By annexing three ciphers to the multiplicand we take it 1000 times, or 1 time more than is required by the given multiplier. We therefore from

this result subtract the multiplicand taken once, and thus obtain the product by 999. In like manner we may multiply by any number of nines. Hence the

RULE. - Annex as many ciphers to the multiplicand as there are nines in the multiplier, and from the number thus produced subtract the given multiplicand.

Note. — To multiply by any number of threes, find the product for the same number of nines, by the rule, and take one third of it by dividing by 3; and to multiply by any number of sixes, take twice the product of the same number of threes, by multiplying it by 2.

2. Multiply 7777777 by 9999.

Ans. 77769992223.

3. Multiply 416231 by 99999.

4. Multiply 987654 by 333333.

Ans. 329217670782.

5. Multiply 876543 by 66666. 6. Multiply 999999 by 9999.

Ans. 58435615638. Ans. 9998990001.

7. Multiply 32567895 by 3333. 8. Multiply 66666 by 66666.

Ans. 4444355556.

9. Multiply 912345678 by 99.

Ans. 9032222122.

10. Multiply 1234567 by 9999.

Ans. 12344435433.

11. Multiply 98123452 by 999999. Ans. 98123353876548.

### CONTRACTIONS IN DIVISION.

95. When the divisor is a convenient part of a number of tens, hundreds, or thousands.

Ex. 1. Divide 19641075 by 25. Ans. 785643.

OPERATION. 19641075

By multiplying both divisor and dividend by 4, which does not change the relation of the one to the other (Art. 83), the divisor becomes 100.

785643|00 Quotient. which enables us to perform the division by simply cutting off two figures

at the right of the dividend (Art. 79). In like manner, if the divisor had been 31, by taking both it and the dividend 3 times as large, we could have performed the division by simply cutting off one figure at the right of the dividend; or, if the divisor had been 125, by taking both it and the dividend 8 times as large, we could have performed the division by simply cutting off three figures at the right of the dividend. Hence the

Rule. - Multiply both divisor and dividend by that number which will change the divisor to a number of tens, hundreds, or thousands, and then divide.

### EXAMPLES.

2. Divide 89630 by 31.

Ans. 26889.

3. Divide 123450 by 164.

4. Divide 18621 by 121.

Ans. 1489 68.

5. Divide 317121 by 21.

Ans. 126848-4.

6. Divide 876735 by 331.

Ans. 26302 150.

7. Divide 123456 by 125.

8. Divide 61678500 by 250.

Ans. 246714.

9. J. Cushing bought a number of horses, at 1663 dollars each, for \$ 9500; required the number bought.

Ans. 57 horses.

- 10. A company has received 12000 dollars from the sale of piano-fortes, at an average price of 3334 dollars each; required the number sold.
- 11. How many shares of the Illinois Central Railroad, at 125 dollars each, can be bought for 150000 dollars?

Ans. 1200 shares.

12. How many cows, at 331 dollars each, may be purchased for 3334 dollars.

- 13. How many books, at 2½ dollars each, may be purchased for 120 dollars?

  Ans. 48 books.
- 14. A certain magazine contains 616350 pounds of powder, in kegs of 25 pounds each; required the number of kegs.

Ans. 24654 kegs.

96. When the divisor is any number of nines.

Ex. 1. Divide 316234 by 99.

Ans. 319428.

3 1 6 2 3 4 3 1 9 6 1 2 7 2 8 3 1 9 4 28 Ans.

We first divide by 100, or 99 + 1, by cutting off two figures at the right of the dividend, obtaining for the first partial quotient 3162. and a remainder 34. Since the divisor used was 1 larger than the given divisor, 99, the quotient obtained denotes that an excess of \$162, or a number equal to the quotient itself, must be added to the 34 for the true remainder.

34 + 3162 = 3196, which exceeding the divisor 99, we write it for a second dividend; and dividing by 100, or 99 + 1, as before, we obtain for the second partial quotient 31, and a remainder 96. To the remainder 96 we add 31, the excess denoted by the last quotient, and obtain 127 for a third dividend, which being divided by 100, or 99 + 1 gives the third partial quotient 1, and a remainder 27. To the remainder 27 adding 1, the excess denoted by the last quotient, we have for the true remainder 28, which is  $\frac{28}{15}$ . The sum of the partial quotients with the final remainder annexed gives  $3194\frac{28}{15}$ , the quotient required.

The above process is based upon the principle that 10 = 9 + 1; 100 = 99 + 1; 1000 = 999 + 1, etc.; consequently  $20 = (2 \times 9) + 2$ , and  $37 = (3 \times 9) + 3 + 7$ ;  $200 = (2 \times 99) + 2$ ;  $3859 = (38 \times 99) + 38 + 59$ ;  $15987 = (15 \times 999) + 15 + 987$ , etc. Hence,  $316234 = 316200 + 34 = (3162 \times 99) + 3162 + 34$ ;  $3162 + 34 = 3196 = 3100 + 96 = (31 \times 99) + 31 + 96$ ;  $31 + 96 = 127 = 100 + 27 = (1 \times 99) + 1 + 27$ ;  $1 + 27 = 28 = \frac{23}{15}$ ; and  $3162 + 31 + 1 + \frac{23}{15} = 3194\frac{23}{15}$ ; the answer, as before obtained. By like process, and upon the same principle, may the quotient be found for any number of nines.

RULE. — Add 1 to the given divisor, and, by it thus increased, divide by cutting off figures at the right of the dividend. To the figures cut off on the right add those on the left for a true remainder, of which, if it equal or exceed the given divisor, make a second dividend, and divide as before. Proceed thus till there shall be no remainder as large as the given divisor; and the sum of the several quotients, with the last remainder, if any, will be the answer required.

Note. — When the last remainder is the same as the given divisor, it must be cancelled, and 1 written as a partial quotient.

### EXAMPLES.

2. Divide 341 by 9.

3. Divide 123332544 by 999.

PERATION.	OPERATION.			
3 4 1	1 2 3 3 3 2 5 4 4			
3 5	123876			
8	1 999			
8 7 § Ans.	123456 Ans.			

4. Divide 12332655 by 999.

Ans. 12345.

5. Divide 987551235 by 9999.

6. Divide 9123456779876543211 by 9999999999.

Ans. 9123456789.

97. Abbreviated method of long division.

Ex. 1. Divide 34634 by 134.

Ans.  $258_{\frac{62}{134}}$ .

134)  $3\frac{4634}{258\frac{62}{134}}$  Ans.  $\frac{783}{1134}$ 

The operation we perform thus: finding the first quotient figure to be 2, we say  $4 \times 2 = 8$ ; 16 - 8 = 8;  $3 \times 2 = 6$ , and 1 carried = 7; 14 - 7 = 7;  $1 \times 2 = 2$ , and 1 carried = 3; 3 - 3 = 0. We now bring down 3, and find

the next quotient figure to be 5, then,  $4 \times 5 = 20$ ; 3 - 0 = 3;  $3 \times 5 = 15$ , and 2 carried = 17; 8 - 7 = 1;  $1 \times 5 = 5$ , and 1 carried = 6; 7 - 6 = 1. We next bring down 4, and find the next quotient figure to be 8; then,  $4 \times 8 = 32$ ; 4 - 2 = 2;  $3 \times 8 = 24$ , and 3 carried = 27; 13 - 7 = 6;  $1 \times 8 = 8$ , and 2 + 1 carried = 11; 1 - 1 = 0; 1 - 1 = 0. Hence, this method is that of ordinary long division (Art. 73), abridged by subtracting each figure of the product of the divisor and a quotient figure, as it is obtained, and writing down only the remainders.

## EXAMPLES.

2. Divide 39006 by 44.

Ans. 88622.

3. A certain orchard contains 1088 trees in 34 rows; how many trees in each row?

Ans. 32 trees.

4. A speculator sold 191 mules at a gain of 5157 dollars; what was the gain on each?

5. The population of Massachusetts, in 1855, was 1,133,123; how many would that be to each of its 7750 square miles of surface?

Ans. 1464923.

## 98. When the divisor or dividend has a fraction annexed.

## Ex. 1. Divide 3692 by 83.

Ans. 426.

-		BERA 3						•
26		11		7 6	(	4 2	6	Ans.
	-		6 7					
		•	1 8	5 6 5 6				

We bring the divisor and dividend to the same fractional parts as those denoted by the given fraction, which are thirds, by multiplying them both by 3, the number of thirds in 1. We thus change the divisor to 26 thirds, and the dividend to 3692 thirds, without changing the quotient (Art. 83); and dividing obtain 426, the answer required.

Rule. — Reduce the divisor and dividend to the same fractional parts as are denoted by the given fraction, and then divide as in whole numbers.

Note. — In case there should be a remainder after the division, its true value may be found as in Art. 76.

## EXAMPLES.

2. Divide 13120 by 9½.

3. Divide 76672\$ by 36.

Ans. 21292\$2.

Ans. 21292\$2.

Ans. 21292\$2.

4. Divide 2090\frac{1}{3} by 205.

Ans.  $10\frac{1}{6}\frac{7}{15}$ .

5. If 161 feet make a red how many reds are there in 106

- 5. If 16½ feet make a rod, how many rods are there in 10626 feet?
- 6. If 69<sup>‡</sup> miles make a degree, how many degrees in 12450 miles?

  Ans. 180 degrees.
- 7. How many barrels of pork, at 17\frac{3}{4} dollars a barrel, may be bought for 5591\frac{1}{4} dollars?
- 8. A merchant has sold 2667 yards of silk in dress patterns of 10½ yards each; required the number of patterns.

Ans. 254 patterns.

- 9. How many plats, each of  $272\frac{1}{4}$  square feet, are equal in extent to 136125 square feet?

  Ans. 500 plats.
- 10. How many days will 119 pounds of bread last a man, if he consume 23 pounds per day?

  Ans. 49 days.
- 11. How many casks of 31½ gallons each will be required to hold 12968¾ gallons of cider?

  Ans. 415 casks.

# PROBLEMS,

#### FOUNDED UPON THE FUNDAMENTAL RULES.

- 99. The following problems are founded upon the general principles of addition, subtraction, multiplication, and division, the fundamental operations of arithmetic, which have already been explained.
- 1. The parts of a number being given, to find the number. Add the parts together (Art. 47).
- 2. The sum of two numbers and one of the numbers being given, to find the other number.—From the sum subtract the given number (Art. 50).
- 3. The difference between two numbers and the larger number being given, to find the smaller. From the larger number subtract the difference (Art. 52).
- 4. The difference between two numbers and the smaller number being given, to find the larger.—Add the smaller number and the difference together (Art. 51).
- 5. The sum and the difference of two numbers being given, to find the numbers. From the sum subtract the difference and divide the remainder by 2, for the smaller number; add the difference to the smaller number, for the larger (Art. 52).

NOTE.—In like manner, when the sum and differences are given, may be found any number of required numbers. After subtracting the differences from the sum, if there are 3 required numbers, divide by 3 for the smaller number; if 4, divide by 4, and so on.

- 6. The product of two numbers, and one of the numbers being given, to find the other numbers. Divide the product by the given number (Art. 62).
- 7. The product of three numbers, and two of the numbers being given, to find the other number. Divide the given product by the product of the two given numbers (Art. 72).
- 8. The dividend and quotient being given, to find the divisor. Divide the dividend by the quotient (Art. 77).
- 9. The divisor and quotient being given, to find the dividend. Multiply the divisor and quotient together (Art. 74).

#### EXAMPLES.

- 1. A carpenter has contracted to build one house for 2763 dollars, another for 4650 dollars, and a third for 8950 dollars. How much is he to receive for them all? Ans. 16363 dollars.
- 2. N. Chandler has invested in railroad stock and a small farm 929 dollars. If the amount invested in the stock was 279 dollars, how much did the farm cost him? Ans. 650 dollars.
- 3. Mount Black, the highest peak of the Blue Ridge, is 6476 feet high, which is 242 feet higher than Mount Washington, the highest peak of the White Mountains. What is the height of Mount Washington?
- 4. The city of Mexico, in 1519, was taken by Cortes, and, 328 years after, by General Scott. In what year did it yield to Scott?

  Ans. 1847.
- 5. Two travellers, A and B, meeting on a journey, found they had both travelled 1963 miles, and that A had travelled 199 miles more than B. What distance had each travelled?

  Ans. A, 1081 miles; B, 882 miles.
- 6. A father gave his three sons 4698 dollars, of which James received 250 dollars more than George, and Edwin 410 dollars more than George. What sum did each receive?

Ans. George \$1346; James \$1596; Edwin \$1756.

- 7. There was paid for 217 chests of tea 8463 dollars. How much was that a chest?
- 8. How many weeks will 684 bushels of oats last 19 horses, each horse consuming 3 bushels a week?
- 9. On dividing 3808 dollars among a certain number of men, it was found that the share of each was 224 dollars. Required the number of men.

  Ans. 17 men.
  - 10. A certain missionary society divides its income among 99 missions, giving to each an average of 575 dollars. What is its income?

    Ans. 56925 dollars.
  - 11. The product of 96, 22, and one other number is 63360. What is the other number?

    Ans. 30.
  - 12. The divisor being 13 and the quotient 1101, what is the dividend?

    Ans. 14313.

## MISCELLANEOUS EXAMPLES.

- 1. What is the distance by railroad from Boston to Galena, it being from Boston to Albany 200 miles, from Albany to Niagara Falls 305, from Niagara Falls to Detroit 230, from Detroit to Chicago 282, from Chicago to Galena 171?
- 2. Sold J. Weimer my best horse for 175 dollars, my secondbest chaise for 87 dollars, and a good harness for 31 dollars. He has paid me in cash 38 dollars, and has given me an order on S. Lantz for 12 dollars. How many dollars remain due?
- 3. Bought 97 barrels of molasses at \$5 a barrel. Gave 17 barrels to support the poor, and the remainder was sold at \$8 a barrel. Did I gain or lose, and how much? Ans. \$155 gain.
- 4. It requires 1728 cubic inches to make one cubic foot; required the number of cubic inches in 3787 cubic feet.
- 5. If a garrison of 987 men are supplied with 175686 pounds of beef, how much will there be for each man? Ans. 178 lbs.
- 6. Albert Peyton sold off from his farm 120 acres, gave his son 80 acres, and had remaining 160 acres; what number of acres did his farm contain before he disposed of any portion of it?

  Ans. 360 acres.
- 7. The annual revenue of a gentleman being \$8395, how much per day is that equivalent to, there being 365 days in a year?

  Ans. \$23.
- 8. What is the difference between half a dozen dozen, and six dozen dozen?

  Ans. 792.
- 9. Bought of F. Johnson 8 barrels of flour at \$7 per barrel, and 3 hundred-weight of sugar at \$8 per hundred. What was the amount of his bill?
- 10. George Adams bought an equal number of cows and oxen for 3952 dollars. For the cows he paid 31 dollars each, and for the oxen 45 dollars each. How many of each kind did he buy?

  Ans. 52.
- 11. If a certain quantity of provisions will sustain 13 men 4 days, how long would it sustain 1 man? Ans. 52 days.
- 12. The Globe Manufacturing Company has a capital of 250,000 dollars, divided into 500 shares. How much is each share?

  Ans. 500 dollars.



- 13. Purchased a farm of 500 acres for \$17,876. I sold 127 acres of it at \$47 an acre, 212 acres at \$96 an acre, and the remainder at \$37 an acre. What did I gain by my bargain?

  Ans. \$14,402.
- 14. There is a certain island 18 miles in circuit, which A and B undertake to travel round, both starting from the same point and going round in the same direction. When A has travelled 17 miles and B 7 miles, how far apart are they?

  Ans. 8 miles.
- 15. If 15 men can reap a certain field in 5 days, how long will it take 1 man to reap the same?

  Ans. 75 days.
- 16. What number is that, which being multiplied by 24, the product divided by 10, the quotient multiplied by 2, 32 subtracted from the product, the remainder divided by 4, and 8 subtracted from the quotient, the remainder shall be 2?

  Ans. 15.
- 17. From  $126 + (\overline{16 + 4}) \times \overline{2}$  take  $(48 \div 2) + (\overline{34 \times 6}) \div (17 5)$ . Ans. 125.
- 18. There are in the library of a certain school 683 books, which number will give 23 books to each pupil, and 16 books over; what is the number of pupils?
- 19. R. Howland in making a journey, after walking 12 miles and travelling 40 miles by stage, went by steamboat 5 times the distance he had travelled by stage, and by cars 6 times as far as he had walked, and 7 miles besides. What was the length of his journey?

  Ans. 331 miles.
- 20. At an election A and B were candidates for the same office, and the whole number of votes cast for them was 9891, of which B received 1211 majority. What number of votes did each receive?

  Ans. A, 4340; B, 5551.
- 21. The product of 3 numbers is 4080; one of the numbers is 15, and another 16. What is the third number?
- 22. How many tons of coal at 6 dollars a ton will be required to pay for 17 yards of cloth at 4 dollars a yard and 32 bushels of wheat at 2 dollars a bushel?

  Ans. 22 tons.
- 23. James Cooper has manufactured in 4 years 5608 pairs of shoes, making each successive year 100 pairs more than the year before; how many pairs did he manufacture each year?

  Ans. 1252; 1352; 1452; 1552.



- 24. How many years will it take a young man earning 45 dollars, and spending 35 dollars of it, every month, if he have already 620 dollars, to lay up enough more to pay for a house costing 1100 dollars?

  Ans. 4 years.
- 25. If the product of the divisor by the quotient be 19782, and the remainder 31, what is the dividend? Ans. 19813.
- 26. John Franklin purchased railroad stock to the amount of 4473 dollars, and sold a part of it for 1885 dollars, obtaining 65 dollars a share, which was at a loss of 6 dollars on every share sold; but the stock advancing much in value, he was able to dispose of the balance so as to gain by the whole operation 812 dollars. What did he get a share for the balance of the stock?

  Ans. 100 dollars.

#### UNITED STATES MONEY.

ART. 100. UNITED STATES MONEY is the legal currency of the United States. It was established by Congress in 1786. Its denominations and their relative values are shown in the following

			$\mathbf{T}$	ABLE.				
10 ]	Mills	make		1 Cent,		marked	(	ct.
10 (	Cents	44		1 Dime,		"		d.
10 ]	Dimes	46		1 Dollar,		"	:	\$.
10 ]	Dollars	"		1 Eagle,		"	1	Ē.
Mills.		Cents.						
. 10	=	1		Dimes.				
100	===	10	===	1		Dollars.		,
1000		100	_	10	-	1		Ragie.
10000	_	1000	_	100	_	10	-	1

In accounts and ordinary business transactions, the only denominations mentioned are dollars and cents, eagles being expressed as dollars, dimes as cents, and mills as a fraction of a cent.

The dollar is the integer, or unit of measure of United States money; and therefore dimes are tenths of a dollar; cents, hundredths; and mills, thousandths.

- 101. Parts of a dollar, or any quantity thus expressed, in tenths, hundredths, etc., are termed DECIMALS, or fractions, whose denominator (Art. 68) is 1, with one or more ciphers annexed. They are usually expressed by simply writing the numerator (Art. 68) with a point (.) before it, called the *decimal point*, or *separatrix*; the first place at the right of the point being tenths; the second place, hundredths; the third place, thousandths; the fourth place, ten-thousandths; and so on. Thus,  $\frac{1}{10}$  is written .1;  $\frac{1}{100}$  is written .01;  $\frac{1}{1000}$  is written .001;  $\frac{1}{10000}$  is written .0001, etc.
- 102. In writing dollars and cents together, the decimal point or separatrix is placed between the dollars and the cents or decimal part; and since cents occupy two places, the place of dimes and of cents, when the number of cents is less than 10, a cipher must be written before them, in the place of dimes. Thus, \$30.375 is read, thirty dollars thirty-seven cents five mills, or thirty dollars and three hundred seventy-five thousandths of a dollar; \$12.05 is read, twelve dollars five cents, or twelve dollars and five hundredths of a dollar, etc.
- 103. The denominations of United States Money increasing from right to left, and decreasing from left to right, in the same manner as do the units of the several orders in simple whole numbers, they may therefore be added, subtracted, multiplied, and divided according to the same rules.
- 104. The coins of the United States are of gold, silver, and copper.

The gold coins are the double eagle, eagle, half-eagle, quarter-eagle, three-dollars, and dollar.

The silver coins are the dollar, half-dollar, quarter-dollar, dime, half-dime, and three-cent-piece.

The copper coins are the cent and half-cent.

NOTE. — All the gold and silver coins of the United States are now made of one purity, 9 parts of pure metal and 1 part alloy. The alloy for the silver is copper, and that for the gold 1 part copper and 1 part silver. The cent and half-cent, old coinage, are of pure copper; the cent, new coinage, is made 88 parts copper and 12 parts nickel.

The standard weight of the eagle, as fixed by present laws, is 258 grains Troy, and the gold coins in proportion according to their values. The weight of the silver dollar is 412½ grains; half-dollar, 192 grains; quarter-dollar, 96

grains; dime 383 grains; half-dime, 191 grains; three-cent piece, 11.52 grains; the cent. old coinage, 168 grains; the cent. new coinage, 72 grains.

The weight of the silver dollar, it will be seen, is greater in proportion to its value than the other silver coins. This is owing to their standard weight having been reduced, while that of the dollar remained unchanged; but since this reduction of weight of the smaller silver coins, no more of the silver dollar appear to have been coined. In circulation, the gold dollar, which of late years has been extensively coined, has almost entirely taken the place of the silver dollar.

The symbol \$, or dollar sign, represents, probably, the letter U written upon an S, denoting U. S. (United States).

#### REDUCTION OF UNITED STATES MONEY.

- 105. REDUCTION of United States Money is changing the units of one of its denominations to the units of another, either of a higher or lower denomination, without altering their value.
- 106. To reduce units from a higher denomination to a lower.
  - Ex. 1. Reduce 58 dollars to cents and mills.

Ans. 5800 cents; 58000 mills.

We multiply the 58 by 100, because 100 cents make 1 dollar; and multiply the 5800 by 10, because 10 mills make 1 cent. Hence,

Or thus: 58000 mills.

To reduce dollars to cents, annex TWO ciphers; to reduce dollars to mills, annex THREE ciphers; and to reduce cents to mills, annex ONE cipher.

NOTE. — Dollars, cents, and mills, expressed by a single number, are reduced to mills by merely removing the separating point; and dollars and cents, by annexing one cipher and removing the separatrix.

- 107. To reduce units from a lower denomination to a higher.
- Ex. 1. Reduce 58000 mills to cents and to dollars.

Ans. 5800 cents; 58 dollars.

 $10) \underbrace{58000}_{5800}$  mills.  $100) \underbrace{5800}_{58}$  cents. 58 dollars.

OPERATION.

We divide the 58000 by 10, because 10 mills make 1 cent; and divide the 5800 by 100, because 100 cents make 1 dollar. Hence,

To reduce mills to cents, cut off ONE figure on the right; to reduce cents to dollars, point off TWO figures; and to reduce mills to dollars, point off THREE figures.

#### EXAMPLES.

2. Reduce \$765 to cents.

3. Change 726 mills to cents. Ans. 72 6 cents.

4. How many dollars are 329 cents? Ans. \$3.29.

5. Change 12345 mills to dollars. Ans. \$12.345.

6. Reduce \$123.56 to mills. Ans. 123560 mills.

7. Reduce 2 eaglés, 2 dollars, and 2 dimes to cents.

Ans. 2220 cts.

#### ADDITION OF UNITED STATES MONEY.

'108. Ex. 1. Add together 17 dollars 13 cents 5 mills; 8 dollars 4 cents 6 mills; 63 dollars 20 cents 3 mills; and 29 dollars 87 cents 5 mills.

Ans. \$118.259.

* cts. m. 17.135 8.046 63.203	We write units of the same denomina- tion in the same column, and add as in addition of simple numbers (Art. 45), and separate the dollars from the cents in the
2 9.8 7 5	separate the dollars from the cents in the answer by the decimal point.

Ans. 118.259

Rulb. — Write dollars, cents, and mills, so that units of the same denomination shall stand in the same column.

Add as in addition of simple numbers, and place the decimal point directly under that above.

*Proof.*—The proof is the same as in addition of simple numbers.

#### EXAMPLES.

· 2.	3.	4.	<b>5.</b>
\$ cts. m.	\$ cts. m.	\$ cts. m.	\$ cts. m.
375.875	7 8.1 9 3	171.013	8 6 1.0 7 3
671.127	1 8.0 1 4	382.094	5 1 6.7 1 6
387.143	9 1.0 3 8	9 9 9.9 0 0	3 4 4.6 7 3
184.189	1 6.8 1 7	155.068	617.814
147.758	8 1.4 7 6	48.153	1 6 9.9 7 3
$6\ 3.0\ 7\ 2$	4 3.1 8 4	4 9.6 1 9	8 1 0.4 2 6
1829164			

N 7

- 6. Add the following sums, \$18.165, \$701.63, \$151.161, \$375.089, and \$471.017. Ans. \$1717.062.
- 7. Bought a horse for eighty-seven dollars nine cents, a pair of oxen for sixty-five dollars twenty cents, and six gallons of molasses for two dollars six cents five mills; what was the amount of my bill?

  Ans. \$154.355.
- 8. Sold a calf for three dollars eight cents, a bushel of corn for ninety-seven cents five mills, and three bushels of rye for three dollars five cents; what was the amount received?

Ans. \$7.105.

#### SUBTRACTION OF UNITED STATES MONEY.

109. Ex. 1. From 106 dollars 7 cents 7 mills, take 92 dollars 83 cents 8 mills. Ans. \$13.239.

OPERATION.				
🌲 . cts. m.				
1	0 6.0 7 7			
	9 2.8 3 8			
Ans.	1 3.2 3 9			

We write the less number under the greater, place mills under mills, cents under cents, and dollars under dollars, and subtract as in subtraction of simple numbers (Art. 50), and separate the dollars from the cents in the answer by the decimal point.

Rule. — Write the several denominations of the subtrahend under the corresponding ones of the minuend.

Subtract as in subtraction of simple numbers, and place the decimal

point directly under that above.

*Proof.*—The proof is the same as in subtraction of simple numbers.

2.	3.	4.	5.
s ets. m.	s ets. m.	\$ cts. m.	\$ cts. m.
871.161	478.477	167.163	163.167
8 9.9 1 8	199.991	9 8.0 9 7	9.0 9 8
781.243		*************	

- 6. Bought a farm for \$1728.90, and sold it for \$3786.98; what did I gain by my bargain?
- 7. Gave \$79.25 for a horse, and \$106.875 for a chaise, and sold them both for \$200; what did I gain?
- 8. Bought a farm for \$8967, and sold it for nine thousand eight hundred seventy-six dollars seventy-five cents; what did I gain?

  Ans. \$909.75.

- 9. Bought a barrel of flour for \$7.50, three bushels of rye for \$2.75, and three cords of wood at \$5.25 a cord; I sold the flour for \$6.18, the rye for \$3.00, and the wood for \$6.75 a cord; what was gained by the bargain?

  Ans. \$3.43.
- 10. A young lady went a "shopping." Her father gave her a twenty-dollar bill. She purchased a dress for \$8.16, a muff for \$3.19, a pair of gloves for \$1.12, a pair of shoes for \$1.90, a fan for \$0.19, and a bonnet for \$3.08; how much money dd she return to her father?

  Ans. \$2.36.

#### MULTIPLICATION OF UNITED STATES MONEY.

110. Ex. 1. What will 365 barrels of flour cost at \$5.75 a barrel?

Ans. \$2098.75.

\$ 5.7 5	We multiply as in multiplication of simple numbers, and, obtaining the product in cents, the lowest denomination of the multiplicand, we reduce them to dollars by pointing off two places at the right for cents (Art. 107).
\$2098.75 Ans.	•

RULE. — Multiply as in multiplication of simple numbers.

The product will be in the lowest denomination in the multiplicand, which must be pointed off as in reduction of United States money.

*Proof.* — The proof is the same as in multiplication of simple numbers.

- 2. What will 126 pounds of butter cost at 13 cents a pound?
- 3. What will 63 pounds of tea cost at 93 cents a pound?
- 4. What will 43 tons of hay cost at 13 dollars 75 cents a ton?

  Ans. \$591.25.
- 5. If 1 pound of pork is worth 7 cents 3 mills, what are 46 pounds worth?

  Ans. \$3.358.
- 6. If 1 hundred of beef cost 3 dollars 28 cents, what are 76 hundred worth?
- 7. What will 96 thousand feet of boards cost at 11 dollars 67 cents a thousand?

  Ans. \$ 1120.32.
- 8. If a barrel of cider be sold for 2 dollars 12 cents, what will be the value of 169 barrels?

  Ans. \$358.28.

- 9. What will be the value of a hogshead of wine, containing 63 gallons, at 1 dollar 63 cents a gallon? Ans. \$ 102.69.
- 10. Sold a sack of hops, weighing 396 pounds, at 11 cents 3 mills a pound; to what did it amount? Ans. \$44.748.
- 11. Sold 19 cords of wood, at \$5.75 a cord; to what did it amount?
- 12. Sold 169 tons of timber, at \$4.68 a ton; what did I receive?
- 13. Sold a hogshead of sugar, weighing 465 pounds; to what did it amount, at 14 cents a pound?

  Ans. \$65.10.
- 14. What will be the amount of 789 pounds of leather at 18 cents a pound?

  Ans. \$ 142.02.
- 15. What will be the expense of 846 pounds of sheet-lead at 5 cents 7 mills a pound?

  Ans. \$48.222.
- 16. When potash is sold for \$132.55 a ton, what will be the price of 369 tons?

  Ans. \$48910.95.
- 17. What will 365 pounds of beeswax cost at 18 cents 4 mills a pound?

  Ans. \$ 67.16.
- 18. If 1 pound of tallow cost 7 cents 3 mills, what are 968 pounds worth?

  Ans. \$70.664.

## DIVISION OF UNITED STATES MONEY.

111. Ex. 1. If 78 barrels of fish cost \$303.42, what will 1 barrel cost?

Ans. \$3.89.

OPERATION.

 $\begin{array}{c} 78) \$303.42 (\$3.89 \\ \underline{234} \\ \hline 694 \\ \underline{624} \\ 702 \\ 702 \end{array}$ 

We divide as in division of simple numbers, and point off as many places for cents in the quotient as there are for cents in the dividend.

Rule. — Divide as in division of simple numbers.

The quotient will be in the lowest denomination of the dividend, which must be pointed off as in reduction of United States money.

Note. — When the dividend consists of dollars only, and is either smaller than the divisor or not divisible by it without a remainder, reduce it to a lower denomination by annexing two or three ciphers, as the case may require, and the quotient will be cents or mills accordingly.

When it is required to find the number of times one sum of money is contained in another, both sums, if of different denominations, must be reduced to the same, before dividing.

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*Proof.*—The proof is the same as in division of simple numbers.

#### EXAMPLES.

- 2. Bought 11 chests of tea for \$234.30. What did I give a chest? Ans. \$21.30.
- 3. How many yards of cloth at 9 cents a yard can be bought for \$28.89?

Ans. 321 yards.

11) \$234.30 Ans. \$21.30 9) 2889 Ans. 321 yards.

- 4. When 19 bushels of fine salt can be bought for \$30.87½, what costs one bushel?
- 5. At 12 cents a pound, how many pounds of sugar can be bought for \$51?
- 6. If 78 barrels of fish cost \$303.42, what will one barrel cost?
- 7. Bought 42 bushels of pears for \$73.50; what cost one bushel? Ans. \$1.75.
- 8. How many yards of broadcloth at \$2.75 a yard can be bought for \$904.75?

  Ans. 329 yards.
- 9. When rye is sold at the rate of 628 bushels for \$471.00, what is that a bushel?

  Ans. \$0.75.
- 10. If it should cost \$1460 to support a family 365 days, what would be the expense a day?

  Ans. \$4.
- 11. How many gallons of oil at \$1.62\frac12 a gallon can be bought for \$234?
- 12. If 1624 pounds of pork cost \$97.44, what cost one pound?

  Ans. \$0.06.
- 13. If 47 thousand shingles cost \$176.25, what is the cost per thousand?

  Ans. \$3#5.
- 14. Bought 148 tons of plaster of Paris for \$337.44; what was it per ton?

  Ans. \$2.28.
- 15. At \$37.75 an acre, how many acres of land may be bought for \$1774.25?

  Ans. 47 acres.
- 16. At 67 cents a pound, how many chests of tea, each weighing 59 pounds, can be bought for \$672.01? Ans. 17 chests.
- 17. How many bushels of wheat at \$1.25 a bushel can be bought for \$863.75?

  Ans. 691 bushels.

- 18. Sold 169 tons of timber for \$790.92; what cost one ton?
- 19. What cost one pound of leather, if 789 pounds cost \$142.02?
- 20. If 369 tons of potash cost \$48910.95, what will be the price of one ton?

  Ans. \$132.55.
- 21. Bought 47 hogsheads of salt, each hogshead containing 7 bushels, for \$368.48; what cost one bushel? Ans. \$1.12.

## GENERAL PRINCIPLES AND APPLICATIONS.

- 112. ANALYSIS is an examination of a question by resolving it into its parts, in order to consider them separately, and thus, by reasoning from the nature of the question, to render each step in the solution plain and intelligible.
- 113. An aliquot part of any number or quantity is such a part as will exactly divide that number or quantity. Thus, 3 is an aliquot part of 6; and 4, an aliquot part of 16.
- 114. The method of performing operations by means of aliquot parts is called PRACTICE, or ANALYSIS by ALIQUOT PARTS.
- 115. In business calculations frequent use is made of the following

# ALIQUOT PARTS OF A DOLLAR.

50 cents = $\frac{1}{2}$ of \$1.	$12\frac{1}{2}$ cents = $\frac{1}{8}$ of \$1.
$33\frac{1}{3}$ cents $=\frac{1}{3}$ of \$1.	10 cents = $\frac{1}{10}$ of \$1.
25 cents = $\frac{1}{4}$ of \$1.	$6\frac{1}{4}$ cents $=\frac{1}{16}$ of \$1.
20 cents = $\frac{1}{5}$ of \$1.	5 cents = $\frac{1}{20}$ of \$1.

# PRACTICAL QUESTIONS BY ANALYSIS.

- 116. The price of a unit of any quantity being given, to find the cost of that quantity.
- Ex. 1. If 1 barrel of flour cost \$7, what will 125 barrels cost?

ANALYSIS. Since 1 barrel costs \$7, 125 barrels will cost 125 times as much:  $$7 \times 125 = $875$ .

- 2. If 1 pound of beef cost 13 cents, what will 914 pounds cost?

  Ans. \$118.82.
- 3. If 1 yard of calico cost 23 cents, what will 31½ yards cost?
- 4. When 44 cents are paid for 1 pound of tea, how much must be paid for 15 chests of tea, each containing 47 pounds?
- 5. At 39½ cents a pound, how much must be paid for 9 bales of wool, each containing 317 pounds? Ans. \$1126.93½.
- 6. If the cost of building 1 mile of railroad be \$41315, what will be that of 113 miles?

  Ans. \$4668595.
- 117. The price of a unit of any quantity being a given aliquot part of a dollar, to find the cost of that quantity.
- 7. At 12½ cents a yard, what will 208 yards of calico cost?

  Ans. \$ 26.

ANALYSIS. Since, at \$1 a yard, the cost would be as many dollars as there are yards, the cost at  $12\frac{1}{2}$  cents, or  $\frac{1}{8}$  of \$1, will be one eighth as many dollars as there are yards, or as many dollars as 8 is contained times in 208;  $208 \div 8 = 26$ . Therefore 208 yards cost \$26. That is, we take such a part of the number denoting the quantity as the price is of 1 dollar.

- 8. What will be the cost of 362 pounds of feathers at 33\frac{1}{3} cents a pound?

  Ans. \\$ 120.66\frac{2}{3}.
- 9. At 50 cents a bushel, what will 7 loads of potatoes cost, each containing 30 bushels?

  Ans. \$105.
- 10. At 6½ cents a pound, how much must be paid for 1163 pounds of shingle nails?
- 11. What will be the cost of 96 yards of broadcloth at \$4.33\frac{1}{3}\$ a yard?

  Ans. \$416.

ANALYSIS. Since, at \$1 a yard, the cost would be as many dollars as there are yards, the cost at \$4.33 $\frac{1}{3}$ , or \$4 $\frac{1}{3}$ , will be  $4\frac{1}{3}$  times as much:  $$4\frac{1}{3} \times 96 = $416$ .

- 12. If the cost of 1 pair of boots be \$3.16\frac{2}{3}, what will be the cost of 20 cases, each case containing 60 pairs?
  - 13. At \$ 2.25 each, what will 150 hats cost?

Ans. \$337.50.

14. What will 98 dozen knives cost at \$5.12½ a dozen?
Ans. \$502.25.

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- 15. If James Spooner spends uselessly every day for cigars 61 cents, how much does he thus spend in 360 days?
- 16. At \$2.20 a bushel, what will 15 car-loads of wheat, each containing 212 bushels, cost?

  Ans. \$6996.
- 118. The price of any article sold by the 100, or 1000, with the quantity, being given, to find the cost of that quantity.
  - 17. What will 732 fishes cost at \$ 5 a hundred?

Ans. \$36.60.

ANALYSIS. Since 100 fishes cost \$5, 1 fish will cost  $\frac{1}{100}$  of \$5, or \$ $\frac{1}{100}$ ; and if 1 fish cost \$ $\frac{1}{100}$ , 732 fishes will cost 732 times as much: \$5 × 732 = \$3660; \$3660 ÷ 100 = \$36.60. That is, we multiply the quantity and price together, and cut off two figures at the right (Art. 79).

- 18. How much must be paid for 950 apple-trees at 20 dollars a hundred?
  - 19. What will 7235 arithmetics cost at \$45 a hundred?

    Ans. \$3255.75.
- 20. What will it cost to excavate 19875 solid feet of earth, at the rate of 52 cents per hundred solid feet? Ans. \$103.35.
- 21. What is the value of 1765 feet of timber at \$3 a hundred?

  Ans. \$52.95.
- 22. What is the value of 2355 cedar rails at \$5.50 per hundred?
  - 23. What will 3100 bricks cost at \$6 a thousand?

Ans. \$18.60.

ANALYSIS. Since 1000 bricks cost \$6, 1 brick will cost  $\frac{1}{1000}$  of \$6, or  $\frac{1}{1000}$ ; and, if 1 brick cost  $\frac{1}{1000}$ , 3100 will cost 3100 times as much:  $\frac{6}{1000} \times \frac{3100}{1000} = \frac{18600}{1000}$ ; \$18600  $\div$  1000 = \$18.60. That is, we multiply the quantity and price together, and cut off three figures at the right (Art. 79).

- 24. At \$142.50 a thousand, what will 6150 chestnut posts cost?

  Ans. \$876.375.
- 25. How much must be paid for 15750 feet of boards at \$30 a thousand?
- 26. At \$2.25 per thousand, how much must be paid for 3550 laths?

  Ans. \$7.983.
  - 27. I have bought of L. T. Robbins 363 feet of plank at

- \$ 2.50 per hundred; 3150 pickets at \$14 per thousand; and 350 feet of boards at \$20 per thousand. What is the amount of the whole?

  Ans. \$60.174.
- 28. What must be paid for 92350 railroad ties, at \$135 per thousand?
- 119. The price of any article sold by the ton of 2000 pounds, with the quantity, being given, to find the cost of that quantity.
  - 29. At \$8 a ton, what will 2550 pounds of coal cost?

    Ans. \$10.20.

ANALYSIS. Since 2000 pounds cost \$8,1000 pounds will cost one half of \$8, or \$4; and, if 1000 pounds cost \$4,1 pound will cost  $_{70}^{1}_{00}$  of \$4, or  $_{70}^{4}_{00}$ , and 2550 pounds will cost 2550 times as much:  $_{54}^{4}\times _{2550} = _{10200}$ ;  $_{10200}^{4}\div _{1000}^{4}= _{10200}^{4}$ . That is, we multiply the number denoting the quantity and half the price together, and cut off three figures at the right (Art. 79).

- 30. How much must be paid for 3760 pounds of hay, when the price is \$15 a ton?

  Ans. \$28.20.
- 31. What is the value of 19 casks of plaster of Paris, each weighing 500 pounds, at \$ 9 a ton?
- 32. How much must be paid for transporting 5163 pounds of freight, at \$3.50 per ton?
- 33. At \$46 a ton, what will 33550 pounds of railroad iron cost?

  Ans. \$771.65.
- 34. G. Reed has bought 30 bags of superphosphate of lime, each bag containing 125 pounds, at \$45 per ton; and 7500 pounds of Peruvian guano, at \$53 per ton. What is the cost of the whole?

  Ans. \$283.125.
- 35. I have bought 6350 pounds of anthracite coal at \$7.12½ per ton, and 3560 pounds of Pictou coal at \$7.25 per ton. What is the cost of the whole?

  Ans. \$35.526\foxup{7}{35.
- 120. The price of any quantity, and the quantity, being given, to find the price of a unit of that quantity.
- 36. If 125 barrels of flour cost \$875, what will 1 barrel cost?

  Ans. \$7.

Analysis. Since 125 barrels cost \$875, 1 barrel will cost

as many dollars as 125 is contained times in 875;  $\$875 \div 125 = \$7$ .

- 37. If 914 pounds of beef cost \$118.82, what will 1 pound cost?

  Ans. \$0.13.
- 38. If 96 yards of broadcloth can be bought for \$400, for what can 1 yard be bought?

  Ans. \$4.163.
- 39. If \$510 are paid for 120 pairs of boots, what is the cost of 1 pair? What is the cost of 17 pairs?
- 40. A merchant paid \$270 for 300 casks of lime; what did he pay a cask? What did he pay for 33 casks?

Ans. \$29.70.

- 41. When boards are \$25 per thousand, what are they per foot? What per hundred? Ans. \$2.50.
- 42. If 3 pieces of calico, each containing 30 yards, cost \$19.80, what does 1 yard cost?

  Ans. \$0.22.
- 43. When 10 firkins of butter, each containing 56 pounds, can be bought for \$112, what is the price per pound?

Ans. \$ 0.20.

- 121. The price of any quantity, and the price of a unit of that quantity, being given, to find the quantity.
- 44. At \$7 per barrel, how many barrels of flour can be purchased for \$875?

  Ans. 125 barrels.

ANALYSIS. Since 1 barrel will cost \$7, as many barrels can be purchased with \$875 as 7 is contained times in 875; 875  $\div$  7 = 125; therefore, there can be purchased 125 barrels.

- 45. At \$3 a yard, how many yards of broadcloth can I purchase with \$456?

  Ans. 152 yards.
- 46. How many cords of wood, at \$4.25 each, can be bought for \$357?
- 47. At 33\frac{1}{3} cents a pound, how many pounds of feathers can be bought for \$120.66\frac{2}{3}?

  Ans. 362 pounds.
- 48. At 50 cents a bushel, how many barrels of apples, each containing 3 bushels, can be purchased for \$40.50?

Ans. 27 barrels.

- 49. Expended for hemp \$11600, at \$232 per ton. How many tons did I buy?

  Ans. 50 tons.
- 50. How many boxes of raisins, at \$3.50 per box, can be bought for \$756?

  Ans. 216 boxes.

#### BILLS.

122. A BILL is a paper, given by merchants, containing a statement of goods sold, and their prices.

An *invoice* is a bill of merchandise shipped or forwarded to a purchaser, or selling agent.

The date of a bill is the time of the transaction.

The bill is against the party owing, and in favor of the party who is to receive the amount due.

A bill is receipted, when the receiving of the amount due is acknowledged by the party in whose favor it is, as in bill 1. A clerk, or any other authorized person, may, in his stead, receipt for him, as in bill 2.

When the items of a bill have been rendered at different dates, the several times may be given at the left hand, as in bill 4.

When the bill is in the form of an account, containing items of debt and credit in its settlement, it is required to find the difference due, or balance, as in bill 4.

NOTE. — A due-bill is a written acknowledgment that a debt is due, and is often given in making settlements to remove cause of subsequent disputes as to a claim. A bill is sometimes settled or receipted by a due-bill, as in bill 4.

What is the cost of each article in, and the amount due of, each of the following bills?

Mr. James Dow.

Boston, July 4, 1857.

AME	່ກບ	ν,		
•		$B_{c}$	ought of D	ennis Sharp,
17	yds.	Flannel,	at	\$ 0.45
19		Shalloon,	"	.37
16	"	Blue Camlet,	"	.46
13	"	Silk Vesting,	"	.87
9	"	Cambric Muslin,	"	.63
<b>2</b> 5	, 66	Bombazine,	"	.56
17	"	Ticking,	"	.31
19	"	Striped Jean,	"	.16
		• ′		<del>\$ 61.33</del>

Received payment,

DENNIS SHARP.

(2.)	Chic	eago, Jan. 1, 1857.
Mr. Samuel Smith,	ought of DAV	id Johnson,
13 lbs. Tea,	at	\$ 0.98
16 " Coffee,	"	.15
36 " Sugar,	"	.13
47 " Cheese,	"	.09
12 " Pepper,	"	.19
7 " Ginger,	"	.17
13 " Chocolate,	"	.61
15 Chocoane,		\$
Received paymer	nt,	
1 0		Johnson,
		by J. Keen.
(3.) Messrs. Wilson, Niles, &	Co.,	ohia, Sept. 5, 1856.
2 doz. National Arithmetic	cs, at	\$ 6.00
3 " Com. Sch. Arithme	•	5.00
5 " Primary Arithmetic	•	1.80
17 Parker's Exercises in Co		.25
13 Maglathlin's National S		.60
19 Leverett's Cæsar,	,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,	.50
3 Greenleaf's Algebra,	"	** *
7 Silliman's Chemistry,	"	
15 Olney's U.S. History,	"	-
5 Imp. Quarto Bibles,	"	
3 Webster's Quarto Diction	maries "	
5 Felton's Greek Readers,		
1 Kane's Arctic Explorat		
1 Hune & Michel Baptera	•••••	\$171.02½
Received payme		PECK & BLISS.

(4.)	New Orle	ns, Jan. 1, 1856.	
Mr. ALBERT CRAWFORD,			
	n, Sherwo	OD, & RICE, Dr.	
1855.			
Jan. 2. For 17 yds. Broadcloth		<b>\$</b> 5.25	
" 15. " 29 " Cassimere,		1.62	
Feb. 3. " 60 " Bleached &		.17	
" 7. " 49 " Ticking,	u	.27	
" 15. " 18 " Blue Cloth		3.19	
June 17. " 27 " Habit do.	ű	2.75	
Aug. 3. " 75 " Flannel,	u	.61	
Sept. 19. " 36 " Plaid Prin		.75	
Dec. 2. " 49 " Brown Sho	seting, "	.18	
•		<b>\$ 372.90.</b>	
Ci	٠.		
Jan. 28. By cash,		<b>\$</b> 83.00	
Feb. 19. " 3 M. Boards,	at	<b>\$</b> 30.00	
May 9. "7 M. Shingles,	"	4.00	
June 17. " 4 days' labor,	"	2.00	
July 3. "5 " "	"	1.75	
Oct. 7. " 7 C. Timber,	46	<b>2.25</b>	
Nov. 4. " Cash,		60.00	
Dec. 29. " Draft,		45.00	
		\$ 338.50	
Bal. due B., S., & R.,		\$34.40	
Jan. 5. Settled by due-bill,			
BALDY	vin, Shert	700D, & RICE.	
	~ -		
(5.)	St. Lo	uis, May 1, 1856.	
Mr. Benjamin Treat, Bought of John True,			
37 chests Green Tea,	at	\$ 25.50	
41 " Black do.	"	16.17	
40 " Imperial Tea,	"	97.75	
13 crates Liverpool Ware,	u	169.37	
•		\$	
Received paym	rent,		
		JOHN TRUE.	

(6.)	Portland, May 16, 1857.				
Mr. J. C. PORTER,					
Bought of Willard & Hale,					
17 bbl. Canada Flour,	at \$8.25				
50 lbs. Dupont's Eagle Gu	npowder, " .50				
140 " Sheet Zinc,	" .08 <del>1</del>				
120 " Prussian Blue,	<b>" .63</b>				
·	\$				
Rece $ived$ $paym$	nent,				
	WILLARD & HALE.				
/m \					
(7.)	New York, July 11, 1856.				
Mr. John Cummings,	71 . CT 9 C				
	th of Lord & Secomb,				
97 bbl. Genesee Flour,	at \$ 6.25 " 5.95				
167 " Philadelphia do.	0.00				
87 " Baltimore do.	0.01				
196 " Richmond do.	0.10				
275 " Howard St. do.	<b>"</b> 7.25				
69 bu. Rye,	" 1.16				
136 " Virginia Corn,	" .67				
68 " North River do.	" .76				
169 " Wheat,	<b>"</b> 1.37				
76 tons Lehigh Coal,	<b>9.67</b>				
89 " Iron,	<b>"</b> 69.70				
49 Grindstones,	<b>"</b> 3.47				
39 Pitchforks,	" 1.61				
197 Rakes,	" .17				
86 Hoes,	<b>"</b> .69				
78 Shovels,	" 1.17				
187 Spades,	<b>" .85</b>				
91 Ploughs,	" 11.61				
* 83 Harrows,	" 17.1 <i>5</i> "				
47 Handsaws,	<b>"</b> 3.16				
35 Mill-saws,	<b>"</b> 18.15				
47 cwt. Steel,	<b>"</b> 9.47				
57 " Lead,	<b>"</b> 6.83				
	<b></b> \$ 17,315.32.				
Received payment,					
E. 1	C. Lowe,				
	for Lord & Secomb.				
	Caagla				

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#### LEDGER ACCOUNTS.

123. The principal book of accounts among merchants is called a ledger. In it are brought together scattered items of accounts, often making long columns. As a rapid way of finding the amounts, accountants generally add more than one column at a single operation (Art. 48). The examples below may be added both by the usual method and by that which is more rapid.

1.	2.	3.	4.
s cts.	\$ cts.	₿ cts.	a cts.
3.3 3	9 1.1 6	$1\ 3\ 4.6\ 2$	1 2 0.0 3
9.1 0	4 0.4 4	511.42	6 3.1 7
1 6.3 7	6 0.6 6	27.34	1 0 0.0 0
4 1.3 0	9 1.3 1	3 3.3 5	781.14
3 1.7 0	4.00	6 0.4 0	1 5 1.6 1
<b>57.7</b> 1	6.3 4	1 4 9.3 1	$3\ 1.2\ 1$
<b>3.6</b> 0	7.1 9	9 2 1.3 1	1 0 5.5 2
5.00	28.37	3 9.1 6	3 0 6.0 6
<b>.</b> 5 0	8 1.0 0	1 1 0.0 5	1 7.7 5
.1 3	6.00	104.03	8 5.2 5
$1.5 \ 5$	1 5.0 0	3 6 3.4 0	1 3.0 0
7.9 1	4 3.0 5	9.00	1 0.0 0
1.19	1 0.0 3	1 1.1 2	1 3 5.7 5
9.00	9 5.0 5	0.1 0	1 1 1.1 5
1 3.4 0	61.18	1 7.9 1	1 6 3.0 5
37.65	1 1 2.1 5	1 1.1 3	7 1 2.1 5
4 5.7 5	1 0 0.6 1	4 5.9 2	· 31.15
5.25	1 6 0.0 0	1 7.7 5	9 5.2 5
7.7 5	3 9.4 1	1 3.8 5	2 5.1 7
5.3 1	2 2.3 9	1 1.0 1	7 7.2 5
8.13	9 2.7 7	6 2.1 9	1 2 3.1 0
4.27	3 3.0 1	9 5.1 7	1 1 6.5 1
2.64	6 4.3 2	1 9.1 0	2 2 3.0 6
1 1.1 2	5 9.1 1	1 1 1.0 0	9 6 6.7 5.
4 1.2 1	17.89	1 9 5.2 5	102.15
3 3.7 9	761.15	6 1 3.2 0	4106.31
47.31	8 2 3.5 4	1 0 8.0 9	6102.19
1 0.0 0	2 3 8.4 7	• 3 3 4.0 5	7119.81

#### COMPOUND NUMBERS.

- 124. A COMPOUND NUMBER is a collection of concrete units of different denominations; as, 5 pounds and 6 ounces; 4 feet and 5 inches.
- 125. A scale expresses the law of relation between the different units of a number.

The different units of simple numbers have a uniform tenfold increase from lower to higher orders, and a like decrease from higher to lower orders. They, therefore, are said to have a uniform scale.

In compound numbers, the names of different measuring units (Art. 9) are included in the expression of a single quantity, so that the relation of the units of one order to those of another is that of a varying scale; as in the expression of pounds, shillings, and pence, it is 4, 12, and 20.

#### REDUCTION OF COMPOUND NUMBERS.

126. REDUCTION is the process of changing numbers from one denomination to another, without altering their values.

It is of two kinds, Reduction Descending and Reduction Ascending.

Reduction Descending is changing numbers of a higher denomination to a lower denomination; as pounds to shillings, &c. It is performed by multiplication.

Reduction Ascending is changing numbers of a lower denomination to a higher denomination; as farthings to pence, &c. It is the reverse of Reduction Descending, and is performed by division.

#### ENGLISH MONEY.

127. English or Sterling Money is the currency of England.

#### TABLE.

4 Farthings (qr. or far.) ·	make	1 Penny,	d.
12 Pence	"	1 Shilling,	s.
20 Shillings	"	1 Pound.	£.

far.		d.			
4	=	1		8.	
48	==	12	===	1	£.
960	•	240	==	20	 1

NOTE 1.— The symbol £. stands for the Latin word libra, signifying a pound; s. for solidus, a shilling; d. for denarius, a penny; qr. for quadrans, a quarter.

NOTE 2.— Farthings are sometimes expressed in a fraction of a penny; thus,  $1 far. = \frac{1}{2} d.$ ; 2 far.  $= \frac{1}{2} d.$ ; 3 far.  $= \frac{1}{2} d.$ 

NOTE 8.— The term *sterling* is probably from Easterling, the popular name of certain early German traders in England, whose money was noted for the purity of its quality.

NOTE 4. — The English coins consist of the five-sovereign piece, the double-sovereign, the sovereign, and the half-sovereign, made of gold; the crown, the half-crown, the shilling, the six-pence, the four-pence, the three-pence, the two-pence, the one-and-a-half-pence, and the penny, made of silver; the penny, the half-farthing, and the half-farthing, made of copper.

The sovereign represents the pound sterling, whose legal value in United States money is \$4.84. The value of the English guinea is 21 shillings sterling. The guinea, the five-guinea, the half-guinea, the quarter-guinea, and the

seven-shilling piece, are no longer coined.

The guinea is so called because the gold of which the first guineas were made was brought from Guinea, in Africa.

The English gold coins are now made of 11 parts of pure gold, and 1 part of copper, or some other alloy; and the silver coin, of 37 parts of pure silver, and 3 parts of copper.

The present standard weight of the sovereign is 12311 grains Troy; the crown, 4864 grains; the copper penny, 2913 grains.

128. To change numbers expressed in one or more denominations to their equivalents in one or more other denominations.

# Ex. 1. In 48£. 12s. 7d. 2far. how many farthings?

We multiply the 48 by 20, be
48 £. 12 s. 7 d. 2 far. cause 20 shillings make 1 pound,
and to this product we add the 12 shillings in the question, and obtain 972 shillings. We then multiply by 12, because 12 pence make 1 shilling, and to the product we add the 7 pence, and obtain 11671 pence. Again, we multiply by 4, because 4 farthings make 1 penny, and to this product we add the 2

farthings, and obtain 46686 farthings, the answer sought.

8 \*

# In 46686 farthings how many pounds?

OPERATION. 4)46686far. 12)11671d. 2 far. 20)972s.7d. 48£.12s.

Ans. 48£. 12s. 7d. 2 far.

We divide by 4, because 4 farthings make 1 penny, and the result is 11671 pence, and 2 farthings remaining. We then divide by 12 because 12 pence make 1 shilling, and the result is 972 shillings, and 7 pence remaining. Lastly, we divide by 20, because 20 shillings Ans. 40x. 12s. (u. 2 lar. make 1 pound, and the result is 48 pounds, and 12 shillings remaining. By annexing to the last

quotient the several remainders, we obtain 48£. 12s. 7d. 2far. as the required result.

From these illustrations, for the two kinds of reduction, we deduce the following

Multiply the highest Rule. — For Reduction Descending. denomination given by the number of units required of the next lower denomination to make one in the denomination multiplied. product add the corresponding denomination of the multiplicand, if there be any. Proceed in this way, till the reduction is brought to the denomination required.

FOR REDUCTION ASCENDING. Divide the lower denomination given by the number of units required of that denomination to make one of the next higher. The quotient thus obtained divide as before, and so proceed until it is brought to the denomination required. The last quotient, with the several remainders, if there be any, annexed, will be

the answer.

#### EXAMPLES.

3. In 127£. 15s. 8d. how many farthings?

4. In 122672 farthings how many pounds?

5. How many farthings in 28£. 19s. 11d. 3 far.?

6. How many pounds in 27839 farthings?

7. In 378£. how many pence?

8. In 90720 pence how many pounds?

9. Reduce 967 guineas to pounds.

10. Reduce 1015£. 7s. to guineas.

## AVOIRDUPOIS WEIGHT.

Avoirdupois or Commercial Weight is used in weighing almost every kind of goods, and all metals except gold and silver.

## TABLE.

16	Drams (d	r.)	n	nake	1	Ounc	e,			oz.	
16	Ounces	•		66	1	Poun	d,			lb.	
25	Pounds			"	1	Quar	ter,		•	qr.	
4	Quarters			"	1	Hund	red	Weigh	ıt,	cw	t.
20	Hundred	Weight		"	1	Ton,		·		T.	
dr.		02.									
16	_	1		lb.							
256	_	16	===	1			qr.	•			
6400	_	400	_	25	,	_	1		cwt.		
25600	==	1600	_	100	)	=	4	_	1		T.
51200	=	32000	=	2000	)	_	80	_	20	-	1

NOTE 1.— The oz. stands for onza, the Spanish for ounce, and in cwt. the c stands for centum, the Latin for one hundred, and wt for weight.

NOTE 2. — The laws of most of the States, and common practice at the present time, make 25 pounds a quarter, as given in the table. But formerly, 28 pounds were allowed to make a quarter, 112 pounds a hundred, and 2240 pounds a ton, as is still the standard of the United States government in collecting duties at the custom-houses.

Note 3. — The term avoirdupois is from the French avoir du poid, signifying

to have weight.

Note 4.— The standard avoirdupois pound of the United States is the weight, taken in the air, of  $27_{1000}^{2000}$  cubic inches of distilled water, at its maximum density, or when at a temperature of  $39_{100}^{32}$  degrees Fahrenheit, the barometer being at 30 inches. It is the same as the Imperial pound avoirdupois of Great Britain, which is the weight of  $27_{10000}^{2024}$  cubic inches of distilled water at the temperature of 62 degrees.

- 1. In 165T. 13cwt. 3qr. 19lb. 14oz. how many ounces?
- 2. In 5302318 ounces how many tons?
- 3. If a load of hay weigh 3T. 16cwt. 2qr. 18lb., required the weight in ounces.
  - 4. In 122688 ounces how many tons?
- 5. Required the number of drams in 2T. 17cwt. 3qr. 16lb. 15oz. 13dr.
  - 6. In 1482749 drams how many tons?
  - What is the value of 7T. 17cwt. at 7 cents per pound?
     Ans. \$ 1099.00.
- 8. What will 19cwt. 3qr. 20lb. of sugar cost at 9 cents per pound?

  Ans. \$179.55.



# TROY OR MINT WEIGHT.

130. Troy or Mint Weight is the weight used in weighing gold, silver, jewels, and liquors; and in philosophical experiments.

#### TABLE.

24 Grains (gr.) 20 Pennyweights	make "		Pennyweig Ounce,	ht,	pwt. oz.
12 Ounces	"	1	Pound,		lb.
gr.	p₩t.			,	
24 =	1		OZ.		
480 ==	20	===	1		lb.
5760 ==	240	_	12	=	1

NOTE 1. — Troy weight was introduced into Europe from Cairo in Egypt, in the 12th century, and was first adopted in Troyes, a city in France, where great fairs were held, whence it may have had its name.

Note 2.— A grain or corn of wheat, gathered out of the middle of the ear, was the origin of all the weights used in England. Of these grains, 32, well dried, were to make one pennyweight. But in later times it was thought sufficient to divide the same pennyweight into 24 equal parts, still called grains, being the least weight now in use, from which the rest are computed.

Note 3.— Diamonds and other precious stones are weighed by what is called Diamond Weight, of which 16 parts make 1 grain; 4 grains, 1 carat. 1 grain Diamond Weight is equal to 4 grains Troy, and 1 carat to 31 grains Troy. In weighing pearls, the pennyweight is divided into 30 grains instead of 24, so that 1 pearl grain is equal to 4 grains Troy. The carat as a weight must not be confounded with the assay carat, a term whose use is to indicate a proportional part of a weight, as in expressing the fineness of gold, each carat means a twenty-fourth part of the entire mass used. Thus, pure gold is termed 24 carat gold, and gold that is not pure is termed 18 carat gold, 20 carat gold, &c., as its mass may be 18 twenty-fourths, 20 twenty-fourths, &c. pure gold. Each assay carat is subdivided into 4 assay grains, and each assay grain into 4 assay quarters.

NOTE 4.—The Troy pound, the standard unit of weight adopted by the United States Mint, is the same as the Imperial Troy pound of Great Britain, and is equal to the weight, taken in the air, of 22,7944 cubic inches of distilled water, at its maximum density, the barometer being at 30 inches.

- 1. How many grains in 28lb. 11oz. 12pwt. 15gr. Troy?
- 2. In 166863 grains Troy how many pounds?
- 3. If a silver pitcher weigh 3lb. 10oz., what is its weight in grains?
  - 4. How many pounds Troy in 22080 grains?

- 5. What is the value of 73lb. 11oz. of standard silver at \$0.062 per pennyweight?
- 6. How many pounds of standard silver can be purchased for \$1099.88, at the rate of \$0.062 per pennyweight?
- 7. A Californian has 57lb. 7oz. of pure gold. What is its value at \$20.593\frac{1}{2} per ounce? Ans. \$14229.901\frac{1}{2}.
- 8. What is the value of a mass of standard gold weighing 19lb. 6oz. 16pwt. at 93 cents per pennyweight?

Ans. \$ 4367.28.

9. I have a lump of pure silver weighing 13lb. 9oz. What is its value at  $1.385 \frac{7}{10}$  per ounce? Ans.  $228.640\frac{1}{2}$ .

## APOTHECARIES' WEIGHT.

131. Apothecaries' Weight is used in mixing medical prescriptions.

#### TABLE.

<ul><li>20 Grai</li><li>3 Seru</li><li>8 Drai</li><li>12 Oun</li></ul>	ples ns		ake " "	1 Scru 1 Dran 1 Ound 1 Pour	n, ce,		sc. or   dr. or   oz. or   lb. or	3. 3.
gr. 20		sc. 1		dr.				
60	==	3	_	1		OŽ.		
480	=	24	_	8	=	1		lb.
5760	===	288	-	96	=	12	_	1

Note 1. — In this weight the pound, ounce, and grain are the same as in Troy Weight.

NOTE 2. - Medicines are usually bought and sold by Avoirdupois Weight.

NOTE 3. — In estimating the weight of fluids, 45 drops, or a common teaspoonful, make about 1 fluid dram; 2 common table-spoonfuls, about 1 fluid ounce; a wineglassful, about 1½ fluid ounces; and a common teacupful, about 4 fluid ounces.

- 1. In 23th 93 03 29 13gr. how many grains?
- 2. In 136853 grains how many pounds?
- 3. How many scruples in 23lb.?
- 4. How many pounds in 6624 scruples?
- 5. In 47th 03 03 19 19gr. how many grains?
- 6. In 270759 grains how many pounds?

7. A physician bought 1 pound of ipecacuanha for \$1.80, and retailed it out in doses of 5 grains, at 12½ cents each. How much did he get for it over the cost?

Ans. \$142.20.

Avoirdupois, Troy, and Apothecaries' Weight Compared.

132. The relative value of the pound, and its subdivisions, of the several weights, in Troy grains, and in denominations of each other, is shown in the following

#### TABLE.

```
1 lb. Av.
                  = 7000 gr. Tr. = 1lb. 2oz. 11pwt. 16gr. Tr.
                                    = 13oz. 2\frac{114}{176}dr. Av.
1 lb. Tr. or Ap. = 5760
                               "
1 oz. Tr. or Ap. = 480
                                     = 1oz. 1_{175}^{97} dr. Av.
1 oz. Av.
                       4371
                                     == 18pwt. 5½gr. Tr.
1 dr. Ap.
                        60
                                     = 2\frac{34}{128} dr. Av.
1 dr. Av.
                        2711
                                     = 1pwt. 311gr. Tr.
                               66
1 pwt. Tr.
                        24
                                     == ₩dr. Av.
                               "
1 sc. Ap.
                        20
                                    == 138dr. Av.
                               "
1 gr. Tr. or Ap. =
                         1
                                    =, 33dr. Av.
```

NOTE. — To change a quantity from one weight to its equivalent in another weight, reduce the given quantity to Troy grains, and then find their value in denominations of the weight required.

- 1. Change 13lb. 6oz. Avoirdupois weight to Troy weight.
- 2. Change 16lb. 3oz. 1pwt. 1gr. Troy weight to Avoirdupois weight.
  - 3. Change 3lb. 8oz. 10pwt. to drams of Apothecaries' weight.

    Ans. 356dr.
  - Change 356 drams Apothecaries' weight to Troy weight.
     Ans. 3lb. 8oz. 10pwt.
  - 5. An apothecary bought by Avoirdupois weight 2lb. 8oz. of quinine at \$2.40 per ounce, which he retailed at 20 cents a scruple. What was his gain on the whole?
  - 6. If I should buy by Avoirdupois weight 12lb. of opium at 37½ cents per ounce, and sell it by Troy weight at 40 cents per ounce, should I gain or lose by so doing?

    Ans. Lose \$ 2.

#### LINEAR OR LONG MEASURE.

133. Linear or Long Measure is used in measuring distances in any direction.

#### TABLE.

12	Inches (in	.)	make	1	Foot,				ft.
3	Feet `		"	1	Yard,				yd.
5 <del>1</del>	Yards, or	16‡ Feet,	"	1	Rod,	or Pole	,		rd.
40	Rods		"	1	Furlo	ng,			fur.
8	Furlongs,	or 320 <b>R</b> o	ds, "	1	Mile,	•			m.
3	Miles		"	1	Leagu	ıe,			lea.
69}	Miles (nea	rly)	"	1	Degre	e on th	e equ	ator, d	eg. or °.
<b>3</b> 60	Degrees	••	"	1	Great	Circle	of th	e Eartl	h.
in		ft.							
1	2 =	1	y	d.					
3	6 ==	3 =	=	1		rd.			
19	8 ==	$16\frac{1}{2} =$	=	5 <u>}</u>	=	1		fur.	
792	0 =	660 =	= 22	90	=	40	=	1	m.

NOTE 1.—12 lines make 1 inch; 4 inches, 1 hand; 6 feet, 1 fathom; 120 fathoms, 1 cable-length;  $7\frac{1}{2}$  cable-lengths, 1 mile;  $\frac{1}{60}$  of a degree of the circumference of the earth, 1 knot, or geographical mile, equal to  $1\frac{1}{12}$  statute miles.

320

1760

63360

5280

Note 2.—The yard adopted by the United States government as the standard unit of linear measure is the same as the imperial yard of Great Britain, which, as compared with a pendulum vibrating seconds in the latitude of London, the pendulum moving in a vacuum, at the level of the sea, and at the temperature of 62° Fahrenheit, should bear the proportion of 36 to 391333 inches. A metre, the unit of linear measure, as established by the French government, is equal to about 3937 English inches.

NOTE 3.—The English statute mile is the same as that of the United States, but that of other countries differs in value from it; as the German short mile is equal to 6857 yards, or about 3. English miles; the German long mile, to 10125 yards, or about 5. English miles; the Prussian mile, to 8237 yards, or about 4. English miles; the Spanish common league, to 7416 yards, or about 4. English miles; the Spanish judicial league, to 4635 yards, or about 2. English miles.

Note 4.—A degree of longitude is  $\frac{1}{360}$  of any circle of latitude. As the circles of latitude diminish in length, the degrees of longitude vary in length under different parallels of latitude. Thus, under the equator, the length of a degree of longitude is about  $69\frac{1}{6}$  statute miles; at 25° of latitude  $62\frac{7}{10}$  miles; at 40° of latitude, 53 miles; at 42° of latitude, 51 $\frac{1}{6}$  miles; at 49° of latitude, 45 $\frac{1}{6}$  miles; at 60°, 34 $\frac{7}{6}$  miles.



#### EXAMPLES.

- 1. In 96deg. 56m. 7fur. 32rd. 12ft. 6in. how many inches?
- 2. In 424320486 inches how many degrees?
- 8. How many feet in 79 miles?
- 4. Required the miles in 417120 feet.
- 5. How many inches in 396 furlongs?
- 6. Required the furlongs in 3136320 inches?
- 7. How many inches from Haverhill to Boston, the distance being 30 miles?
  - 8. Required the miles in 1900800 inches.

#### CLOTH MEASURE.

134. Cloth Measure is used in measuring cloth, ribbons, lace, and other articles sold by the yard or ell.

#### TABLE.

4 N 4 C 8 C	nches (ir Vails Quarters Quarters Quarters	n.)	mak " "	e e	1 Y 1 E	ail, uarter ard, il Flen il Eng	nish,	ard,	i	na. qr. yd. E. F. E. E.
in. 2‡	===	na. 1		qr.						
9	===	4	-	1		E. F.				
27	=	12	-	3	=	1		yd.		
36	=	16	-	4	_	1}	-	1		E. I.
45	==	20		5	=	13	-	14	-	1

Note 1. - The Ell French is 6 quarters; the Ell Scotch, 4qr. 1lin.

NOTE 2. — Cloth measure is a species of linear measure, and the yard and inch are the same in both.

#### EXAMPLES.

- 1. In 17yd. 3qr. 2na. how many nails?
- 2. In 286 nails how many yards?
- 3. In 365yd. 1qr. 3na. how many nails?
- 4. In 5847 nails how many yards?
- 5. In 71E. E. 4qr. how many nails?
- 6. In 1436 nails how many ells English?
- 7. What cost 47yd. 3qr. of silk velvet at \$1.25 per quarter?
- 8. A merchant bought a roll of cloth containing 31 LE. E. and paid for it at the rate of \$3 per yard. What did it cost him?

  Ans. \$117.

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## SURFACE OR SQUARE MEASURE.

# 135. Square Measure is used in measuring surfaces of all kinds.

#### TABLE.

144	Square	inches	(sq. in	.) make	1	Square	foot,		ft.	
9	Square	feet		"	1	Square	yard,		yd.	
30	& Square	yards		"		Square			p.	
40	Square	rods		"	1	Rood,		•	Ř.	
4	Roods			"	1	Acre,			A.	
640	Acres			44	1	Square	mile,		S. M	И.
	in.		ft.			_				
•	144 :	==	1	2	yd.					
	1596 =		9 =	==	1		p.			
	39204 =	_	2721 =	= 3	04 =	=	1	R.		
	1568160 =	<b>–</b> ∙1	.0890 =	= 12	10 =	= 4	10 ==	1	A.	
	6272640 =	= 4	3560 =	= 484	40 =	= 16	i0 <del></del>	4 ==	1 g.:	M.
401	4489600 =	<del></del> 2787	/8 <b>4</b> 00 =	= 30976	00 =	= 10240	00 == 2	2560 <b>—</b>	640 <del></del>	1

Note. — A square is a figure bounded by four equal lines, perpendicular to each other.

When the four lines are each 1 foot in length, the space enclosed is 1 square foot; when 1 yard in length, 1 square yard; when 1 rod in length, 1 square rod; and so for any other dimension.

	8	ft. = 1yd	ι
d.			Square foot.
3ft. = 1yd.			
8			

In this diagram the large square represents a square yard, and each of the smaller squares within it represents one square foot. Now, since there are three rows of small squares, and three square feet in each row, there will be 3 sq. ft.  $\times 3 = 9$  sq. ft. in the large square. But the large square is 3ft. in length and 3ft. in breadth; hence,

To find the contents of a square, multiply the numbers denoting its length and breadth together.

#### EXAMPLES.

- 1. In 57A. 3R. 27p. 21yd. 8ft. 57in. how many square inches?
  - 2. In 363331893 square inches how many acres?
  - 3. How many square feet in 25 acres?
  - 4. How many acres in 1089000 square feet?
  - 5. How many square rods in 365 square miles?
  - 6. How many square miles in 37376000 square rods?
  - 7. How many acres in 12345678 square inches?

Ans. 1A. 3R. 34p. 27yd. 4ft. 54in.

8. Bought 89A. 2R. 16p. of land for \$3.75 per square rod, and sold the same for \$0.25 per square foot. What did I gain by my bargain?

Ans. \$407,484.00.

#### SURVEYORS' MEASURE.

136. This measure is used by surveyors in measuring land, roads, &c.

#### TABLE.

7.33	Inches (in.)	make	1	Link		1.
25	Links	"	1	Pole,	•	p.
100	Links, 4 Poles, or 66 Feet,	"	1	Chain,		ch.
10	Chains	46	1	Furlong,		fur.
8	Furlongs, or 80 Chains,	"	1.	Mile,		m.

Inches.		Link.								
7 93 100	=	1.		Pole.						
198	=	25	=	1		Chain.				
792	===	100	=	4	=	1	1	urlon'	g.	
7920	=	1000	=	40	=	10	-	1		Mile.
<b>63</b> 360	-	8000	•	320	=	80	-	8	=	1

NOTE 1. — Gunter's chain, in length 4 poles, or 66 feet, and divided into 100 links, is that mostly used in ordinary land surveys; but in locating roads, and like public works, an engineer's chain is usually 100 feet in length, containing 120 links, each 10 inches long.

NOTE 2.—A section of government lands is 1 square mile, or 640 acres. An acre, as a square piece of land, will measure on each side about 209 feet or 70 paces. 625 square links make 1 square rod or pole; 16 square rods make 1 square chain, and 10 square chains make 1 acre.

Note 3.— A rod or pole is sometimes called a perch, and each of the names given to this measure is expressive of the instrument by which it was formerly measured.

- 1. How many links in 46m. 3fur. 5ch. 25l.?
- 2. In 371525 links how many miles?
- 3. In 97m. Ofur. how many links?
- 4. In 776000 links how many miles?
- 5. The extent of a certain farm is found, by survey, to be 1377 square chains (Note 2). How many acres does it contain?

  Ans. 137A. 2R. 32p.
- 6. What will be the cost of a field measuring 2,126,250 square links, at \$80 per acre?

  Ans. \$1701.00.

#### CUBIC OR SOLID MEASURE.

137. Cubic or Solid Measure is used in measuring such bodies or things as have length, breadth, and thickness; as timber, stone, &c.

#### TABLE.

				LAD	11111					
1728	Cubic	inches	(cu. in.)	mak	e 1	Cubic	foot	,		cu. ft.
27	"	feet		"	1	"	yard	i,	(	eu. yd.
40	"	feet		"	1	Ton,	_			T.
16	44	feet		4	1	Cord	foot,			e. ft.
	Cord Cubic	feet, or feet,	}	u	1	Cord	of wo	ood,	•	€.
	in.		ft.							
1	1728	-	1		yd.					
46	3656		27	=	1			T.		
GS	120	-	40	=	113	=	=	1		σ.
221	184	-	128	-	427		=	31	=	1

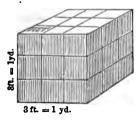
NOTE 1. — A pile of wood 8ft. in length, 4ft. in breadth, and 4ft. in height, contains a cord.

Also, one ton of timber, as usually surveyed, contains  $50^{92}_{100}$  cubic or solid feet.

Sawed timber, joists, plank, and scantlings are now generally bought and sold by what is called board measure.

Note 2. — A cube is a solid bounded by six square and equal sides.

If the cube is 1 foot long, 1 foot wide, and 1 foot high, it is called a cubic or solid fingt. If the cube is 3 feet long, 3 feet wide, and 3 feet thick, it is called a cubic or solid yard.



Now, since each side of a cubic yard, as represented in the diagram, contains 9 sq ft. of surface (Art. 93), it is plain, if a block be cut off from one side, one foot thick, it can be divided into 9 solid blocks, with sides 1 foot in length, breadth, and thickness, and therefore will contain 9 solid feet; and since the whole block or cube is three feet thick, it must contain 3 solid feet  $\times$  9 = 27 solid feet; for 3 solid feet  $\times$  3  $\times$  3 =

27 solid feet. Hence,

To find the contents of a cubic body, multiply the numbers denoting its length, breadth, and thickness together.

Note 3.—A cubic foot of distilled water at the maximum density, at the level of the sea, and the barometer at 80 inches, is equal in weight to 62½lb. or 1000cz. avoirdupois.

NOTE 4.—A cubic foot of lead weighs 708½lb.; of brass, 534½lb.; of copper, 555lb.; of wrought-iron, 486½lb.; of cast-iron, 450½lb.; of marble, 171lb.; of granite, 165lb.; of clay, 180lb.; of common soil, 124lb.; of bricks, 124lb.; of sand, 95lb.; of sea-water, 64,3lb.; of oak wood, 55lb.; of Anthracite coal, 54lb.; of Bituminous coal, 50lb.; of red-pine wood, 42lb.; and of white-pine wood, 80lb.

## EXAMPLES.

- 1. In 29 cords of wood how many solid inches?
- 2. In 6414336 cubic inches how many cords?
- 3. In 19 tons of timber how many solid inches?
- 4. How many tons of timber in 1313280 cubic inches?
- 5. How many cubic feet of wood in 128 cords?
- 6. How many cords of wood in 16384 cubic feet?
- 7. How many cubic feet in a pile of wood, 40 feet long, 4 feet wide, and 7 feet high?
  - 8. How many cords of wood in 8650 cubic feet?

Ans. 67 cords, 74 cubic feet.

9. How many cubic feet in a granite block, 17 feet long, 11 feet wide, and 9 feet high?

Ans. 1683 cubic feet.

# LIQUID OR WINE MEASURE

138. Liquid or Wine Measure is used in measuring all kinds of liquids, except, in some places, beer, ale, porter, and milk.

#### TABLE.

4 Gills (gi.)				make	1 Pint,				pt.		
2 Pints				"	1 Quart,				qt.		
4 Quarts				"	1 Gallon,				gal.		
63 Gallons				"	1 Hogshead,				hhd.		
2	Hogsh	eads		"	1 Pipe, or Butt,				pi.		
2	Pipes			"	1	Tun,			-		tun.
gi.		pt.									
4	===	1		qt.							
8	=	2	=	1		gal.					
32		8	=	4	==	1		hhd.			
2016	=	504	==	<b>252</b>	==	63	-	1		pi.	
4032		1008	=	504	_	126	_	2	=	1	tun.
8064		2016	=	1008	=	<b>252</b>	_	4	=	2	== 11°

Note 1.—By laws of Massachusetts, 32 gallons make 1 barrel. In some States 31½ gallons, and in others from 28 to 32 gallons, make 1 barrel. 42 gallons make 1 tierce, and 2 tierces make 1 puncheon.

Note 2. — The term hogshead is often applied to any large cask that may contain from 50 to 120 gallons, or more.

Note 3.— The Standard Unit of Liquid Measure adopted by the government of the United States is the Winchester Wine Gallon, which contains 231 cubic inches, and is of a capacity to hold  $8^{1000}_{1000}$ lb. Avoirdupois of distilled water, at its maximum density, weighed in air, the barometer being at 30 inches. It has the name Winchester, from its standard having been formerly kept at Winchester, Eugland. The Imperial Gallon, now adopted in Great Britain, contains 277 1274 cubic inches; so that 6 Winchester gallons make about 5 Imperial gallons.

Note 4. —1 gallon of alcohol weighs 7lb.; of camphene,  $7_1^5$ lb.; of proof spirits,  $7_{16}^{5}$ lb.; of spirits of turpentine,  $7_{16}^{5}$ lb.; of sperm oil,  $7_2^4$ lb.; of olive oil,  $7_2^4$ lb.; and of molasses,  $11_2^2$ lb.

NOTE 5.—The fluid measure of apothecaries, used by them in measuring liquids of medical prescriptions, divides the gallon (marked Cong.) into \$\mathbb{S}\$ pints (0.); the pints into 16 fluid ounces (f\mathbb{S}); the fluid ounces into 8 fluid drams (f\mathbb{S}); and the fluid drams into 60 minims (m) or drops. The abbreviation Cong. stands for conginarium, the Latin for gallon, and the O. is the initial of octans, the Latin for an eighth, the pint being an eighth of a gallon.

## EXAMPLES.

- 1. In 57T. 3hhd. 50gal. 3qt. how many pints?
- 2. In 116830 pints how many tuns?
- 3. Reduce 96hhd. 47gal. 2qt. to gills.
- 4. How many hogsheads in 195056 gills?
- 5. What cost 40 hogsheads of wine at \$0.37½ per pint?
- 6. How much may be gained by buying 2 hogsheads of molasses, at 40 cents a gallon, and selling it at 12 cents a quart?

  Ans. \$10.08.

#### BEER MEASURE.

139. Beer Measure is used in measuring beer, ale, porter, and milk.

		<b>T</b> .	ABLE.				•
2 Pints (pt.)		make	1	Quart,	qt.		
4 Quarts		66	1	Gallon,		gal	
54 Gallons	•	"	1	1 Hogshead,			hhd.
pt.		qŧ.					
2	-	1		gal.			
8	sies	4	-	1		hhd.	
452		216	1930	54	-	1	
		9 *					

NOTE 1. — The gallon of beer measure contains 282 cubic inches; and has been usually reckoned, 36 gallons equal 1 barrel; 2 hogsheads, or 108 gallons, 1 butt: 2 butts, or 216 gallons, 1 tun.

Note 2. — Beer Measure is becoming obsolete. Milk and malt liquors, at the present time, are bought and sold, very generally, by wine or liquid

measure.

### EXAMPLES.

- 1. How many pints in 46hhd. 49gal.?
- 2. In 20264 pints how many hogsheads?
- 3. In 368hhd. how many pints?
- 4. In 158976 pints how many hogsheads?
- At 29 cents per gallon, what cost 76 hogsheads of ale?
   Ans. \$ 1190.16.
- 6. How much may be obtained by selling 47hhd. 36gal. of lager-bier at 5 cents a quart?

  Ans. \$514.80.

#### DRY MEASURE.

140. This measure is used in measuring grain, fruit, salt, &c.

#### TABLE.

2 Pints (	(pt.)	make	1	Quart,		qt.
8 Quarte		44	1	Peck		pk.
4 Pecks		"	1	Bushel,		ъ́и.
pts.		gal.				•
. 8	-	1		pk.		
16	=	2		1		bu.
64	_	8		<b>4</b>	-	1

Note 1.— The Standard Unit of Dry Measure adopted by the United States government is the Winchester bushel, which is in form a cylinder, 18½ inches in diameter, and 8 inches deep, containing 2150 100 200 cubic inches. The Standard Imperial bushel of Great Britain contains 2218 100 cubic inches, so that 32 Imperial bushels equal about 33 Winchester bushels. The gallon in Dry Measure contains 268½ cubic inches.

Note 2.— Of wheat a standard bushel is 60lb.; of shelled corn, 56lb.; of corn on the cob, 70lb.; of rye, 56lb.; of barley, 48lb.; of buckwheat in Pa., 50lb.; in Kentucky, 52lb.; in Mass., 48lb.; of oats in Ohio, Ill., Mass., &c., 32lb.; of oats in Ky., 38½lb.; of oats in Me., 30lb.; of oats in Pa., 30lb.; of clover-seed, 60lb.; of flax-seed, 56lb.; of Timothy-seed, 45lb.; of bran, 20lb.; of beans, 60lb.; of onions, in Pa., Ky., &c., 57lb.; of onions in Mass., 52lb.; of salt in Ky., 56lb.; of salt in Ill., 50lb.; of dried apples in Pa., 22lb.; of dried apples in Ill., 24lb.; of dried peaches in Pa., 33lb.; of dried peaches in Ill., 32lb.; of stove coal in Ill., 50lb.; of bituminous coal in the Western States, 76lb.; and of hard-wood charcoal, 30lb. The weight by law, of a few

of the articles named, to a bushel, is not uniform in all the States, and therefore may vary slightly from the above, in a few States not mentioned.

Note 3.—In some places it is customary, in measuring toal, potatoes, and like articles, to "heap" the bushel, as it is called, and in that case 5 even pecks are about equal to 1 "heaped bushel." The "coal bushel," as established by laws of Massachusetts, Ohio, and some other States, is of greater capacity than the Winchester bushel. In some parts of the United States a chaldron, a measure of coal, consists of 36 bushels; and in other parts of the country it consists of 32 bushels, or of 4 quarters, each consisting of 8 bushels. The quarter, however, in England is 8 Imperial bushels, a measure of grain equal to 560lb., or one quarter of a ton of 2240lbs.

### EXAMPLES.

- 1. How many pints in 35bu. 3pk.?
- 2. In 2288 pints how many bushels?
- 3. In 676 chaldrons, of 36 bushels each, how many pecks?
- 4. How many chaldrons, of 36 bushels each, in 97344 pecks?
- 5. A grocer purchased 50 bushels of potatoes, by "heaped" measure, at 60 cents a bushel, and sold the same, by "even" measure, at 15 cents a peck; did he gain or lose by the operation?

  Ans. Gain \$7.50.
- 6. If I purchase by measure 96 bushels of oats, weighing 2304 pounds, at 42 cents a bushel, and sell the same by weight in Ohio, at 45 cents a bushel, shall I gain or lose by so doing?

  Ans. Lose \$7.92.

# DRY, LIQUID, AND BEER MEASURES COMPARED.

141. The relative value of the gallon and its subdivisions, of the several measures, in cubic inches, and in denominations of each other, are shown in the following

### TABLE.

cu. in.	
1 gal. B. M. = $282$ = 1gal. 1pt. $3\frac{5}{77}$ gi. L. M	$L = 1$ gal. $\frac{11}{28}$ pt. D. M.
1 gal. D. M. = $268_1^4$ = 1gal. 1pt. $1\frac{13}{58}$ gi. L. M	$1 = 3qt. 1\frac{147}{233}pt. B. M.$
1 gal. L. M. = $231$ = $3qt$ . $pt$ . D. M.	= 3qt. 26pt. B. M.
1 qt. B. M. = $70\frac{1}{2}$ = 1qt. 0pt. $1\frac{59}{79}$ gi. L. M.	$= 1qt. \frac{11}{112}pt. D. M.$
1 qt. D. M. = $67\frac{1}{8}$ = 1qt. $1\frac{1}{85}$ pt. L. M.	= 1pt. 3191gi. B. M.
1 qt. L. M. = $57\frac{3}{4}$ = $1\frac{23}{32}$ pt. D. M.	$= 1\frac{30}{47}$ pt. B. M.
1 pt. B. M. = 35½ = 153pt. L. M.	$=1_{\frac{11}{2924}}$ pt. D. M.
1 pt. D. M. = $33\frac{3}{5}$ = 1pt. $\frac{17}{15}$ gi. L. M.	$=3_{\frac{661}{9400}}^{661}$ gi. B. M.
1 pt. L. M. = $28\frac{7}{8}$ = $\frac{53}{52}$ pt. D. M.	= 17pt. B. M.
1 gi. L. M. = $7\frac{7}{32} = \frac{55}{256}$ pt. D. M.	= 7/ <sub>5</sub> pt. B. M.

Note 1.—By the table, it is evident that each of the measures of capacity is a species of cubic measure; and to change cubic measure, expressed in cubic inches, to any denomination of either dry, liquid, or beer measure, divide by the number of cubic inches required to make a unit of the proposed denomination. Thus, to reduce 14ft. 294in. cubic measure to gallons of liquid measure: 14 cu. ft. 294 cu. in. = 24486 cu. in.; 24486 cu. in.; 231 = 106 gallons, liquid measure.

NOTE 2. — To change a quantity from one measure of capacity to its equivalent in another, reduce the given quantity to cubic inches, and then find their value in denominations of the proposed measure.

### EXAMPLES.

- 1. Change 4hhd. 15 gal. beer measure to liquid measure?
- 2. Change 4hhd. 30 gal. liquid measure to beer measure?
- 3. If a milkman buy 2820 gallons of milk at 4 cents per quart, beer measure, and sell the same at 6 cents per quart, liquid measure, what will he gain?

Ans. \$ 375.0225.

- 4. If 2538 gallons of milk have been purchased by liquid measure, at 4 cents per quart, and the same has been sold by beer measure at 6 cents per quart, what has been the gain?

  Ans. \$92.88.
- 5. A merchant bought 385 bushels of seed peas at \$4.00 per bushel, dry measure. He sold the same at 20 cents per quart, liquid measure. What did he gain by the purchase?

  Ans. \$1327.20.
- 6. J. Day bought 1000 bushels of corn at \$1.05 per bushel, dry measure, and sold the same at \$1.12 per bushel, liquid measure. Did he gain or lose by the operation, and how much?
- 7. My hogshead contains 30 cubic feet. How many more gallons of dry measure will it contain, than of beer measure?
- 8. Bought of my neighbor, John Smith, 365 gallons of milk, at 5 cents per quart; but by mistake he measured it in his liquid measure. How much did I lose?

### MEASURE OF TIME.

142. This measure is applied to the various divisions and subdivisions into which time is divided.

Table.	,
--------	---

60 Secon	nds (se	ec.)		mak	e 1	Minute	Э,		m.
60 Minu	tes `	•		"	1	Hour,			h.
24 Hour	S			"	1	Day,			d.
7 Days				. "	1	Week,	,		w.
3651 Days,	or 52	weeks 1	day	s, "	1	Julian	Year	,	<b>y</b> .
12 Caler	idar M	lonths (m	0.)	"	1	Year,		-	y.
sec.		m.	•			-			•
60	==	1		h.					
3600	=	60	==	. 1		đ.			
86400	-	1440	=	24	=	1		₩.	
604800		10080	=	168	=	7	==	1	у.
<b>315</b> 57600	===	525960	-	8766	_	365 <del>1</del>		=	1

Note 1.— The true Solar or Tropical Year is the time measured from the sun's leaving either equinox or solstice to its return to the same again, and is 865d. 5h. 48m. 497sec.

The Julian Year, so called from the calendar instituted by Julius Cæsar, contains 365½ days, as a medium; three years in succession containing 365 days, and the fourth year 366 days; which, as compared with the true solar year, produces an average yearly error of 11m. 10½ sec., or a difference that would amount to 1 whole day in about 120 years.

The Gregorian Year, or that instituted by Pope Gregory XIII., in the year 1582, and which is now the Civil or Legal Year in use among most nations of the earth, contains, like the Julian year, 385 days for three years in succession and 386 for the fourth, excepting centennial years whose number cannot be exactly divided by 400. The Gregorian year is so nearly correct as to err only 1 day in 3866 years, a difference so little as hardly to be worth taking into account.

The manner of reckoning time according to the Julian Calendar is termed Old Style, and that according to the calendar of Gregory, New Style. England did not adopt the new style till 1752, when, according to an act of Parliament, the difference between the two styles, which then amounted to 11 days, was removed, by the day following the 2d of September of that year being accounted the 14th day. The difference now between old and new style is 12 days.

A Common Year is one of 365 days, and a Leap or Bissextile Year is one of 366 days. Any year is Leap Year whose number can be divided by 4 without a remainder, except years whose number can be divided without a remainder by 100, but not by 400.

A Sidereal Year is the time in which the earth revolves round the sun, and is 865d. 6h. 9m. 95 sec.

Note 2.—The names of the 12 calendar months, composing the civil year, are January, February, March, April, May, June, July, August, September, October, November, December, and the number of days in each may be readily remembered by the following lines:—

"Thirty days hath September,
April, June, and November;
And all the rest have thirty-one,
Save February, which alone
Hath twenty-eight; and this, in fine,
One year in four hath twenty-nine."

TABLE

SHOWING THE NUMBER OF DAYS FROM ANY DAY OF ONE MONTH TO THE SAME DAY OF ANY OTHER MONTH IN THE SAME YEAR.

FROM ANY				TO	THE	Same	DAY	OF 1	NEXT			
DAY OF	Jan.	Feb.	Mar.	Apr.	May.	June	July.	Aug.	Sept.	Oct.	Nov.	Dec.
JANUARY	365	31	59	90	120	151	181	212	243	273	304	334
FEBRUARY	334	365	28	59	89	120	150	181	212	242	273	303
MARCH	806	337	865	81	61	92	122	153	184	214	245	275
APRIL	275	806	334	865	30	61	91	122	153	183	214	244
MAY	245	267	804	885	365	31	61	92	123	153	184	214
JUNE	214	245	273	304	334	365	80	61	92	122	153	183
JULY	184	215	243	274	804	335	365	81	62	92	123	153
August	153	184	212	243	273	304	334	365	31	61	92	122
SEPTEMBER	122	153	181	212	242	273	303	334	365	80	61	91
OCTOBER	92	123	151	182	212	243	273	804	835	365	81	61
NOVEMBER	61	92	120	151	181	212	242	273	304	334	365	80
DECEMBER	81	62	90	121	151	182	212	243	274	304	335	365

For example, suppose we wish to find the number of days from April 4th to November 4th, we look for April in the left-hand vertical column, and for November at the top, and where the lines intersect is 214, the number sought. Again, if we wish the number of days from June 10th to September 16th, we find the difference between June 10th and September 10th to be 92 days, and add 6 days for the excess of the 16th over the 10th of September, and so we have 98 days as the exact difference.

If the end of February be included between the points of time, a day mustbe added in leap year.

When the time includes more than one year, there must be added 365 days for each year.

#### EXAMPLES.

- 1. How many seconds in a solar year?
- 2. In 31556929 seconds how many days?
- 3. How many seconds from the deluge, which took place 2348 years before Christ, to the year 1856, a year being 3654 days?

  Ans. 132636592800.
  - 4. In 74726807872 seconds how many solar years?

Ans. 2368 years.

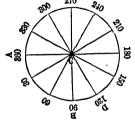
5. How long did the last war with England continue; it having commenced June 18th, 1812, and ended February 17th, 1815?

Ans. 974 days = 2 years 244 days.

#### CIRCULAR ANGULAR MEASURE.

143. Circular Angular Measure is applied to the measurement of circles and angles, and is used in reckoning latitude and longitude, and the revolutions of the planets round the sun.

60 Seconds 60 Minutes 30 Degrees 12 Signs, or	` ,	Degrees.	make " "	1 Minute, 1 Degree, 1 Sign, The Circle	of the	<b>Z</b> odis	<b>1</b> C	/. e. S. C.
60 3609 108009	=	1 60 1800	=	° 1 30		5. 1	<b></b> ,	о. О.
1296000	270	21600	bou	860  OTE 1.—A  nded by a cu  equally distir	rve line	, all pa	rts of	which



The Greumference of a circle is the line which bounds it, as shown by the diagram.

An Arc of a circle is any part of its circumference; as AB.

A Radius of a circle is a straight line drawn from its centre to its circumference; as CA, CB, or CD.

Every circle, large or small, is supposed to be divided into 860 equal parts, called degrees.

A Quadrant is one fourth of a circle, or an arc of 900; as AB.

An Angle, as ACB, is the inclination or opening of two lines which meet at a point, as C. The point is the vertex of the angle. If a circle be drawn around the vertex of an angle as a centre, the two sides of the angle, as radii of the circle, will include an arc, which is the measure of the angle; as the arc AD = 120° is the measure of the angle ACD, and AB = 90°, the measure of the angle ACB; hence the one is called an angle of 1200, and the other an angle of 900.

Note 2. - As the earth turns on its axis from west to east every 24 hours, the sun appears to pass from east to west 1 of 3600 of longitude every hour. or 150 of longitude in 1 hour's time, or 15 in 4 minutes of time, and 1 in 4 seconds of time; so that, for instance, when it is noon at any place, it is 1 hour earlier for every 150 of longitude westward, and 1 hour later for every 150 of longitude eastward. Thus, Boston being 710 4' west of Greenwich, and San Francisco 51º 17' west of Boston, when it is noon at Boston, it is 4h. 44m. 16sec. past noon at Greenwich, and wanting 3h. 25m. 8sec. of noon at San Francisco.

- 1. How many minutes in 27S. 27° 43'?
- 2. In 50263 minutes how many signs?
- 3. How many seconds in 44S. 18° 57' 23"?
- 4. How many signs in 4820243"?
- 5. How many seconds in 360°?
- 6. How many degrees in 1296000"?

### MISCELLANEOUS TABLE.

144. The following denominations, frequently used, are not embraced in the preceding tables.

12	units	make	1 dozen.
12	dozen	66	1 gross.
12	gross	"	1 great gross.
20	units	"	1 score.
24	sheets of Paper	"	1 quire.
	quires	46	1 ream.
2	reams	"	1 bundle.
5	bundles	"	1 bale.
14	pounds of Iron or Lead	"	1 stone.
214	stone	"	1 pig.
8	pigs	66	1 fother
		"	1 cubit.
24	cubic feet of Stone	66	1 perch.
60	pairs of Shoes	"	1 case.
25	pounds of Powder	66	1 keg.
56		66	1 firkin.
100	pounds of Fish	46	1 quintal.
196	pounds of Flour	66	1 barrel.
200	pounds of Beef	. "	1 barrel.
200	pounds of Pork	66	1 barrel.
200	pounds of Shad or Salmon, in N. Y. & Ct.	66	1 barrel.
220	pounds of Fish, in Md.	"	1 barrel.
256		66	1 barrel.
800	pounds of Hydraulic Cement	46	1 barrel.
30	gallons of Fish, in Mass.	46	1 barrel.
5	bushels of Shelled Corn, in Southern States	٠ ، ،	1 barrel.
8	bushels of Salt	"	1 hogshead.

Note 1.— A sheet of paper folded into 2 leaves forms a folio; a sheet folded into 4 leaves, a quarto; a sheet folded into 8 leaves, an octavo; a sheet folded into 12 leaves, a 12mo; a sheet folded into 18 leaves, an 18mo; a sheet folded into 24 leaves, a 24mo.

Note 2. — Of shoemaker's measure, No. 1 of small size is  $4\frac{1}{8}$  inches in length; and No. 1, large size, is  $8\frac{1}{24}$  inches in length; and each succeeding number of either size is  $\frac{1}{8}$  of an inch additional length.

- 1. In 4 bales 4 bundles 1 ream 10 quires of paper how many sheets?

  Ans. 23760.
  - 2. In 23760 sheets of paper how many bales?
  - 3. In 10 fothers 6 pigs 8 stones of iron how many pounds?
  - 4. In 25998 pounds of iron how many fothers?

- 5. At 23 cents a pound, what will 12 firkins of butter cost?
- 6. If \$22 is paid for a barrel of pork, how much is that by the pound?

  Ans. 11 cents.
- 7. How much must be paid for 302 hogsheads of salt at \$0.30 a bushel? Ans. \$724.80.
- 8. At \$4 per quintal, how many pounds of fish may be bought for \$50.24?

  Ans. 1256 pounds.
- 9. If the wholesale price of one writing-book be 4½ cents, what will be the cost of a great-gross of writing-books?
- 10. A dairyman sells 2 firkins of butter at 20 cents a pound, and takes in pay half a barrel of flour at 5 cents a pound, and the balance in cash. How much cash does he receive?

Ans. \$17.50.

### MISCELLANEOUS EXAMPLES IN REDUCTION.

- 1. In 57£. 15s. how many half-pence? Ans. 27720.
- 2. In 59lb. 13pwt. 15gr. how many grains?
- 3. In 340167 grains how many pounds?
- 4. How many ells English in 761 yards?

Ans. 608 E. E. 4qr.

- 5. How many yards in 61 ells Flemish? Ans. 45yd. 3qr.
- 6. How many bottles, that contain 3 pints each, will it take to hold a hogshead of wine?

  Ans. 168.
- 7. How many steps, of 2ft. 8in. each, will a man take in walking from Bradford to Newburyport, the distance being fifteen miles?

  Ans. 29700.
- 8. How many spoons, each weighing 2oz. 12pwt., can be made from 5lb. 2oz. 8pwt. of silver?

  Ans. 24.
- 9. How many times will the wheel of a coach revolve, whose circumference is 14ft. 9in., in passing from Boston to Washington, the distance being 436 miles?

  Ans. 156073
- 10. I have a field of corn, consisting of 123 rows, and each row contains 78 hills, and each hill has 4 ears of corn; now if it takes 8 ears of corn to make a quart, how many bushels does the field contain?

  Ans. 149bu. 3pk. 5qt. 0pt.
- 11. If it take 5yd. 2qr. 3na. to make a suit of clothes, how many suits can be made from 182 yards?
  - 12. A goldsmith wishes to make a number of rings, each N 10

weighing 5pwt. 10gr., from 3lb. 1oz. 2pwt. 2gr. of gold; how many will there be?

Ans. 137.

- 13. How many shingles will it take to cover the roof of a building, which is 60 feet long and 56 feet wide, allowing each shingle to be 4 inches wide and 18 inches long, and to lie one third to the weather?

  Ans. 20160.
- 14. There is a house 56 feet long, and each of the two sides of the roof is 25 feet wide; how many shingles will it take to cover it, if it require 6 shingles to cover a square foot?

Ans. 16800.

- 15. If a man can travel 22m. 3fur. 17rd. a day, how long would it take him to walk round the globe, the distance being about 25000 miles?

  Ans. 11144833 days.
- 16. If a family consume 7lb. 10oz. of sugar in a week, how long would 10cwt. 3qr. 16lb. last them? Ans.  $143_{51}^{5}$  weeks.
  - 17. What will 7 hogsheads of wine cost, at 9 cents a quart?
- 18. What will 15 hogsheads of beer cost, at 3 cents a pint?

  Ans. \$194.40.
- 19. What will 73 bushels of meal cost, at 2 cents a quart?

  Ans. \$46.72.
- 20. A merchant has 29 bales of cotton cloth; each bale contains 57 yards; what is the value of the whole at 15 cents a yard?

  Ans. \$247.95.
- 21. There is a certain pile of wood 120 feet long, 4½ feet high, and 4 feet wide; what is its value at \$4.00 per cord?

  Ans. \$67.50.
- 22. How much must be paid, at twenty cents a square yard, for plastering overhead in a room which is 33 feet long and 18 feet wide?
- 23. An apothecary, in compounding 20 boxes of pills, each box containing 25 pills, used 6 grains of aloes, 5 grains of rhubarb, and 4 grains of calomel in each pill. What was the entire quantity used?

  Ans. 7500 grains.
- 24. Purchased a cargo of molasses, consisting of 87 hogsheads; what is the value of it at 33 cents a gallon?

Ans. \$ 1808.73.

25. If a cubic foot of white-oak wood weighs 880 ounces, and a cubic foot of white-pine wood weighs 480 ounces, how much will a load weigh, which is composed of half a cord of white-oak, and of a cord of white-pine?

Ans. 7360lbs.

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- 26. How many chests of tea, weighing 24 pounds, at 43 cents a pound, can be bought for \$1548?
- 27. Joseph Eldredge received \$10, as a birthday present, from his father, on every 29th day of February, from 1837 to 1857. How much less than \$200 did he receive, in all?

Ans. \$150.

- 28. If 25\frac{1}{25} grains of standard gold be worth \$1, how many pounds avoirdupois of standard gold will be worth \$1,000,000?

  Ans. 3685\frac{1}{25} pounds.
- 29. A merchant, who had bought 188 gallons of molasses, at 40 cents a gallon, intended to have it sold at the rate of 50 cents a gallon; but his shop-boy retailed half of the quantity at 12½ cents a quart, beer measure, when, finding he had made a blunder, he sold the balance at 14 cents a quart, wine measure, thereby expecting to exactly make up for the mistake. How much less did the whole bring than was intended?

  Ans. \$2.86.

### ADDITION OF COMPOUND NUMBERS.

145. Addition of Compound Numbers is the process of finding the amount of two or more compound numbers.

Ex. 1. Required the amount of 31£. 17s. 9d. 2far.; 16£. 16s. 6d. 1far.; 16£. 11s. 11d. 1far.; 19£. 19s. 9d. 3far.; 61£. 17s. 1d. 2far.

Ans. 147£. 3s. 2d. 1far.

		OPER#	TION.	
	£.	8.	d.	fa
	31	17	9	2
	16	16	6	1
	16	11	11	1
	19	19	9	3
	61	17	1	2
Ans.	147	3	2	1

Having written units of the same denomination in the same column, we find the sum of farthings in the right-hand column to be 9 farthings = 2d. 1far. We write the 1far. under the column of farthings, and carry the 2d. to the column of pence; the sum of which is 38d. = 3s. 2d. We write the 2d. under the column of pence, and carry the 3s. to the column of

and carry the 3s. to the column of shillings; the sum of which is  $83s. = 4\pounds$ . 3s. Having written the 3s. under the column of shillings, we carry the  $4\pounds$ . to the column of pounds, and find the entire amount sought to be  $147\pounds$ . 3s. 2d. 1far.

The same result can be arrived at by reducing the numbers as they are added in their respective columns. Thus, beginning with farthings, we can add, in this way: 2far. + 3far. = 5far. = 1d. 1far.;

and 1far. = 1d. 2far., and 1far. = 1d. 3far., and 2far. = 2d. 1far. Writing the 1far. under the column of farthings, we carry the 2d. to the column of pence; and add, 2d. (carried) + 1 = 3d., and 9d. = 12d. = 1s., and 11d. = 1s. 11d., and 6d. = 2s. 5d., and 9d. = 3s. 2d. Writing the 2d. under the column of pence, we carry the 3s. to the column of shillings; and add, 3s. (carried) + 17s. = 20s. = 1£. and 19s. = 1£. 19s., and 11s. = 2£. 10s., and 16s. = 3£. 6s., and 17s. = 4£. 3s. Writing the 3s. under the column of shillings, we carry the 4£. to the column of pounds, and so find the whole amount to be, as before, 147£. 3s. 2d. 1far.

The last method of operation may be rendered more concise, as it should always be in practice, by merely naming results as the adding

is performed (Art. 45).

From the illustrations given, it is evident that the adding of compound numbers is like that of simple numbers, except in carrying; which difference holds also in subtracting, multiplying, and dividing compound numbers.

Rule. - Write all the given numbers, so that units of the same de-

nomination may stand in the same column.

Add as in addition of simple numbers; and carry, from column to column, one for as many units as it takes of the denomination added to make a unit of the denomination next higher.

*Proof.* — The proof is the same as in addition of simple numbers.

EXAMPLES.

			2.						3.		
Ton.	cwt.		lb.	OS.	dr.		cwt.	qr.	lb.	.05.	đr.
61	19	3	17	15	15		61	2	11	. 11	14
63	18	3	16	11	11		16	3	15	15	11
51	1 2	3	17	7	6		41	3	13	9	9
61	16	3 1	11	12	12		38	2	11	10	10
13	1 8	3	1 2	13	15		42	1	9	8	13
7 1	18	2	13	14	14		31	3	17	11	12
324	1 5	2	15	12	9	,					
		4.							5.		
	lb.	oz.	pwt.	gr.			11		05.	pwt.	gr.
	16	11	19	23			12	3	9	7	13
	31	10	1.8	16			9	8	11	17	14
	63	9	12	15			4	9	7	13	21
	17	8	13	12			1	3	10	10	20
(	61	7	12	16			4	7	9	19	23
	17	6	17	22			5	1	5	15	15
2	0 9	7	1.5	8							

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		6	<b>.</b>						7			
	Тb	3	3 <b>Э</b>	gr.			b	3	3	B		gr.
	27	11	7 2	19		3	7	9	6	1	1	. 8
	16	10	6 1	13		1	4	4	4	: 2	1	1
	41	9	3 2	16		6	1	6	3	2		6
	38	10	5 2	14		4	1	4	7	2	1	. 6
	41	4	4 1	11		3	9	8	4	. 1	1	. 2
	16	6	6 2	6		5	1	11	. 7	2	1	. 9
. 1	8 3	6	3 1	19		_						
		8	•						9.			
Deg.	m.	fur.		ft.	in.	m.			rđ.	yd.	· ft.	in.
17	69	7	39	16	11	6 9	) '	7 8	3 1	5	2	11
61	62	3	17	12	9	1 (	3	6 ]	l 6	4	1	6
16	16	6	16	13	10	6 1	l !	7 8	3 2	3	2	10
48	19	3-	15	15	9	7 8	3	3 1	L 6	4	2	9
17	58	6	33	14	7	19	)	4	l 4	1	1	8
33	35	5	19	9	9	7 8	5 ,	5 5	2 5	5	2	7
195	5 5	<del>1</del> 1	24	1	7							
		$\frac{1}{2} = 4$		1	<del>-</del> 6							
195	5 5	5	24	1	1							

Note. — As half a mile is equal to 4 furlongs, we add them to the 1 furlong, which makes 5 furlongs. And as half a foot is equal to 6 inches, we add them to the 7 inches, which makes 13 inches; and these are equal to 1 foot 1 inch.

- 10. Add together 37yd. 3qr. 3na. 2in.; 61yd. 3qr. 1na. 1in.; 13yd. 2qr. 2na. 2in.; 32yd. 1qr. 1na. 1in.; 61yd. 2qr. 2na. 2in.; and 22yd. 1qr. 3na. Ans. 229yd. 3qr. 3na. 1\frac{1}{2}in.
- 11. Add together 671 E.E. 1qr. 1na. 1in.; 161 E.E. 3qr. 3na. 2in.; 617 E.E. 3qr. 1na. 2in.; 178 E.E. 3qr. 2na. 1in.; 717 E.E. 2qr. 1na. 2in.; and 166 E.E. 3qr. 2na. 1in.
- 12. Add together 761A. 3R. 37p. 260ft. 125in.; 131A. 2R. 16p. 135ft. 112in.; 613A. 1R. 14p. 116ft. 131in.; 161A. 3R. 13p. 116ft. 123in.; 321A. 2R. 31p. 97ft. 96in.; and 47A. 3R. 19p. 91ft. 48in.

   Ans. 2038A. 1R. 13p. 2ft. 95in.
- 13. Add together 38A. 1R. 39p. 272ft.; 61A. 3R. 38p. 167ft.; 35A. 3R. 19p. 198ft.; 47A. 3R. 16p. 271ft.; 86A. 2R. 13p. 198ft.; and 46A. 1R. 14p. 269ft.
  - 14. Add together 17m. 7fur. 9ch. 3p. 241.; 16m. 3fur. 4ch. 10\*

1p. 15l.; 27m. 4fur. 6ch. 2p. 17l.; 18m. 6fur. 8ch. 3p. 21l.; 61m. 7fur. 7ch. 2p. 16l.; and 17m. 1fur. 8ch. 2p. 19l.

Ans. 160m. 0fur. 1ch. 1p. 12l.

- 15. Required the sum of 27m. 4fur. 3ch. 1p. 21l.; 29m. 3fur. 1ch. 3p. 23l.; 67m. 3fur. 3ch. 1p. 19l.; 21m. 7fur. 1ch. 3p. 16l.; 16m. 7fur. 9ch. 3p. 13l.; and 31m. 4fur. 8ch. 1p. 20l.
- 16. Required the value of 29T. 36ft. 1279in. + 69T. 19ft. 1345in. + 67T. 18ft. 1099in. + 71T. 14ft. 1727in. + 48T. 35ft. 916in. + 53T. 17ft. 1719in.

Ans. 335T. 23ft. 1173in.

- 17. Add together 61C. 127ft. 1161in.; 37C. 89ft. 1711ia.; 61C. 98ft. 1336in.; 43C. 56ft. 1678in. 91C. 119ft. 1357in. and 81C. 115ft. 1129in.
- 18. Required the value of 61 tuns 3hhd. 62gal. 3qt. 1pt. + 39 tuns 2hhd. 16gal. 1qt. 1pt. + 68 tuns 3hhd. 57gal. 2qt. 1pt. + 87 tuns 3hhd. 45gal. 3qt. 1pt. + 47 tuns 2hhd. 59gal. 3qt. 1pt. + 47 tuns 3hhd. 39gal. 2qt. 1pt.

Ans. 354 tuns 0hhd. 30gal. 1qt.

- 19. Required the value of 67hhd. 15gal. 3qt. 1pt. + 16hhd. 16gal. 3qt. + 39hhd. 16gal. 3qt. + 47hhd. 62gal. 1qt. 1pt. + 43hhd. 57gal. 3qt. + 71hhd. 61gal. 3qt. 1pt.
- 20. Add together of beer measure 161hhd. 53gal. 3qt. 1pt.; 371hhd. 52gal. 3qt. 1pt.; 98hhd. 19gal. 1qt.; 47hhd. 43gal. 1qt. 0pt.; 61hhd. 43gal. 1qt. 1pt.; and 42hhd. 27gal. 3qt. 1pt.
- 21. Find the amount of 37bu. 3pk. 5qt. 1pt. + 61bu. 2pk. 7qt. 1pt. + 32bu. 3pk. 2qt. + 71bu. 1pk. 6qt. 1pt. + 61bu. 1pk. 3qt. 1pt. + 32bu. 3pk. 3qt. 1pt.

Ans. 298bu. 0pk. 4qt. 1pt.

- 22. Required the value of 31bu. 3pk. 3qt. + 31bu. 3pk. 1qt. + 16bu. 3pk. 1qt. + 15bu. 3pk. + 17bu. 3pk. 1qt. + 14bu. 3pk. 1qt.
- 23. Required the value of 57y. 11mo. 27d. 23h. 29m. 55s. + 31y. 11mo. 18d. 19h. 19m. 39s. + 46y. 9mo. 23d. 17h. 28m. 56s. + 43y. 10mo. 16d. 18h. 17m. 48s. + 32y. 9mo. 19d. 16h. 23m. 28s. + 14y. 1mo. 29d. 21h. 28m. 16s.

Ans. 227y. 7mo. 16d. 21h. 28m. 2s.

24. Required the value of 23w. 6d. 28h. 59m. 58s. + 51w. 3d. 18h. 51m. 17s. + 29w. 5d. 21h. 47m. 49s. + 28w. 4d.

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- 23h. 56m. 18s. + 19w. 6d. 10h. 18m. 53s. + 86w. 1d. 29h. 40m. 51s.
- 25. Add together 4S. 29° 59′ 59′′; 6S. 17° 17′ 29″; 11S. 16° 56′ 58″; 9S. 13° 46′ 51″; 5S. 27° 16′ 42″; and 2S. 25° 17′ 17″.

  Ans. 5S. 10° 35′ 16″.
- 26. Add together 11S. 11° 16′ 51″; 6S. 6° 6′ 16″; 9S. 14° 56′ 56″; 3S. 29° 29′ 49″; 9S. 17° 18′ 58″; and 6S. 13° 13′ 52″.

Norz. — We divide the sum of the signs, in the last two questions, by 12, and write down the remainder only. Since the circumference of a circle cannot exceed 12 signs (Art. 143).

27. Bought of a London tailor a vest for  $1\pounds$ . 13s. 4d., a coat for  $7\pounds$ . 12s. 9d., pantaloons for  $2\pounds$ . 3s. 9d., and surtout for  $9\pounds$ . 8s. 0d.; what was the whole amount?

Ans. 20£. 17s. 10d.

28. Bought a silver tankard, weighing 1lb. 8oz. 17pwt. 14gr., a silver can, weighing 1lb. 2oz. 12pwt., a porringer, weighing 1loz. 19pwt. 20gr.; and 3 dozen of spoons, weighing 1lb. 9oz. 15pwt. 10gr.; what was the whole weight?

Ans. 5lb. 9oz. 4pwt. 20 gr.

- 29. What is the weight of a mixture of 3th 43 23 29 14gr. of aloe, 2th 73 63 19 13gr. of picra, and 1th 103 13 29 17gr. of saffron?

  Ans. 7th 103 33 19 4 gr.
- 30. Sold 4 loads of hay; the first weighed 27cwt. 3qr. 18lb.; the second, 31cwt. 1qr. 15lb.; the third, 19cwt. 1qr. 15lb.; and the fourth, 38cwt. 2qr. 24lb.; what is the weight of the whole?
- 31. Bought 5 pieces of broadcloth; the first contained 17yd. 3qr. 2na.; second, 13yd. 2qr. 1na.; the third, 87yd. 1qr. 3na.; the fourth, 27yd. 1qr. 2na.; and the fifth, 29yd. 1qr. 2na.; what was the whole quantity purchased?

Ans. 175yd. 2qr. 2nc.

- 32. A pedestrian travelled, the first week, 371m. 3fur. 37rd. 5yd. 2ft. 10in.; the second week, 289m. 2fur. 18rd. 3yd. 1ft. 9in.; and the third week, 399m. 7fur. 3ft. 11in.; how many miles did he travel?

  Ans. 1060m. 5fur. 16rd. 5yd. 1ft.
- 33. A man has three farms; the first contains 186A. 3R. 14p.; the second, 286A. 17p.; and the third, 115A. 2R.; how much do they all contain?

  Ans. 588A. 1R. 31p.



34. The Moon is 5S. 18° 14′ 17" east of the Sun; Jupiter is 7S. 10° 29′ 28" east of the Moon; Mars is 11S. 12° 11′ 56" east of Jupiter; and Herschel is 7S. 18° 38′ 15" east of Mars; how far is Herschel east from the Sun?

Ans. 7S. 29° 33′ 56″.

### SUBTRACTION OF COMPOUND NUMBERS.

146. Subtraction of Compound Numbers is the process of finding the difference between two compound numbers.

Ex. 1. From 617£. 11s. 8d. take 181£. 15s. 5d.

Ans. 435£. 16s. 3d.

OPERATION.								
	£	8.	d.					
Min.	617	11	8					
Sub.	181	15	5					
Rem.	435	16	3					

Having placed the less number under the greater, pence under pence, shillings under shillings, &c., we begin with pence, thus: 5d. from 8d. leaves 3d., which we set under the column of pence. As we cannot take 15s. from 11s., we add 20s. = 1£. to

the 11s., making 31s., and then subtract the 15s. from it, and set the remainder, 16s., under the column of shillings. Then, having added  $1\pounds$ . = 20s. to the  $181\pounds$ ., to compensate for the 20s. added to the 11s. in the minuend, we subtract the pounds as in subtraction of simple numbers, and obtain  $435\pounds$ . for the remainder, and as the result complete,  $435\pounds$ . 16s. 3d.

Rule. — Write the less compound number under the greater, so that units of the same denomination shall stand in the same column.

Subtract as in subtraction of simple numbers.

If any number in the subtrahend is larger than that above it, add to the upper number as many units as make one of the next higher denomination before subtracting, and carry one to the next lower number before subtracting it.

*Proof.*—The proof is the same as in simple subtraction.

	2.		3.	
£	s.	d. •	£ s. d.	
87	16	3 <u>1</u>	617 11 5	ļ
19	17	9 1	181 15 8	ł
67	18	5 <del>3</del>		_

4.										<b>5</b> .			
T	٠. ر	wt.	qr.	lb.	oz.	dr.		cw	t. q	r. lb.		z.	
7	1 1	18	1	13	1	13		7 8	3 1	1 8	5 1	3	•
1	9 1	t 9	2	16	8	5		19	) 1	19	1	5	
5	1 ]	18	2	21	9	8				•		_	
			6.							7.			
	lb.	O		pwt.	gr.			Ib.	. (	ж. рч	rt. e	ŗ.	
	71		3	12	15			5 8	3 5	1 1 2	2 1	0	
	16	1	0	17	20			1 9	9	17	7 2	1	
	54		4	14	19							_	
			8.							9.			
	ib 71	3 1	3 3	Ð 1	gr. 14			<b>fb</b> 1 5	3 2	3 E		7. 5	
	18	6	7	2	19			9	9	1 1	. 1	8	
	<b>52</b>	6	3	1	15		,	•				_	
			10.							11.			
m.	fur.	rd.		t.	in.		dog.	m.	. far.		7d.	ſt.	ín.
6	7	18	;	3	1		38	41	3	29	2	1	7
9	7	19	1	6	-8		29	36	5	<b>31</b>	8	1	9
6	7	38		21	5								
				<del>1</del> =	= 6								
6	7	38		2	11			•					

NOTE. — As half a foot is equal to 6 inches, we add them to the 5 inches, which make 11 inches.

- 12. From 67yd. 1qr. 1na. 1in. take 18yd. 2qr. 2na. 2in. Ans. 48yd. 2qr. 2na. 14in.
- 13. From 51E.E. 2qr. 3na. take 19E.E. 3qr. 1na.

1

- 14. From 56A. 1R. 19p. 119ft. 110in. take 17A. 3R. 18p. 127ft. 113in. Ans. 38A. 2R. 5p. 264ft. 33in.
- 15. Find the value of 13A. 1R. 15p. 19yd. 1st. 17in. 9A. 3R. 16p. 30yd. 5st. 17in.
- 16. Subtract 19m. 2fur. 1ch. 3p. 21l. from 21m. 1fur. 3ch. 2p. 19l. Ans. 1m. 7fur. 1ch. 2p. 23l.
- 17. From 28m. 6fur. 1ch. 2p. 18l. take 15m. 7fur. 8ch. 1p. 19l.
- 18. Required the value of 49T. 13ft. 1611in. 18T. 15ft. 1719in. Ans. 30T. 37ft. 1620in.

- 19. Required the value of 361C. 47ft. 1178in. 197C. 121ft. 1617in.
- 20. Subtract 11tun 1hhd. 28gal. 2qt. 1pt. from 79tun 3hhd. 19gal. 1qt. 1pt. Ans. 68tun 1hhd. 53 gal. 3qt.
- 21. Subtract of beer measure 191hhd. 19gal. 3qt. from 769hhd. 18gal. 1qt.
  - 22. From 56ch. 2bu. 1pk. take 38ch. 3bu. 1pk.

Ans. 17ch. 35bu.

- Required the value of 25bu. 3pk. 1qt. 12bu. 3pk. 5qt.
   Ans. 12bu. 3pk. 4qt.
- 24. Required the difference between 6mo. 16d. 13h. 27m. 19s. and 1mo. 22d. 16h. 41m. 37s.

Ans. 4mo. 23d. 20h. 45m. 42s.

- 25. From 48y. 0mo. 15d. 19h. 27m. 31s. take 19y. 10mo. 29d. 21h. 38m. 56s.
  - 26. From 6S. 11° 12' 48" subtract 9S. 8° 15' 56".

Ans. 9S. 2° 56′ 52″.

- 27. Take 1S. 22° 19′ 28″ from 4S. 19° 41′ 22″.
- 28. I have 73A. of land; if I should sell 5A. 3R. 1p. 7ft., how much should I have left? Ans. 67A. 0R. 38p. 2651ft.
- 29. A owes B 100£.; what will remain due after he has paid him 3s.  $6\frac{1}{2}$ d.?

  Ans. 99£. 16s.  $5\frac{1}{2}$ d.
- 30. It is about 25,000 miles round the globe; if a man shall have travelled 43m. 17rd. 9in., how much will remain to be travelled?

  Ans. 24,956m. 7fur. 22rd. 15ft. 9in.
- 31. Bought 7 cords of wood; 2 cords 78ft. having been stolen, how much remained?
- 32. I have 15 yards of cloth; having sold 3yd. 2qr. 1na., what remains?

  Ans. 11yd. 1qr. 3na.
- 33. If a wagon loaded with hay weighs 43cwt. 2qr. 18lb., and the wagon is afterwards found to weigh 9cwt. 3qr. 23lb., what is the weight of the hay?

  Ans. 33cwt. 2qr. 20lb.
- 34. Bought a hogshead of wine, and by an accident 8gal. 3qt. 1pt. leaked out; what remains?
- 35. I had 10A. 3R. 10p. of land; and I have sold two houselots, one containing 1A. 2R. 13p., the other 2A. 2R. 5p.; how much have I remaining?

  Ans. 6A. 2R. 32p.
- 36. The Moon moves 13° 10′ 35" in a solar day, and the Sun 59′ 8"; now supposing them both to start from the

same point in the heavens, how far will the Moon have gained on the Sun in 24 hours?

Ans. 12° 11′ 27″.

37. A farmer raised 136bu. of wheat; if he sells 49bu. 2pk. 7qt. 1pt., how much has he remaining?

Ans. 86bu. 1pk. 0qt. 1pt.

38. If from a stick of timber containing 2T. 18ft. 1410in. there be taken 38ft. 1720in., how much will be left?

Ans. 1T. 19ft. 1418in.

### MULTIPLICATION OF COMPOUND NUMBERS.

- 147. Multiplication of Compound Numbers is the process of taking a compound number any proposed number of times.
- Ex. 1. What will 6 bales of cloth cost, at 7£. 12s. 7d. per bale?

  Ans. 45£. 15s. 6d.

Multiplicand 7 12 7
Multiplier 6
Product 45 15 6

Having written the multiplier under the lowest denomination of the multiplicand, we multiply thus: 7d. × 6 = 42d. = 3s. 6d. We write the 6d. under the number multiplied, and reserve the 3s. to be added to the pro-

duct of the shillings. Then,  $12s. \times 6 = 72s.$ , and 8s. (carried) =  $75s. = 3\pounds$ . 15s. We write the 15s. under the column of shillings, and reserve the  $3\pounds$ . to be added to the product of the pounds. Again,  $7\pounds. \times 6 = 42\pounds$ ., and  $3\pounds.$  (carried) =  $45\pounds$ . This, placed under the column of pounds, gives  $45\pounds$ . 15s. 6d.

RULE. — Multiply each denomination of the compound number as in multiplication of simple numbers, and carry as in addition of compound numbers.

Proof. — Write down by themselves the several products obtained by multiplying each denomination of the multiplicand by the multiplier, and these partial products added together will equal the entire product, if the work be right. (Art. 60.)

NOTE. — Going a second time carefully over the work is a good way of testing its accuracy. On learning Division of Compound Numbers, the pupil will find that rule a better method of proving multiplication of compound numbers.

### EXAMPLES.

2. Multiply 1£. 8s. 7d. 2far. by 7. Ans. 10£. 0s. 4d. 2far.

	OPE	RATION	r.	PROOF BY ADDITION.							
	£. s. d.										
	2	8	91	9 <b>3</b> d.	× 8	-	76d.	=	0£.	<b>6s.</b>	<b>4</b> d.
			8	8s.	× 8	==	64s.	-	3£.	4s.	0d.
Ans.	19	10	4	2£.	× 8	-	16£.	=	16£.	0s.	0d.
				2	£. 8s.	9 <u>1</u> d	l. 🗙 8	_	19£.	10s.	4d.

NOTE. — The answers to the following examples may be found in corresponding numbers of examples in Division of Compound Numbers.

3.	4.						
T. cwt. qr. lb. cs. dr. 61 19 3 17 15 15	1b. ez. pwt. gr. 7 11 14 15 5						
<b>5.</b> .	6.						
1b. cz. pwt. gr. 32 8 17 12	ib 3 3 9 sr. 38 10 5 2 14						
8	.11						
7.	8.						
deg. m. fur. rd. ft. in.	- m. fur. ch. p. 1.						
71 38 2 13 14 4	17 7 9 3 23						
1 2	12						

- 9. Multiply 16A. 2R. 4p. 19yd. 7ft. 79in. by 11.
- 10. Multiply 10yd. 3qr. 3na. by 5.
- 11. Multiply 17tun 2hhd. 50gal. 1qt. by 7.
- 12. Multiply 29hhd. 61gal. 3qt. 1pt. 3gi. by 7.
- 13. Multiply 19bu. 2pk. 7qt. 1pt. by 6.
- 14. What is the value of 13y. 316d. 15h. 27m. 39s.  $\times$  8?
- 15. Multiply 16deg. 39m. 8fur. 39rd. 5yd. 2ft. by 9.
- 16. If a man gives each of his 9 sons 23A. 3R. 194p., what do they all receive?
- 17. If 12 men perform a piece of labor in 7h. 24m. 30s., how long would it take 1 man to perform the same task?
- 18. If 1 bag contain 3bu. 2pk. 4qt., what quantity do 8 bags contain?

- 148. When the multiplier is a composite number, and none of its factors exceed 12.
- Ex. 1. What will 35 loads of coal weigh, if each load weighs 2T. 1cwt. 2qr. 6lb.?

	OPERAT	ion.		We find the num-
T.	cwt.	qr.	1b.	
2	1	2	6 =  weight of 1 load.	ber 35 equal to the product of 7 and 5; we therefore multiply
14	10	3	$\overline{17}$ = weight of 7 loads.	by 7, and then that
72	14	2	$\overline{10}$ = weight of 35 loads.	product by 5; and the last product is the answer. Hence, when
the :	multipl	ier i	s a composite number,	

Multiply by its factors in succession.

- 2. Bought 90 hogsheads of sugar, each weighing 12cwt. 2qr. 11lb.; what was the weight of the whole?
  - 3. What cost 18 sheep at 5s. 91d. apiece?
  - 4. What cost 21 yards of cloth at 9s. 11d. per yard?
  - 5. What cost 22 hats at 11s. 6d. each?
- 6. If 1 share in a certain stock be valued at 13£. 8s. 9½d., what is the value of 96 shares?
- 7. If 1 spoon weighs 3oz. 5pwt. 15gf., what is the weight of 120 spoons?
- 8. If a man travel 24m. 7fur. 4rd. in 1 day, how far will he go in 1 month?
- 9. If the earth revolve 0° 15' per minute, how far does it revolve per hour?
  - 10. Multiply 39A. 3R. 17p. 30yd. 8ft. 100in. by 32.
- 11. If a man be 2d. 5h. 17m. 19sec, in walking 1 degree, how long would it take him to walk round the earth, allowing 3651 days to a year?
- 149. When the multiplier is not a composite number, and exceeds 12; or when a composite number one of whose factors exceeds 12.
- E2. 1. What is the value of 453 tons of iron at 18£. 17s. 11d. a ton?

2. s. d. f. s. d. 1 ton = 
$$18\ 17\ 11, \times 3 = 56\ 13\ 9 =$$
 Value of 8 tons.  $10$ 

10 tons =  $10$ 

Since 453 is not a composite number, we cannot resolve it into factors; but we may separate it into parts, and find the value of each part separately: Thus, 453 = 400 + 50 + 3. In the operation, we first multiply by 10, and obtain the value of 10 tons, and this product we multiply by 10, and obtain the value of 100 tons. Then, to find the value of 400 tons, we multiply the last product by 4; and to find the value of 50 tons, we multiply the value of 10 tons by 5; and to find the value of 3 tons, we multiply the value of 1 ton by 3. Adding the several products, we obtain 8559 £. 16s. 3d. for the answer. Hence,

Having resolved the multiplier into any convenient parts, as of units, tens, &c., multiply by these several parts, and add together the products thus obtained for the required result.

- 2. Multiply 2hhd. 19gal. 0qt. 1pt. by 39.
- 3. Multiply 3bu. 1pk. 4qt. 1pt. 1gi. by 53.
- 4. Multiply 16ch. 7bu. 2pk. 0qt. 0pt. by 17.
- 5. What will 57 gallons of wine cost at 8s. 31d. per gallon?
- 6. Bought 29 lots of wild land, each containing 117A. 3R. 27p.; what were the contents of the whole?
- 7. Bought 89 pieces of cloth, each containing 37yd. 3qr. 2na. 2in.; what was the whole quantity?
- 8. Bought 59 casks of wine, each containing 47gal. 3qt. 1pt.; what was the whole quantity?
- 9. If a man travel 17m. 3fur. 13rd. 14ft. in one day, how far will he travel in a year?
- 10. If a man drink 3gal. 1qt. 1pt. of beer in a week, how much will he drink in 52 weeks?
- 11. There are 17 sticks of timber, each containing 87ft. 978in.; what is the whole quantity?
- 12. There are 17 piles of wood, each containing 7 cords 98 cubic feet; what is the whole quantity?

## DIVISION OF COMPOUND NUMBERS.

- 150. DIVISION of Compound Numbers is the process of dividing compound numbers into any proposed number of equal parts.
- Ex. 1. Divide 139£. 13s. 11d. 2far. equally between 5 persons.

  Ans. 27£. 18s. 9d. 2far.

Having divided 139£. by 5, we find the quotient to be 27£., and 4£. remaining. We place the quotient 27£. under the 139£., and the remainder 4£. reduced to shillings = 80s.; 80s. + the 13s. in the dividend = 93s.;

93s.  $\div$  5 = 18s. and a remainder of 3s. We write the quotient 18s. under the shillings in the dividend; and the remainder 3s. reduced to pence = 36d.; 36d. + 11d. in the dividend = 47d.; 47d.  $\div$  5 = 9d. and a remainder of 2d. We write the quotient 9d. under the pence in the dividend; and the remainder 2d. reduced to farthings = 8far., + the 2far. in the dividend = 10far.; 10far.  $\div$  5 = 2far. The quotient 2far. we write under the farthings in the dividend; and thus find the answer to be 27£. 18s. 9d. 2far.

Rule. — Divide as in division of simple numbers, each denomination in its order, beginning with the highest.

If there be a remainder, reduce it to the next lower denomination, adding in the number already contained in the dividend of this denomination, if any, and divide as before.

PROOF. — The same as in simple numbers.

Note. — When the divisor and dividend are both compound numbers, they must be reduced to the same denomination, and the division then is that of simple numbers.

#### EXAMPLES.

NOTE. — The answers to the following examples are found in the corresponding numbers of examples in Multiplication of Compound Numbers.

6.						7.						
$11) \frac{1}{427}$	<b>5</b>	<b>3</b> 0	9 2	gr. 14	1 2	deg ) 8 5	8	m. 44	fur. 4	rd.,	n. 7	<b>in.</b>
i 2) 2 1 5	8. fur. el 7 9	n. p.	ı. 1		11)	A. 181	R. 3	9. 1	. y 1 (	d. 1	r. 4	in. 41

- 10. Divide 54vd. 2gr. 3na. equally among 5 persons.
- 11. Divide 123tun 3hhd. 36gal. 3qt. by 7.
- 12. Divide 209hhd. 55gal. 3qt. 0pt. 1gi. by 7.
- 13. What is the value of 118bu. 1pk. 5qt. ÷ 6?
- 14. What is the value of 110y. 343d. 3h. 41m. 12s.  $\div$  8?
- 15. Divide 149deg. 9m. 5fur. 13rd. 3yd. 1ft. by 9.
- 16. A man divides his farm of 214A. 3R. 12p. equally among his 9 sons; how much does each receive?
- 17. If one man perform a certain piece of labor in 3d. 16h. 54m., how long would it take 12 men to perform the same work?
- 18. A farmer has 29 bushels of oats which he wishes to put in 8 sacks; how much must each sack contain?
- 151. When the divisor is a composite number, and none of its factors exceed 12.
- Ex. 1. If 35 loads of coal weigh 72T. 14cwt. 2qr. 10lb., what will 1 load weigh?

We find the factors of 35 to be 5
5) 72 14 2 10 = weight of 35 loads.
7) 14 10 3 17 = weight of 7 loads.
2 1 2 6 = weight of 1 load.
We find the factors of 35 to be 5 and 7. We therefore divide the weight of 35 loads by 5, and obtain the weight of 7

loads; and the weight of 7 loads we divide by 7, and thus find the weight of 1 load. Hence, when the divisor is a composite number,

Divide by its factors in succession.

- 2. If 90 hogsheads of sugar weigh 56T. 14cwt. 3qr. 15lb., what is the weight of 1 hogshead?
  - 3. What will be the price of 1 sheep, if 18 cost 5£. 4s. 3d.?

- 4. If 21 yards of cloth cost 10£. 8s. 3d., what is the price of 1 yard?
  - 5. What is the value of 1 hat, when 22 cost 12£. 13s. 0d.?
- 6. When 96 shares of a certain stock are valued at 1290£.
  4s. Od., what would be the cost of I share?
- 7. If 120 spoons weigh 32lb. 9oz. 15pwt., what does 1 weigh?
- 8. If a man in 1 month travels 746m. 5fur., how far does he go in 1 day?
- 9. If the earth revolves 15° on its axis in 1 hour, how far does it revolve in 1 minute?
- 10. Divide 1275A. 2R. 16p. 22yd. 8ft. 32in. equally among 32 men.
- 11. If a man walk round the earth in 2y. 68d. 19h. 54m., how long would it take him to walk 1 degree, allowing 365½ days to a year?
- 152. When the divisor is not a composite number, and exceeds 12, or when a composite number one of whose factors exceeds 12, the whole operation can be written, as in the following example.
  - Ex. 1. Divide 360£. 8s. 4d. by 173. Ans. 2£. 1s. 8d.

OPERATION.

s. d.

173) 360 8 4 (2£.

346

14

20

173) 288 (1s.

173

115

12

173) 1384 (8d.

1384

We divide the pounds by 173, and obtain  $2\pounds$ . for the quotient, and  $14\pounds$ . remaining, which we reduce to shillings, and add the 8s., and again divide by 173, and obtain 1s. for the quotient. The remainder, 115s., we reduce to pence, and add the 4d., and again divide by 173, and obtain 8d. for the quotient. Thus, the method is the same as by general rule (Art. 150). By writing the several quotients, we obtain  $2\pounds$ . 1s. 8d. for the answer.

- 2. Divide 89hhd. 52gal. 3qt. 1pt. by 39.
- 8. Divide 179bu. 8pk. 5qt. 0pt. 1gi. by 53.
- 4. Divide 275ch. 19bu. 2pk. equally among 17 men.

- 5. If 57 gallons of wine cost 23£. 11s. 5½d., what cost 1 gallon?
  - 6. Divide 3419A. 2R. 23p. by 29.
- 7. If 89 pieces of cloth contain 3375yd. 3qr. 1na. 01in., how much does 1 piece contain?
- 8. If 59 casks contain 44hhd. 52gal. 2qt. 1pt. of wine, what are the contents of 1 cask?
- 9. If a man travel in 1 year (365 days) 6357m. 5fur. 14rd. 11½ft., how far is that per day?
- 10. When 175gal. 2qt. of beer are drunk in 52 weeks, how much is consumed in 1 week?
- 11. When 17 sticks of timber measure 15T. 38ft. 1074in., how many feet does 1 contain?
  - 12. Divide 132 cords 2ft. by 17.
  - 13. Divide 697T. 18cwt. 3qr. 14lb. by 146.

Ans. 4T. 15cwt. 2qr. 10+28lb.

14. Divide 916m. 3fur. 30rd. 10ft. 6in. by 47.

Ans. 19m. 3fur. 39rd. 13ft. 234in.

- 15. Divide 718A. 3R. 37p. by 29. Ans. 24A. 3R. 633p.
- 16. Divide 815A, 1R, 17p, 200ft. by 87.

Ans. 9A. 1R. 19p. 13985ft.

17. Divide 144A. 3R. 18p. 3yd. 1ft. 36in. by 11.

Ans. 13A. 0R. 27p. 3yd. 0ft. 45<sub>17</sub>in.

# PRINCIPLES AND APPLICATIONS.

### DIFFERENCE BETWEEN DATES.

- 153. To find the time between two different dates.
- Ex. 1. What is the difference of time between May 16, 1819, and March 4, 1857?

  Ans. 37y. 9mo. 18d.

FIRST OPERATION.									
	у.	mo.	d.						
Min.	1857	2	4						
Sub.	1819	4	16						
Rem.	37	9	18						

Commencing with January; the first month in the year, and counting the months and days in the later date up to March 4th, we find that 2mo. and 4d. have elapsed. We therefore write the numbers for subtraction as in the first operation.

Min. 1857 3 4
Sub. 1819 5 16
Rem. 37 9 18

The same result, however, could be obtained, as some prefer, by reckoning the number of the given months instead of the number of months that have elapsed since the beginning of

the year, which would require the numbers to be written as in the second operation. Written either way, the earlier date, being placed under the later, is subtracted from it.

Note: — In finding the difference between two dates, and in computing interest for less than a month, 30 days are considered a month. In legal transactions, however, a month is reckoned from any day in one month to the corresponding day of the following month, if it has a corresponding day, otherwise to its end. The above process, which is that ordinarily used by business men, does not give always the exact time between two different dates. The result obtained by it may deviate sometimes a day, and, less often, two days, from the exact difference. But for practical purposes it is generally regarded as sufficiently accurate.

### EXAMPLES.

- 2. What is the time from June 3d, 1854, to April 19th, 1857?
- 3. A note was given October 26th, 1856, and paid June 12th, 1857; how long was it on interest?
- 4. The Pilgrims landed at Plymouth December 22d, 1620, N. S., and the Declaration of Independence was made July 4th, 1776; what is the difference of time between these events?
- 5. General Washington was born February 22d, 1732, and died December 14th, 1799; how long did he live?

Ans. 67y. 9mo. 22d.

154. To find the exact number of days between two different dates.

Ex. 1. How many days from January 28 to July 30, common year?

Ans. 183 days.

January to July = 6mo.  $6 \times 31 = 186$  days. For Feb. 3d., April 1d., June 1d.,  $3+1+1=\frac{5}{181}$  "

For difference between 30 and 28, 30 - 28 = 2Ans. 183 days.

The difference between January and July we find to be 6mo., which, multiplied by 31, the greatest number of days in any month in the year, gives 186 days. But since in the interval of time included between the given dates several months end that do not con-

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tain 31 days, we make deductions for these, which, in all, amount to 5 days, and have left 181 days; and as the difference between the given dates is the difference between 30 and 28 more than exactly 6mo., we add 2 to the 281 days, thus obtaining 183 days, the difference of time required. Hence, to find the number of days between two different dates.

Find the number of months ending between the given dates, and multiply that number by 31, and from the product make the necessary deduction for the months counted that do not contain 31 days, if any. Should the later date end later in the month than the earlier, add the difference of days; but should it end earlier, subtract the same.

Note. — The exact difference in days between two different dates can also be obtained by use of the table in Note 2, Art. 142.

### EXAMPLES.

- 2. How many days has a note to run dated November 15, 1856, and made payable February 13, 1857? Ans. 90 days.
  - 3. How many days from June 18, 1855, to May 1, 1856?
- 4. How many days from March 4 to May 3 of the same year?
- 5. From November 4, 1856, to April 4, 1857, how many days?

  Ans. 151 days.
- 6. In a leap year, how many days are there from the 7th of January to the 11th of December?

  Ans. 339 days.
- 155. To find the day of the week corresponding to any given day of the month, when the day of the week of some other date is given.
- Ex. 1. If the 16th day of May be on Saturday, what day of the week will the next 25th of December be?

Ans. Friday.

OPERATION.

From May 16 to December 25 = 223 days. 223 days  $\div 7 = 31$  weeks and 6 days. 6 days after Saturday = Friday, Ans.

Having found the difference of time in days between the given dates, we bring the days to weeks by dividing by 7, and obtain 31 weeks and 6 days. The 25th of December, therefore, must come 6 days after Saturday, or on Friday. Hence, we

Reduce the days between the given dates to weeks. Should there be no remainder, the day given will be the same as that sought, but should there be a remainder, it will indicate the number of days that the day sought is after the day given.

## EXAMPLES.

- 2. If the 2d day of April be on Wednesday, what day of the week will the following 4th of July be? Ans. Friday.
- 3. If a leap year commence on Tuesday, on what day will the 17th of June, the anniversary of the battle of Bunker Hill, happen?
- 4. If in a common year the 25th of December, or Christmas, be on Tuesday, on what day did the year commence?

Ans. Monday.

- 5. If the 4th day of November be on Tuesday, what day of the next February will be the second Monday of that month?

  Ans. The 9th.
- 6. A bill was dated on Thursday, December 20th, 1855, and made payable 90 days after date. In what year and month, and on what day of the month and week, did it become due?

  Ans. Wednesday, March 19, 1856.

### DIFFERENCE OF LATITUDE.

156. LATITUDE is the distance of any place from the equator, north or south. It is reckoned in degrees, minutes, and seconds, from the equator to either pole of the earth; and cannot exceed 90 degrees, or one fourth of the earth's circumference. Places north of the equator are said to be in north latitude, and those south of the equator, in south latitude.

Note. — The shape of the earth not being that of a perfect sphere, but somewhat flattened toward the poles, the degrees of latitude differ a little from each other in length toward either pole. Thus the 1st degree is about  $68\frac{9}{10}$  statute miles in length; the 40th degree, about  $68\frac{98}{100}$  miles; and the 89th degree about  $69\frac{98}{100}$  miles.

- 157. To find the difference of latitude of any two places.
- Ex. 1. The latitude of London is 51° 31' north, and that of Boston is 42° 23' north. What is their difference of latitude?

  Ans. 9° 8'.

OPERATION.

Lat. of London =  $5 \cdot 1^{\circ} \cdot 3 \cdot 1' \cdot N$ . Lat. of Boston =  $4 \cdot 2^{\circ} \cdot 2 \cdot 3' \cdot N$ .

Dif. of Latitude = 9° 8' N.

The two places being both in north latitude, we subtract the less latitude from the greater. If the latitude of the one place had been north and that of

the other south, we should have added the two latitudes together for the difference required. Hence

When the latitudes of two places are either both north or both south, subtract the less latitude from the greater for their difference; and when the latitude of the one place is north and the other south, add the latitudes together for their difference.

Note. — In north latitude, when the sailing is southerly, or in south latitude, when the sailing is northerly, if the difference of latitude be subtracted from the latitude LEFT, the remainder will be the latitude IN; or, in north latitude, when the sailing is northerly, or in south latitude, when the sailing is southerly, if the difference of latitude be added to the latitude LEFT, the sum will give the latitude IN.

### EXAMPLES.

- 2. The latitude of Quebec is 46° 48' north, and that of New Orleans 29° 57' north. What is their difference of latitude?

  Ans. 16° 51'.
- 3. The latitude of Washington City is 38° 53' north, and that of Cape Horn 55° 58' south. What is their difference of latitude?

  Ans. 94° 51'.
- 4. Valparaiso is in latitude 33° 2' south, and San Francisco 37° 48' north. What is their difference of latitude?
- 5. Captain James Francis, sailing southerly from New York City, whose latitude is 40° 42′ north, found on reaching Havana that his latitude differed 17° 33′ from that he left. What latitude was he then in?

  Ans. 23° 9′ north.
- 6. Philadelphia is 9° 15′ of latitude north of Mobile, whose latitude is 30° 41′ north; what is the latitude of Philadelphia?

  Ans. 39° 56′ north.

# DIFFERENCE OF LONGITUDE.

158. Longitude is the distance of any place from a given meridian, east or west. It is reckoned in degrees, minutes, and seconds, and cannot exceed 180 degrees, or one half of a circle.

Note 1.—A degree of longitude on the equator is about  $69\frac{1}{6}$  statute miles, and, in general, a degree is  $\frac{1}{360}$  of any circle of latitude. The meridians all converge from the equator to the poles to a point, so that the degrees of longitude under different parallels of latitude vary, diminishing with the circles of latitude, till at the poles the longitude becomes nothing. (Art. 133, Note 4.)

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- NOTE 2. The mariners of Great Britain and the United States reckon longitude from the meridian of Greenwich in England, as do the nautical books of both countries for the most part. American maps, however, very generally have longitude reckoned both from the meridian of Greenwich, and from the meridian of Washington.
  - 159. To find the difference of longitude of any two places.
- Ex. 1. What is the difference of longitude between Boston, which is 71° 4′ west, and Detroit, which is 82° 58′ west.

Ans. 11° 54′.

OPERATI	The			
Long. of Detroit	===	8 2°	5 8′	both in
Long. of Boston	-	7 1°.	4′	subtract from the
Dif. of Longitude		1 1°	5 4'	ever. on

The two places being both in west longitude, we subtract the less longitude from the greater. If, however, one of the places had been in east longitude, and

the other in west, we should have added the two longitudes together for the difference required. Hence,

When the longitudes of two places are either both east or both west, subtract the less longitude from the greater for their difference; and when the longitude of the one place is east, and that of the other west, add the longitudes together for their difference.

Note. — In adding together two longitudes, should their sum exceed 180 degrees, subtract it from 360 degrees, and the remainder will be the correct difference of longitude.

- 2. What is the difference of longitude between the city of Washington, whose longitude is 77° 16′ west, and Paris, whose longitude is 2° 20′ east?

  Ans. 79° 36′.
- 3. The United States extend from the St. Croix River, longitude 67° 2′ west, to Cape Flattery, longitude 124° 43′ west. Over how many degrees of longitude does the Union extend?
- 4. What is the difference of longitude between Raleigh, whose longitude is 78° 48' west, and Sacramento City, whose longitude is 120° west?

  Ans. 41° 12'.
- 5. What is the difference of longitude between Hartford, whose longitude is 72° 40′ west, and Fort Leavenworth whose longitude is 94° 44′ west?
- 6. The longitude of Honolulu is 157° 52' west, and that of Canton 113° 14' east. What is their difference of longitude?

  Ans. 88° 54'.

### LONGITUDE AND TIME.

- 160. To find the time corresponding to degrees and minutes of longitude.
- Ex. 1. The difference of longitude between Boston and London being 71° 4′, what is their difference of time?

Ans. 4h. 44m. 16sec.

Dif. of longitude = 71° 4′ 4

Dif. of time = 284m. 16sec. 284m. 16sec. = 4h. 44m. 16sec. Ans. Since 1' of longitude corresponds to 4sec. of time, and 1° of longitude to 4m. of time (Art. 143, Note 2), 4' corresponds to 16sec., and 71° to 284m.; and 284m. 16sec.

= 4h. 44m. 16sec., the answer required. The apparent motion of the sun being west, the time at Boston is as much earlier than that of London as the difference of time between them. Thus, when it is noon at London, it wants 4h. 44m. 16sec. of noon at Boston (Art. 143, Note 2). Therefore, to find the time corresponding to degrees and minutes of longitude,

Multiply the minutes of longitude by 4, for seconds of time. Multiply the degrees of longitude by 4, for minutes of time.

Note 1.— Should the seconds of time, when found, be 60 or more, they may be reduced to minutes; and should, also, the minutes be 60 or more, they may be reduced to hours. The difference of time between two places is exactly as much as the true clock time of the one is fast or slow as compared with that of the other.

#### EXAMPLES.

- 2. Galveston in Texas is 14° 43' west of Pittsburg. When it is 12 o'clock at Galveston, what is the time at Pittsburg?
- 3. The longitude of Valparaiso is 71° 37′ west, and the longitude of Rome is 20° 30′ east. When it is 11h. 15m. A. M. at Valparaiso, what is the time at Rome?

Ans. 23m. 28sec. past 5 o'clock, P. M.

4. The longitude of Jerusalem is 35°, 32′ east, and the longitude of Baltimore 76° 37′ west. When it is 9 o'clock, A. M. at Jerusalem, what time is it at Baltimore?

Ans. 1h. 31m. 24sec. A. M.

161. To find the longitude corresponding to hours, minutes and seconds of time.

Ex. 1. The difference of time between Boston and London is 4h. 44m. 16sec.; what is the difference of longitude?

Ans. 71° 4'.

Since 1h. of time corresponds to 15° of longitude, 4m. of time to 1° of longitude, and 4sec. of time to 1' of longitude (Art. 143, Note 2), 4h. corresponds to 60°, 44m. to 11°, and 16sec. to 4'; and 60° + 11° + 4' = 71° 4', the answer required.

Hence, to find the longitude corresponding to hours, minutes, and seconds of time,

Multiply the hours of time by 15, and divide the minutes of time by 4, for degrees of longitude; and divide the seconds of time by 4, for minutes of longitude. The several results added together will be the difference of longitude.

- 2. The difference of time between Washington and Cincinnati is 29m. 36sec.; what is the difference of longitude?
- 3. A ship-captain sailing from New York to Europe, after being at sea some days, on taking an observation, found that the sun at noon was 2h. 20m. 40sec. earlier than the New York time, as shown by his chronometer. How many degrees east had he sailed?

  Ans. 35° 10′.
- 4. A gentleman travelling west from Philadelphia, whose longitude is 75° 10′ west, found, on arriving at St. Louis, that his watch, an accurate timekeeper, which was right when he left Philadelphia, was 1h. 20sec. earlier than the time at St. Louis. What, then, is the longitude of St. Louis? Ans. 90° 15′ west.
- 5. The difference of time between Baltimore and New Orleans is 53m. 30sec.; what is the difference of longitude?
- 6. When it is noon at St. Paul's, longitude 93° 5′ west, it is at Bangor 1h. 37m. 12sec. P.M.; what is the longitude of Bangor?

  Ans. 68° 47′ west.
- 7. When it is 11 A. M. at a place 30° east of Greenwich, it is 3h. 44m. 20sec. A. M. at Buffalo; what is the longitude of Buffalo?

  Ans. 78° 55′ west.
- 8. The difference of time between Cambridge Observatory and Greenwich is 4h. 44m. 32sec.; what is the difference of longitude?

  Ans. 71° 8′.

# MISCELLANEOUS EXAMPLES. .

- 1. If the population of the world be as follows: America, 57,550,000; Europe, 263,517,496; Asia, 626,400,000; Africa, 100,000,000; Australia, 1,445,000; Polynesia, 1,500,000; and the average length of life be 33 years, what must be the average number of deaths annually?

  Ans. 31,833,712.
- 2. A farmer has in 3 bins 755 bushels of grain; there being in the first 125 bushels, and in the second 96 bushels, more than in the third; how many bushels in the second and third?

  Ans. 363 in the second, 267 in the third.
- 3. There is a certain island 30 miles in circumference. If A and B commence travelling round it, A at the rate of 3 miles an hour, and B at the rate of 5 miles an hour, how far apart will they be at the end of 30 hours?
- 4. Having money to invest, I purchased two farms at \$1,750 each, and 19 shares of bank stock at \$108 per share, and have left \$113; how much money had I?

  Ans. \$5,570.
- 5. It has been agreed by 12 men to gather 960 bushels of cranberries, and receive for their labor one half of the quantity gathered; after one half was gathered, one third of the men withdrew, leaving the others to complete the job. How many bushels should each man receive? Ans. Those who left, 20 bushels each; those who remained, 50 bushels each.
- 6. A drover made \$652.00 by selling a lot of sheep, at a profit of 50 cents each; how many did he sell?
- 7. What will it cost to carpet a floor that is 18 feet wide and 27 feet long, provided the carpeting cost \$2.25 per yard?
- 8. If a young man, by early rising and economy of time, can save for study and improvement of mind two and a half hours a day, how many years' study, of 12 hours per day, can thus be gained in 20 years?

  Ans. 4y. 60d. 101h.

  9. From 4 piles of wood, the first containing 7c. 76ft. 1671in.,
- 9. From 4 piles of wood, the first containing 7c. 76ft. 1671in., the second 16c. 28ft. 56in., the third 29c. 127ft. 1000in., the fourth 29c. 10ft. 1216in., I have sold 45 cords and 6 cord feet; how much remains?

  Ans. 37c. 19ft. 487in.
- 10. Boston is in north latitude 42° 21'; Portland is in latitude 1° 15' north of Boston; and Charleston is in latitude

- 10° 40' south of Portland. What is the latitude of Charleston?

  Ans. 32° 56' north.
- 11. If 1 cubic foot of anthracite coal weighs 54 pounds, how many cubic feet of space are required to stow 2 tons of 2000 pounds each?

  Ans. 7427 cubic feet.
- 12. Two engineers, A and B, surveyed a certain house-lot. A made its contents 3R. 18p. 0yd. 6ft. 64sq. in., but B made its contents 3R. 17p. 30yd. 8ft. 100sq. in. How much did the one differ from the other?
- 13. The products of the industry of 250,000 persons in Massachusetts, during the year 1855, amounted to \$295,300,000. What was the average amount to each individual, and how much was added to the capital of the State, if one fourth of the whole amount was saved? Ans. \$1,181.20 to each individual; \$73,825,000 added to the capital.
- 14. The capacity of a certain cistern is 216 cubic feet; how many hogsheads of water will it contain?
- 15. What day of the month and what day of the year is the second Monday of May, in a common year commencing on Thursday? Ans. 11th day of May; 131st day of the year.
- 16. Purchased 18T. 17cwt. 3qr. 20lb. of copperas, at 4 cents per pound. I sold 4T. 6cwt. 1qr. 14lb. at 5 cents per pound, and 7T. 1cwt. 3qr. 10lb. at 6 cents per pound. Moses Atwood purchased one fourth of the remainder at 6 cents per pound. One half of what then remained I sold to J. Gale at 10 cents per pound. The remainder I sold to J. Smith at 12 cents per pound; but he has become a bankrupt, and I lose half my debt. What have I gained by my purchase?

  Ans. \$894.07½.
- 17. The distance between Boston and San Francisco is 2691 miles. If Nathan Swift of San Francisco and Oliver Fleet of Boston, on Thursday, the first day of January, 1857, set out to meet each other, Swift travelling 3 miles 7 furlongs 29 rods 15 feet per hour, and Fleet 5 miles 10 rods and 1½ feet per hour, both travelling 6½ hours per day, commencing at 8 o'clock, A. M., provided they rest on the Sabbath, in what year and month, and on what day of the month, and at what time of the day, will they meet, and how far will each have travelled?

Ans. On Monday, February 23, 1857, 2h. 30m. P. M. Swift, 1186m. 4fur. 22rd. 13ft. 6in.; Fleet, 1504m. 3fur. 17rd. 3ft.

### EXAMPLES BY ANALYSIS.

- 1. If 7 pairs of shoes cost \$8.75, what will one pair cost?

  Ans. \$25.00.
- 2. If 5 tons of hay cost \$ 85, what will 1 ton cost? what will 17 tons cost?

  Ans. \$ 289.00.
- 3. When \$0.75 are paid for 3gal. of molasses, what is the value of 1gal.? What cost 37 gal.?
- 4. Gave \$1.92 for 4lb. of tea; what cost 1lb.? what cost 37lb.?

  Ans. \$17.76.
- 5. For 12lb. of rice I paid \$1.08; what was paid for 1lb.; and what must I give for 25lb.?

  Ans. \$2.25.
- 6. Gave S. Smith \$63.00 for 9 tubs of butter; what was the cost of 1 tub? What cost 27 tubs? Ans. \$189.00.
- 7. T. Swan can walk 20 miles in 5 hours; how far can he walk in 1 hour? How long would it take him to walk from Bradford to Boston, the distance being in a straight line 28 miles?
- 8. If a hungry boy would eat 49 crackers in 1 week, how many would he eat in 1 day? how many would be sufficient to last him 19 days?

  Ans. 133 crackers.
- 9. Gave \$20 for 5 barrels of flour; what cost 1 barrel? what cost 40 barrels?

  Ans. \$160.00.
- 10. For 3lb. of lard there were paid 36 cents; what was the cost of 37lb.?
- 11. Paid F. Johnson 72 cents for 9 nutmegs; how many cents were paid for 1 nutmeg; and what should be charged for 37 nutmegs?

  Ans. \$ 2.96.
- 12. Paid 2£. 17s. 5d. for 52lb. of sugar; what cost 1lb.? what cost 76lb.?
  - 13. Paid 4£. 3s. 11d. for 76lb. of sugar; what cost 52lb.?
- 14. If a man walk 17m. 4fur. 28rd. in 6 days, how far will he walk in 100 days?

  Ans. 293m. 1fur.
- 15. If a farmer feed to his stock in 7 months 41bu. 3pk. 4qt. 1pt. of grain, how, much is required for 1 month? how much for 7 years?

  Ans. 502bu. 2pk. 6qt.
- 16. A field containing 39A. 2R. 5p. 8yd. 6ft. 108in. will pasture 8 cows during the season. How large a field will pasture 1 cow? How large a field 72 cows?

- 17. If 4 casks of vinegar contain 63gal. 3qt., what are the contents of one cask? What are the contents of 37 casks?

  Ans. 589gal. 2qt. 1pt. 2gi.
- 18. When 5yd. 3qr. 1na. of cloth cost \$ 4, how much cloth can be bought for \$ 1? How much for \$ 36?

Ans. 52yd. 1qr. 1na.

- 19. If 11T. 3cwt. 2qr. of hay be sufficient to keep 4 horses 7 <sup>3</sup> months, how much will keep 1 horse the same time? How much 23 horses?

  Ans. 64T. 5cwt. 12lb. 8oz.
- 20. If 12 men can dig a certain ditch in 286 days 4h. 33m., how long will it require 1 man to do the same labor? How long 72 men?

  Ans. 47 days 16h. 45m. 30sec.
- 21. If 27yd. 1qr. of cloth be required to make 21 coats, how many yards will be required to make 11 coats?
- 22. If a train of cars move at the rate of 174m. 26rd. in 7 hours, how far will it move in 1 hour? How far in 10 hours?

  Ans. 248m. 5fur. 20rd.
- 23. If 4 cases of shoes, containing 60 pairs each, cost \$192, what will 1 pair cost? What will 25 cases cost?
- 24. When 3A. 2R. 20rd. of land will buy 4 hogsheads of molasses, how much land will buy 1 hogshead? How much 30 hogsheads? Ans. 27A. 0R. 30rd.
- 25. If a man can travel 20deg. 49m. 5fur. 35rd. 5yd. 3in. in 9 weeks, how far would he travel in 1 week? How far in 90 weeks? Ans. 207deg. 13m. 1fur. 25rd. 5yd.

# PROPERTIES OF NUMBERS.

#### DEFINITIONS.

- 162. An integer is a whole number; as 1, 7, 16. All whole numbers are either *prime* or *composite*.
- 163. A prime number is a number which can be exactly divided only by itself and 1; as 1, 3, 5, 7, 11.

A composite number is a number which can be exactly divided by some number besides itself and 1; as 6, 9, 14, 18.

- 164. A factor of a number is such a number as will, by being taken an entire number of times, produce it; as, 3 is a factor of 9, and 4 a factor of 16.
- 165. A prime factor of a number is a prime number that will exactly divide it; thus the prime factors of 10 are the prime numbers 1, 2, and 5.

NOTE. — Unity or 1 is not generally regarded as a prime factor, since multiplying or dividing any number by 1 does not alter its value. It therefore will be omitted when speaking of the prime factors of numbers.

A composite factor of a number is a composite number that will exactly divide it; thus, 6 and 8 are composite factors of 48.

- 166. Numbers are prime to each other when they have no factor in common; thus, 4, 9, and 23 are prime to each other.
- 167. An aliquot part of a number is such a part as will exactly divide it; as, 1, 3, and 5 are aliquot parts of 15.

NOTE. — The aliquot parts of a number include all its factors, prime and composite.

An aliquant part of a number is such a part as will not exactly divide it; as 2, 4, 5, 7, and 8 are aliquant parts of 9.

- 168. The *reciprocal* of a number is the quotient arising from dividing 1 by the number; thus, the reciprocal of 2 is  $\frac{1}{2}$ .
- 169. The *power* of a number is the product obtained by taking the number a certain number of times as a factor; thus 25 is a power of 5.

Note. — When the number is taken once, it is called its first power; when taken twice, as a factor, the product is called its second power; and so on. The second power of a number is sometimes termed its square, and the third power, its cube.

170. The exponent of a power is a figure written at the right of a number, and a little above it, to show how many times it is taken as a factor; thus, in the expression  $4^2$ , the exponent is the 2, and the whole is read 4 second power; and in  $7^3$ , it is the 3, and the whole read 7 third power.

Note. — The first power of a number being always the number itself, its exponent is not expressed.

#### PROPERTIES OF PRIME NUMBERS.

- 171. No direct process of detecting prime numbers has been discovered.
- NOTE. A few facts, such as are given below, if kept in mind, will aid somewhat in ascertaining whether a number is prime or not.
- 172. The only even prime number is 2; since all other even numbers, as 4, 6, 8, and 10, it is evident, can be exactly divided by 2, and therefore must be composite.
- 173. The only prime number having 5 for a unit or right-hand figure is 5; since every other whole number thus terminating, as 15, 25, 35, and 45, can be exactly divided by 5, and therefore must be composite.
- 174. Every prime number, except 2 and 5, must have 1, 3, 7, or 9 for the right-hand figure; since all other numbers are composite.
- 175. Every prime number above 3, when divided by 6, must leave 1 or 5 for a remainder; since every prime number above 3 is either 1 greater or 1 less than 6, or some exact number of times 6.
- 176. In a series of odd numbers written in their proper or natural order, if beginning with 3 every THIRD number, with 5 every FIFTH, with 7 every SEVENTH, be cancelled, as composite, the remaining numbers, with 2, will be the prime numbers of the natural series. Thus, in the series 1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27, 29, 31, 33, 35, 37, 39, 41, 43, 45, 47, 49, every third number from the 3, every fifth from the 5, every seventh from the 7, every ninth from the 9, and so on, being cancelled, the remaining numbers, with 2, are all the prime numbers under 50.
- NOTE 1. In the series, every third number from the 3 contains that number as a factor; every fifth number from the 5, that number as a factor; and so on.
- Note 2. The whole number of prime numbers from 1 to 100,000 is 9,593. Although all of these, except 2 and 5, end in 1, 3, 7, or 9, there are, within the same range, no less than 30,409 composite numbers terminating with some one of the same figures.

177. All the prime numbers not larger than 4049 are included in the following

TABLE OF PRIME NUMBERS.

1         233         557         888         1249         1607         2008         2389         2797         3251         3631           2         239         563         887         1259         1609         2011         2398         2797         3251         3643           5         251         571         911         1279         1619         2027         2411         2803         3257         3659           7         257         577         919         1283         1621         2029         2417         2819         3259         3671           11         263         587         929         1289         1621         2039         2423         2838         3271         3671           11         263         587         3929         1803         1667         2083         2447         2851         3307         3667           19         277         601         947         1201         1663         2069         2447         2851         3307         3667           29         283         613         967         1807         1669         2083         2467 <t>2861         3819         3760</t>											
2         289         563         887         1259         1609         2011         2898         2797         8251         8637           3         241         569         907         1277         1619         2027         2411         2803         3253         3659           7         257         577         919         1283         1621         2029         2417         2819         3259         3671           11         263         587         929         1289         1627         2089         2423         2883         3271         8673           13         269         5847         1291         1637         2063         2441         2843         3301         3691           19         277         601         947         1801         1667         2081         2469         2857         3313         3701           29         288         613         967         1807         1669         2083         2467         2867         3313         3701           31         298         6137         971         1319         1693         2087         2473         2867         3323         3719           <	1	233	557	888	1249	1607	2008	2389	2791	8229	8681
3         241         569         907         1277         1618         2017         2899         2801         3253         3643           6         251         571         911         1279         1619         2027         2411         2803         3257         3659           7         257         577         919         1283         1621         2029         2417         2819         3271         3271         3673           13         269         588         937         1291         1637         2053         2427         2887         3299         3677           17         271         599         941         1297         1657         2063         2447         2861         3301         3691           23         281         607         953         1803         1667         2081         2469         2857         3318         370         3697           29         283         613         967         1807         1669         2083         2467         2861         3819         3719           37         807         619         977         1821         1697         2089         2473         3829 <t< th=""><th></th><th></th><th></th><th></th><th></th><th></th><th></th><th></th><th></th><th></th><th></th></t<>											
6         251         571         911         1279         1819         2027         2417         2803         2825         3671           11         263         587         929         1289         1627         2039         2423         2838         3271         3671           13         269         598         937         1291         1687         2068         2487         2837         3299         3677           17         271         599         941         1297         1667         2063         2441         2433         3031         3691           19         277         601         947         1807         1669         2083         2447         2851         307         3697           29         288         618         967         1807         1669         2081         2467         2861         3819         3719           37         807         619         977         1321         1697         2089         2477         2887         3333         3719           37         807         619         977         1321         1697         2569         2477         2861         3819         3719 <th></th> <th>8643</th>											8643
7 257 577 919 1283 1621 2029 2417 2819 8259 8671 11 263 587 929 1289 1627 2039 2423 2833 8271 8673 13 269 593 927 1291 1697 2063 2427 2837 8299 3677 17 271 599 941 1297 1667 2063 2447 2851 8307 3697 19 277 601 947 1801 1663 2069 2447 2851 8307 3697 23 281 607 953 1303 1667 2081 2459 2557 3313 3701 29 283 613 967 1807 1669 2083 2467 2861 8819 3769 31 293 617 971 1319 1693 2087 2473 2579 8823 3719 37 807 619 977 1821 1697 2089 2477 2887 3329 3727 41 311 681 988 1327 1699 2099 2503 2897 8381 8783 43 313 641 991 1361 1709 2111 2521 2903 3343 3739 47 87 643 997 1867 1721 2113 2531 2909 8347 8761 53 381 647 1009 1873 1723 2129 2539 2917 8359 3767 65 387 658 1018 1831 1738 2181 2548 2927 3861 3769 61 347 659 1019 1399 1741 2187 2549 2939 3371 3779 67 349 661 1021 1409 1747 2141 2551 2953 8373 8798 61 347 659 1019 1399 1747 2141 2551 2953 8373 8798 79 867 683 1031 1423 1753 2143 2557 2957 3889 3797 73 859 677 1033 1427 1759 2153 2579 2968 8391 8908 89 379 701 1061 1439 1787 2208 2609 2999 3488 3883 97 883 709 1061 1447 1789 2207 2617 3001 3449 3847 101 389 719 1063 1451 1801 2213 2621 3011 3469 3847 101 389 719 1063 1451 1801 2213 2621 3011 3469 3847 101 389 719 1063 1451 1801 2213 2633 3019 3461 3858 107 401 733 1087 1459 1832 2237 2647 3023 3463 3863 107 401 733 1087 1459 1832 2237 2647 3023 3463 3863 107 401 733 1087 1459 1831 2229 2657 3087 347 3851 103 387 727 1069 1453 1811 2221 2633 3019 3461 3858 107 409 789 1091 1471 1831 2239 2657 3067 3511 3911 139 443 773 1123 1499 1877 2261 2683 3049 3491 3869 131 441 757 1103 1487 1867 2267 2677 3067 3511 3911 149 443 773 1123 1499 1877 2281 2687 3083 3529 3923 163 461 809 1153 1527 1901 2297 2699 3119 3539 3931 167 463 811 163 1529 1907 2809 2707 3121 3541 3943 173 467 821 1171 1533 1919 1227 2679 3079 3121 3541 3943 189 499 899 1201 1553 1961 2287 2689 3089 3529 3923 163 461 809 1153 1527 1901 2297 2699 3119 3539 3931 167 463 811 163 1529 1907 2809 2707 3121 3541 3943 179 479 823 1181 1539 1931 2833 2719 3169 3571 4001 199 509 857 1217 1571 1979 2857 2749 3191 3589 3931											8659
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18											
17											
19											
23         281         607         963         1803         1667         2081         2459         2857         8318         3701           29         288         613         967         1807         1669         2083         2467         2861         3819         3701           31         298         617         971         1321         1697         2089         2477         2887         3822         3719           37         307         619         977         1321         1697         2089         2477         2887         3829         3727           41         311         681         981         1327         1699         2099         2608         2897         3831         3739           47         847         643         997         1367         1721         2113         2531         2909         3447         3761           53         381         647         1009         1373         1722         2113         2531         2909         3447         3761           50         337         658         1019         1399         1741         2137         2549         2939         3271         3779											
29         288         613         967         1807         1669         2083         2467         2861         3819         3769           31         298         617         971         1319         1693         2087         2473         2879         3823         3719           37         307         619         977         1321         1697         2089         2473         2887         3829         3727           41         311         631         988         1327         1699         2099         2608         2897         3831         3739           47         847         643         997         1867         1721         2113         2521         2903         3347         3761           53         381         647         1009         1873         1722         2112         2539         2917         3359         3761           53         387         663         1013         1381         1733         2181         2543         2927         3861         3769         3779         3661         347         6661         1021         1409         1747         2141         2551         2963         3871         3799 <th></th> <th></th> <th></th> <th></th> <th></th> <th></th> <th></th> <th></th> <th>2857</th> <th></th> <th></th>									2857		
31         298         617         971         1319         1693         2087         2473         2879         8822         3719           37         807         619         977         1321         1699         2099         2477         2887         3329         3727           41         311         631         983         1327         1699         2099         2608         2897         3831         3783           43         318         641         991         1361         1709         2111         2521         2908         3343         3739           53         381         647         1009         1873         1723         2129         2539         2917         3369         3767           59         337         658         1018         1381         1738         2181         2643         2927         3361         3769           61         347         669         1019         1399         1741         2181         2551         2953         3373         3798           71         358         673         1031         1427         1759         2163         2557         2957         3389         3797											
37         807         619         977         1821         1697         2089         2477         2887         8329         3727           41         311         631         988         1327         1699         2099         2608         2897         3831         3739           47         847         643         997         1367         1721         2113         2581         2909         3347         3761           53         381         647         1009         1873         1722         2129         2539         2917         3359         3761           69         387         688         1018         1381         1738         2181         2639         2917         3359         3761           61         347         669         1019         1399         1741         2187         2549         2939         3871         3779           67         349         661         1021         1409         1747         2141         2551         2953         3873         3799         377         3369         677         1033         1427         1759         2153         2579         2963         3891         3893         379 <th></th> <th></th> <th></th> <th></th> <th></th> <th></th> <th></th> <th></th> <th>2879</th> <th></th> <th>3719</th>									2879		3719
41         311         681         988         1327         1669         2099         2608         2897         8381         3733           43         318         641         997         1367         1721         2113         2521         2909         3343         7361           53         381         647         1009         1878         1723         2129         2539         2917         3359         3767           59         387         663         1018         1381         1733         2181         2543         2927         3361         3769           61         347         669         1019         1399         1741         2187         2549         2939         3871         3779           67         349         661         1021         1409         1747         2141         2551         2963         3871         3779           73         365         677         1033         1427         1759         2153         2577         2963         3891         8903           79         367         683         1089         1429         1777         2161         2591         2969         3433         3893											
48         818         641         991         1361         1709         2111         2521         2908         8343         873           47         81         643         997         1867         1721         2118         2581         2909         8347         3761           53         831         647         1009         1873         1723         2129         2539         2917         3359         3767           59         337         658         1018         1381         1738         2181         2648         2927         3361         3769           61         347         669         1019         1399         1741         2141         2551         2953         3873         8798           71         358         673         1031         1428         1758         2143         2557         2963         3873         8793           73         369         677         1033         1427         1759         2163         2577         2968         3891         3797         73         369         6783         1093         1429         1777         2161         2591         2968         3407         3821         389							2099	2508	2897	8381	8783
47         84         643         997         1867         1721         2118         2531         2909         8347         3761           53         831         647         1009         1873         1723         2129         2539         2917         3359         3767           59         337         658         1019         1399         1741         2187         2549         2939         8271         3779           67         349         661         1021         1409         1747         2141         2551         2953         3873         3793           71         385         673         1031         1422         1759         2153         2579         2963         8391         8903         379         73         366         677         1033         1427         1759         2153         2579         2963         8991         8903         799         367         688         1089         1429         1777         2161         2591         2969         8407         8821         893         379         701         1051         1439         1787         2202         2667         3001         3449         3843         3833	48										
53         831         647         1009         1873         1728         2129         2539         2917         8359         376           59         337         658         1018         1381         1733         2181         2648         2927         3861         3769           61         347         669         1019         1399         1741         2187         2649         2939         3871         3779           67         349         661         1021         1409         1747         2141         2551         2953         3873         3798           71         3856         677         1033         1428         1753         2143         2557         2963         3891         389         379           79         367         683         1089         1429         1777         2161         2591         2969         8407         3821           89         879         701         1061         1447         1789         2207         2617         3001         3449         3847           101         389         719         1063         1451         1801         221         2621         3001         3449											
59         337         653         1018         1381         1788         2181         2648         2927         8361         3769         61         347         669         1019         1399         1741         2187         2649         2939         3871         3779         67         349         661         1021         1409         1747         2141         2551         2953         3873         3793         71         358         673         1031         1428         1758         2143         2557         2957         2389         3797         73         369         677         1033         1427         1759         2153         2579         2968         3891         3899         3797         73         69         867         683         1039         1429         1777         2161         2591         2969         3497         3413         3823           89         379         701         1051         1439         1787         2203         2609         2999         3483         3823           97         383         709         1061         1447         1789         2207         2617         3001         3449         3847 <t< th=""><th></th><th></th><th></th><th></th><th></th><th></th><th></th><th></th><th></th><th></th><th></th></t<>											
61         847         659         1019         1399         1741         2187         2549         2939         8371         3779           67         349         661         1021         1409         1747         2141         2551         2953         3373         3798           71         385         673         1031         1423         1753         2143         2557         2963         8391         3898           79         867         683         1089         1429         1777         2161         2591         2969         8407         3803           88         878         691         1049         1433         1783         2179         2593         2971         8413         3823           89         379         701         1051         1439         1787         2203         2609         2999         3438         3833           97         383         709         1061         1447         1789         2207         2617         3001         3449         3847           101         389         719         1068         1453         1811         2221         2633         3019         3461         3853 <th></th> <th></th> <th></th> <th></th> <th></th> <th></th> <th></th> <th></th> <th></th> <th>8361</th> <th>3769</th>										8361	3769
67         349         661         1021         1409         1747         2141         2551         2953         3373         3798           71         385         677         1031         1428         1753         2143         2557         2957         3889         379           79         867         683         1089         1429         1777         2161         2591         2969         8407         3821           89         379         701         1051         1439         1787         2203         2691         2969         8449         3823           97         383         709         1061         1447         1789         2207         2617         3001         3449         3847           101         389         719         1063         1451         1801         2213         2621         3011         3449         3847           103         897         719         1069         1453         1811         2221         2621         3011         3449         3847           103         499         739         1091         1471         1831         22237         2647         3023         3463         3863<						1741		2549	2939	8371	3779
71         858         672         1081         1428         1758         2143         2557         2957         3889         3797           73         369         677         1033         1427         1759         2153         2579         2968         3891         3890           79         867         688         1089         1429         1777         2161         2591         2969         3497         3813         3823           89         379         701         1051         1439         1787         2203         2609         2999         3483         3833           97         383         709         1061         1447         1789         2207         2617         3001         3449         3847           101         389         719         1068         1451         1801         2213         2621         3011         3449         3847           103         387         727         1069         1453         1811         2221         2633         3019         3461         3583           107         401         733         1087         1459         1823         2237         2647         3023         3467<							2141	2551	2953	8373	8798
73         859         677         1033         1427         1759         2153         2579         2968         8891         8801           79         867         683         1039         1429         1777         2161         2591         2969         3407         3821           88         878         691         1049         1433         1783         2179         2593         2971         3413         3823           89         379         701         1051         1439         1787         2208         2609         2999         3483         3823           97         383         709         1061         1447         1789         2207         2617         3001         3449         3847           101         389         719         1068         1453         1811         2221         2633         3019         3461         3853           107         401         738         1087         1459         1823         2237         2647         3023         3461         3853           109         409         739         1091         1481         1847         2243         2659         3041         3463         3861<							2143	2557	2957	8389	8797
79         867         688         1089         1429         1777         2161         2561         2969         8407         3821           83         378         691         1049         1438         1783         2179         2593         2971         3413         3823           89         379         701         1061         1447         1789         2207         2617         3001         3449         3847           101         389         719         1063         1451         1801         2213         2621         3011         3457         3851           103         397         71069         1453         1811         2221         2621         3011         3457         3851           103         401         733         1087         1459         1823         2237         2647         3023         3463         3863           109         499         739         1091         1471         1831         2239         2657         3037         3467         3851           113         419         743         1093         1481         1861         2251         2663         3049         3491         3889					1427	1759	2153	2579	2968	8391	8803
88         878         691         1049         1433         1783         2179         2593         2971         3413         3823           89         379         701         1051         1439         1787         2203         2609         2999         3483         3833           97         383         709         1061         1447         1789         2207         2617         3001         3449         3847           101         389         719         1068         1451         1801         2213         2621         3011         3449         3841           103         387         727         1069         1453         1811         2221         2633         3019         3461         3853           109         409         789         1061         1471         1831         2239         2657         3037         3467         3871           113         419         743         1093         1481         1847         2243         2659         3041         3467         3871           127         421         751         1097         1483         1861         2243         2659         3041         3469         389					1429	1777	2161	2591	2969	8407	8621
89         379         701         1051         1439         1767         2208         2609         2999         8488         3838           97         383         709         1061         1447         1789         2207         2617         3001         3449         3847           101         389         719         1063         1451         1801         2213         2621         3011         3447         3851           103         397         727         1069         1453         1811         2221         2633         3019         3461         3858           109         409         739         1091         1471         1831         2237         2647         3023         3463         3867           113         419         743         1093         1481         1847         2243         2659         3041         3469         3881           127         421         761         1097         1483         1861         2251         2663         3049         3491         3889           131         431         757         1103         1487         1867         2267         2673         3067         3511         39				1049	1433	1783	2179	2593	2971	8418	8823
101   388   719   1068   1451   1801   2213   2621   3011   3457   3851   108   387   727   1069   1453   1811   2221   2633   3019   3461   3853   107   401   738   1087   1459   1823   2237   2647   3023   3463   3863   109   409   739   1091   1471   1831   2239   2657   3037   3467   3877   113   419   743   1093   1481   1847   2248   2659   3041   3469   3881   127   421   751   1097   1483   1861   2251   2663   3049   3491   3481   131   431   757   1103   1487   1867   2267   2677   3067   3511   3911   139   439   769   1117   1493   1873   2273   2683   3079   3511   3911   139   439   769   1117   1493   1873   2273   2683   3079   3513   3919   151   443   773   1123   1499   1877   2281   2687   3083   3527   3919   151   449   767   1129   1511   1879   2287   2689   3089   3529   3923   163   461   809   1153   1527   1901   2297   2699   3119   3533   3929   163   461   809   1153   1527   1901   2297   2699   3119   3533   3931   167   463   811   1163   1529   1907   2309   2707   3121   3541   3943   178   467   821   1171   1538   1913   2311   2211   3187   3547   3947   179   479   822   1187   1547   1933   2333   2713   3163   3557   3947   179   479   823   1181   1539   1931   2333   2713   3163   3557   3989   191   491   829   1193   1551   1949   2841   2729   3169   3571   4001   193   499   839   1201   1553   1951   2347   2743   3181   3581   4007   199   509   857   1217   1571   1979   2357   2749   3191   3593   4013   223   523   863   1229   1583   1998   2377   2758   3208   3607   4013   223   523   863   1229   1583   1998   2377   2758   3208   3607   4013   223   523   863   1229   1583   1998   2377   2757   3217   3617   4027   227   541   877   1231   1597   1997   2381   2777   3217   3617   4027   2277   541   877   2317   3617   4027   2277   541   877   2317   3617   4027   2377   3217   3617   4027   3277   3217   3617   4027   3277   3217   3617   4027   3277   3217   3617   4027   3277   3217   3617   4027   3277   3217   3617   4027   3277   3217   3617		879	701	1051	1439	1787	2208		2999	8488	3833
108         897         727         1069         1458         1811         2221         2633         3019         8461         3858           107         401         738         1087         1459         1823         2237         2647         3023         3463         3863           109         409         789         1091         1471         1881         2239         2657         3037         3467         8877           113         419         743         1093         1481         1847         2243         2659         3041         3469         3881           127         421         761         1097         1483         1861         2251         2663         3049         3491         3889           131         431         757         1103         1487         1867         2267         2671         3061         3499         3907           137         483         761         1109         1489         1871         2269         2677         3067         3511         3917           149         443         773         1123         1499         1877         2281         2683         3079         3517	97	883	709		1447		2207		8001		
108         897         727         1069         1453         1811         2221         2633         3019         3461         3853           107         401         733         1087         1459         1823         2237         2647         3023         3463         3863           109         409         739         1091         1471         1831         2239         2657         3037         3467         8877           113         419         743         1098         1481         1847         2243         2659         3041         3469         3881           127         421         761         1097         1483         1861         2261         2663         3049         3491         3889           131         431         757         1103         1489         1871         2269         2677         3067         3511         3917           149         437         773         1123         1499         1871         2269         2677         3067         3511         3917           149         443         773         1123         1499         1877         2281         2687         3083         3527	101	389	719		1451		2213		8011		
109   409   729   1091   1471   1881   2289   2657   3037   3467   8877   113   419   743   1093   1481   1847   2248   2659   3041   3469   3881   127   421   751   1097   1483   1861   2251   2663   3049   3491   3889   131   431   757   1103   1487   1867   2267   2671   3061   3499   3897   137   488   761   1109   1489   1871   2269   2677   3067   3511   3911   1399   439   769   1117   1493   1873   2273   2683   3079   3517   3917   149   443   773   1123   1499   1877   2281   2687   3083   3527   3917   149   443   773   1123   1499   1877   2281   2687   3083   3527   3917   151   1497   1511   1879   2287   2689   3089   3529   3923   157   457   797   1151   1523   1889   2293   2698   3109   3533   3929   163   461   809   1153   1527   1901   2297   2699   3119   3533   3929   163   461   809   1153   1529   1907   2309   2707   3121   3541   3917   179   479   823   1181   1539   1931   2311   2211   3187   3547   3947   179   479   823   1181   1539   1931   2333   2713   3163   3557   3967   181   487   827   1187   1547   1933   2389   2719   3167   3559   3989   191   491   829   1193   1551   1949   2341   2729   3169   3671   4001   193   499   839   1201   1553   1951   2347   2741   3187   3543   4007   199   509   857   1217   1571   1979   2357   2749   3191   3593   4013   211   521   869   1223   1579   1987   2371   2758   3203   3607   4019   223   523   663   1229   1583   1993   2377   2767   3209   3613   4021   227   541   877   1231   1597   1997   2381   2777   3217   3617   4027   3617   4027   3617   3617   4027   3617   3617   4027   3617   3617   4027   3617   3617   4027   3617   3617   4027   3617   3617   4027   3617   3617   4027   3617   3617   4027   3617   3617   4027   3617   3617   4027   3617   3617   4027   3617   3617   4027   3617   3617   4027   3617   3617   4027   3617		897		1069	1458						
113         419         743         1098         1481         1847         2243         2659         3041         3469         3881           127         421         761         1097         1488         1861         2251         2663         3049         3491         3880           131         431         757         1103         1487         1867         2267         2671         3067         3511         3917           137         483         761         1109         1489         1871         2269         2677         3067         3511         3917           149         443         773         1123         1499         1877         2281         2687         3068         3527         3919           151         449         787         1151         1523         1889         2287         2689         3089         3529         3923           167         467         797         1151         1523         1889         2293         2698         3109         3533         3929           163         461         809         1153         1527         1901         2297         2699         3119         3533	107	401	733	1087			2237				
127         421         751         1097         1488         1861         2251         2663         3049         3491         3889           131         431         757         1103         1487         1867         2267         2671         3061         3499         3899         3899           137         438         761         1109         1489         1871         2267         2671         3061         3411         3911           139         439         769         1117         1493         1873         2273         2683         3079         3517         3917           149         443         773         1123         1499         1877         2281         2687         3083         3527         3917           151         449         767         7129         1511         1879         2287         2689         3089         3527         3919           151         449         767         7151         1523         1889         2293         2698         3109         3533         3929           163         461         809         1153         1527         1901         2297         2699         3119	109	409	789				2289				
131         431         757         1103         1487         1867         2267         2671         3061         8499         3907           137         483         761         1109         1489         1871         2269         2677         3067         3511         3911           139         439         769         1117         1493         1873         2273         2683         3079         3517         3917           149         443         773         1123         1499         1877         2281         2687         3083         3527         3919           151         449         767         1129         1511         1879         2287         2689         3089         3529         3923           157         457         797         1151         1523         1889         2292         2899         3109         3539         3931           167         463         811         1163         1529         1907         2309         2707         3121         3541         3943           167         463         811         1163         1529         1907         2309         2707         3121         3541		419									
137	127	421									
189											
149         443         773         1123         1499         1877         2281         2687         3083         3527         3919           151         449         767         1129         1511         1879         2287         2689         3089         3529         3923           157         457         797         1151         1523         1889         2293         2698         3109         3533         3929           163         461         809         1153         1527         1901         2297         2699         3119         3539         3931           167         463         811         1168         1529         1907         2309         2707         3121         3541         3943           173         467         821         1171         1583         1913         2311         2211         3187         3541         3943           179         479         822         1181         1539         1981         2333         2713         3163         3567         3967           181         487         827         1187         1547         1933         2339         2719         3167         3559											
151         449         767         1129         1511         1879         2287         2689         3089         3529         3923           157         457         797         1151         1523         1889         2293         2698         8109         3533         3933           163         461         809         1153         1527         1901         2297         2699         8119         8393         3931           167         463         811         1163         1529         1907         2309         2707         3121         3541         3943           178         467         821         1171         1538         1913         2311         2211         3187         3541         3943           179         479         823         1181         1539         1931         2333         2713         3163         3557         3967           181         487         827         1187         1547         1983         2339         2719         3167         3559         3989           191         491         829         1193         1551         1949         2841         2729         3169         3571											
157         457         797         1151         1523         1889         2293         2698         8109         8533         8929           163         461         809         1153         1527         1901         2297         2699         8119         3639         3931           167         463         811         1168         1529         1907         2309         2707         3121         3641         3943           173         467         821         1171         1583         1913         2811         2211         8187         8547         3947           181         487         827         1187         1547         1983         2839         2719         3167         3557         3967           181         487         827         1187         1547         1943         2389         2719         3167         3557         3967           191         491         829         1193         1551         1949         2841         2729         3169         3571         4001           193         499         839         1201         1553         1951         2847         2731         3181         3581											
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#### FACTORING.

- 178. FACTORING is the process of resolving a quantity into its factors.
- 179. Every number that is not prime is composed of prime factors, since all numbers are either prime or composite; and, if composite, can be separated into factors, which, if themselves composite, can be further separated into those that shall be prime.
  - 180. To resolve a composite number into its prime factors. Ex. 1. It is required to find the prime factors of 42.

Ans. 2, 3, 7.

2 4 2 3 2 1 We divide by 2, the least prime number greater than 1, and obtain the quotient 21; and, since 21 is a composite number, we divide this by 3, and obtain for a quotient 7, which is a prime number. The several divisors and the last quotient, all being prime, constitute all the prime factors of 42, which, multi-

plied together, they equal. Hence

Divide the given number by any prime number that will exactly divide it, and the quotient, if a composite number, in the same manner; and so continue dividing, until a prime number is obtained for a quotient. The several divisors and the last quotient will be the prime factors required.

NOTE 1. — The composite factors of any number may be found by multiply-

ing together two or more of its prime factors.

NOTE 2. — Such prime factors as two or more numbers may have alike, are termed prime factors common to them; and these may be readily determined after the numbers are resolved into their prime factors.

#### EXAMPLES.

- 2. What are the prime factors of 105? Ans. 3, 5, 7.
- 3. Resolve 220 into its prime factors.
- 4. What are the prime factors of 936?

Ans. 2, 2, 2, 3, 3, 13.

- 5. What are the prime factors of 1953?
- 6. Resolve 12462 into its prime factors. Ans. 2, 3, 31, 67.
- 7. Resolve 19987 into its prime factors. Ans. 11, 23, 79.
- 8. What are the prime factors common to 225, 435, and 540?

  Ans. 3, 5.

- 9. What are the prime factors common to 960, 1568, and 5824?
- 10. What are the prime factors common to 2340, 11934, 12987, and 14859?

  Ans. 3, 3, 13.
- 11. A man has 105 apples, which he wishes to distribute into small parcels, each of equal numbers; what are the smallest whole numbers, greater than 1, into which they may be exactly divided?

  Ans. 3, 5, and 7.

#### DIVISIBILITY OF NUMBERS.

- 181. One number is said to be divisible by another, when the latter will divide the former without a remainder. Thus, 9 is divisible by 3.
- 182. One number is divisible by another, when it contains all the prime factors of that number. Thus, 12, which contains all the factors of 4, is divisible by 4.
- 183. All even numbers, or such as terminate with 0, 2, 4, 6, or 8, are divisible by 2, since each of them contains 2 as a factor. Thus, 10, 24, 36, 58, are each divisible by 2.
- 184. All numbers which terminate with 0 or 5 are divisible by 5, since each of them contains 5 as a factor. Thus, 20, 25, 50, are each divisible by 5.
- 185. Every number is divisible by 4, or any other number that will exactly divide 100, when its two right-hand figures are divisible by the same. For any figure on the left of the two right-hand figures must express one or more hundreds, and a factor of one hundred is a factor of any number of hundreds; so, if the sum exactly divides the units and tens of a number, the entire number will be divisible by it. Thus, 116 is divisible by 4; 140, by 20; 225, by 25; and 450, by 50.
- 186. Every number is divisible by 8, or any other number that will exactly divide 1000, when its three right-hand figures are divisible by the same. For any figure on the left of the three right-hand figures must express one or more thousands, and a factor of one thousand is a factor of any number of thousands; so, if the sum exactly divides the units, tens, and

hundreds of a number, the entire number will be divisible by it. Thus, 1824 is divisible by 8; 1840, by 40; 3375, by 125; 2750, by 250; and 4500, by 500.

187. Every number the sum of whose digits 3 or 9 will exactly divide, is divisible by 3 or 9. For 10, or any power of 10, less 1, gives a number, as 9, 99, 999, &c., which is divisible by 3 and by 9. Hence, any number of tens, hundreds, thousands, &c., less as many units, must be divisible by 3 and by 9; and if the excess of units denoted by the significant figures, in the aggregate, is likewise divisible by 3 and by 9, it follows that the entire number is thus divisible. For example, 7542 is a number, the sum of whose digits is divisible by 3 and by 9; and separated into tens, hundreds, and thousands, it is equal to 7000 + 500 + 40 + 2. Now,  $7000 = 7 \times$  $1000 = 7 \times (999 + 1) = 7 \times 999 + 7$ ;  $500 = 5 \times 100$  $= 5 \times (99 + 1) = 5 \times 99 + 5$ ; and  $40 = 4 \times 10 = 4 \times (9 + 1) = 4 \times 9 + 4$ . Therefore,  $7542 = 7 \times 999 + 4$  $5 \times 99 + 4 \times 9 + 7 + 5 + 4 + 2$ . The remainders 7 + 5 + 4 + 2, corresponding with the significant figures of the number, added together, equal 18, which sum being divisible by 3 and by 9, it is evident that 7542 is divisible in like manner.

NOTE. — Upon the property of 9 now explained depends the method of proving, by excess of nines, multiplication (Art. 63), and division (Art. 75).

The same method of proof may be resorted to in addition and in subtraction. Thus,

To prove Addition. Find the excess of nines in each number added, and then the excess of nines in the sum of these results; which, if the work be right, will equal the excess of nines in the answer.

To prove Subtraction. Find the excess of nines in the subtrahend, and also in the remainder, and then the excess of nines in the sum of these results; which, if the work be right, will equal the excess of nines in the minuend.

. 188. Every number occupying four places, in which two like significant figures have two ciphers between them, is divisible by 7, 11, and 13. Thus, 9009, 1001, 3003, 4004, &c., are each divisible by 7, 11, and 13.

- 189. Every number is divisible by 11, in which the sum of the digits in the odd places is equal to the sum of the digits in the even places, or in which the difference of their sums can be exactly divided by 11. Thus, 8305, in which 3+5=8+0, and 628001, in which 2+0+1 and 6+8+0 differ by 11, are each divisible by 11.
- . 190. Every number divisible by two or more numbers, which are prime to each other, is divisible by their product. For, being prime to each other, dividing by one of the numbers does not cancel the others as factors. Thus, 770, being divisible by 2, 5, and 7, which are prime to each other, is divisible by 70, their product.
- 191. Every even number, the sum of whose digits 6 will exactly divide, is divisible by 6. For being even, it is divisible by 2 (Art. 183), and its digits being divisible by twice 3, or 6, are evidently divisible by once 3, so that the number is also divisible by 3; and as the 2 and the 3 are prime to each other, the number is divisible by their product, or 6 (Art. 190). Thus, 174, 6312, are each divisible by 6.
- 192. Every number terminating with 0 or 5 that 3 will exactly divide, is divisible by 15, and every number that 9 will exactly divide, is divisible by 45. For, terminating with 0 or 5, it is divisible by 5 (Art. 184), and, as 3 and 9 are each prime to 5, if it can be exactly divided by 3 or by 9, it must be divisible by  $5 \times 3 = 15$ , or by  $5 \times 9 = 45$ . Thus, 75, which 3 will exactly divide, is divisible by 15; and 90, which 9 will exactly divide, is divisible by 45.

# DIVISORS OR MEASURES.

- 193. A divisor or measure of a number is any number that will divide it without a remainder. Thus, 3 is a divisor or measure of 6, and 5 a divisor or measure of 10.
  - 194. To find all the divisors or measures of a number.
  - Ex. 1. Required all the divisors of 60.

    Ans. 1, 2, 3, 4, 5, 6, 10, 15, 20, 30, and 60.

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$$\text{Divisors} \begin{cases} 6\ 0 = 2\ \times\ 2\ \times\ 3\ \times\ 5. & \text{number into its}\\ 1\ 2\ 4 = 2\ \times\ 2\ & \text{we find } 60 = \\ 3\ 6\ 1\ 2 = 2\ \times\ 2\ \times\ 3\ & \text{2}\ \times\ 2\ \times\ 3\ \times\ 5. \\ 5\ 1\ 0\ 2\ 0 = 2\ \times\ 2\ \times\ 5\ & \text{Now, any number is divisible}\\ 1\ 5\ 3\ 0\ '\ 6\ 0 = 2\ \times\ 2\ \times\ 3\ \times\ 5. & \text{Now, any number is divisible}\\ \text{Ans. } 1,\ 2,\ 3,\ 4,\ 5,\ 6,\ 10,\ 12,\ 15,\ 20,\ 30,\ \text{and } 60. \end{cases}$$

prime factors, and every product these factors can form. Now 1, being a divisor of every number, is a divisor of 60, and, since 2 enters twice as a factor into 60, it is evident that 2, and  $4 = 2 \times 2$ , are also divisors of 60. These divisors we arrange on a horizontal line, and determine the other divisors by multiplying those on this line by the factor 3, for the second line of divisors, by 5 for the third line, and by  $3 \times 5$  for the fourth line, and thus obtain all the possible divisors of the given number.

The whole number of divisors is 12, which corresponds to the product arising from multiplying together the exponents, each increased by 1, of the different prime factors of 60; thus, of the different prime factors, since 2 enters twice, its exponent is 2, +1 = 3; 3 enters once, its exponent is 1, +1 = 2; 5 enters once, its exponent is 1, +1 = 2; and  $3 \times 2 \times 2 = 12$ , the number of divisors. The same holds true in all cases. Hence, in any composite number,

To find the Number of Divisors. — Multiply together the exponents, each increased by 1, of the different prime factors of the given number, and the product will be the number of divisors required. And

TO FIND THE SEVERAL DIVISORS.—Form from the prime factors of the number all the products possible, and these factors (including 1) and products will be the divisors required.

#### EXAMPLES.

2. What are the divisors of 72?

Ans. 1, 2, 3, 4, 6, 8, 9, 12, 18, 24, 36, and 72.

- 3. Required the divisors of 105?
- 4. How many divisors has 1764?
- 5. How many divisors has 3528?
- 6. How many divisors has 5880? Ans. 48.

# COMMON DIVISORS OR MEASURES.

195. A common divisor or measure of two or more numbers is any number that will divide them without a remainder; thus, 2 is a common divisor of 4, 8, and 10.

Ans. 27.

- 196. A common divisor of two numbers is a divisor of their sum, and also of their difference. Thus, 6, a common divisor of 12 and 18, is a divisor of their sum, 30, and of their difference, 6.
- 197. A common divisor of the remainder and the divisor is a divisor of the dividend. Thus, in a division having 8 for a remainder, 16 for divisor, and 24 for dividend, 8, a common divisor of 8 and 16, is also a divisor of the 24.
- 198. To find all the divisors common to two or more numbers.
  - Ex. 1. Required all the common divisors of 45 and 135.

    Ans. 1, 3, 5, 9, 15, and 45.

$$\begin{array}{c} 4 \ 5 = 3 \ \times \ 3 \ \times \ 5. \\ 1 \ 3 \ 5 = 3 \ \times \ 3 \ \times \ 3 \ \times \ 5. \end{array}$$

Common  $\begin{cases} 1 & 3 & 9 = 3 \times 3 \\ 5 & 15 & 45 = 3 \times 3 \times 5 \end{cases}$  and these COMMON Prime factors with 1, and all the products we are able to form from

Resolving the given numbers into their prime factors, we find they have of these 3, 3, and 5 in common, and these COMMON prime factors with 1, and all the products we are able to form from them (Art. 194), give

them (Art. 194), give all the common divisors required. When only the number of common divisors is required, it may readily be found by multiplying together the exponents, each increased by 1, of the different COMMON prime factors. (Art. 194.)

#### EXAMPLES.

- 2. What are the common divisors of 51, 153, and 255?
- Required the several common divisors of 180 and 360.
   Ans. 1, 2, 3, 4, 5, 6, 9, 10, 12, 15, 18, 20, 30, 36, 45, 60, 90, and 180.
- 4. How many common divisors have 2025, 6075, and 8100?

  Ans. 15.
- 5. How many common divisors have 4500 and 9000?

  Ans. 36.

THE GREATEST COMMON DIVISOR OR MEASURE.

199. The greatest common divisor or measure of two or more numbers is the greatest number that will divide each of

them without a remainder. Thus, 4 is the greatest common divisor of 8, 12, and 16.

- 200. To find the greatest common divisor of two or more numbers.
- Ex. 1. Required the greatest common divisor or measure of 24 and 88.

  Ans. 8.

$$\begin{array}{c} \textbf{24} = 2 \times 2 \times 2 \times 2 \times 3 \\ \textbf{88} = 2 \times 2 \times 2 \times 11. \\ \textbf{2} \times 2 \times 2 = \textbf{8.} \quad \textbf{Ans.} \end{array}$$

Resolving the numbers into their prime factors, thus,  $24 = 2 \times 2 \times 2 \times 3$ , and  $88 = 2 \times 2 \times 2 \times 11 = 88$ , we find the factors  $2 \times 2 \times 2 \times 2 \times 11 = 88$ , we find the factors  $2 \times 2 \times 2 \times 2 \times 11 = 88$ , we find the factors  $2 \times 2 \times 2 \times 2 \times 11 = 88$ , we find the factors  $2 \times 2 \times 2 \times 2 \times 11 = 88$ , we find the factors  $2 \times 2 \times 2 \times 2 \times 11 = 88$ , we find the factors only these common factors, or the pro-

duct of two or more of such factors, will exactly divide both numbers, it follows that the product of all their common prime factors must be the greatest factor that will exactly divide both of them. Therefore,  $2 \times 2 \times 2 = 8$ , the greatest common divisor required.

The same result may be obtained by a sort of trial process, as by

the second operation.

It is evident, since 24 cannot be exactly divided by a number greater than itself, if it will also exactly divide 88, it will be the greatest common divisor sought. But, on trial, we find 24 will not exactly divide 88, there being a remainder, 16. Therefore 24 is not a common divisor of the two numbers.

We know that a common divisor of 16 and 24 will, also, be a common divisor of 88 (Art. 197). We next try to

find that divisor. It cannot be greater than 16. But 16 will not exactly divide 24, there being a remainder, 8; therefore 16 is not the greatest common divisor.

As before, the common divisor of 8 and 16 will be the common divisor of 24 and 88 (Art. 197); we make trial to find that divisor, knowing that it cannot be greater than 8, and find 8 will exactly divide 16. Therefore 8 is the greatest common divisor required.

The last method may be often contracted, if there should be observed to be any prime factor in a remainder which is not common to the preceding divisor, by canceling said factor. Thus, in the third operation, the factor 2 being found in the remainder 16 once more than in the divisor 24, we cancel one 2 from 16, and, having left the composite factor 8, we divide 24 by that factor. There being no remainder, 8 is the greatest common divisor, as before obtained.

- RULE 1.—Resolve the given numbers into their prime factors. The product of all the factors common to the several numbers will be the greatest common divisor. Or,
- RULE 2. Divide the greater number by the less, and if there be a remainder divide the preceding divisor by it, and so continue dividing until nothing remains. The last divisor will be the greatest common divisor.
- NOTE 1.—When the greatest common divisor is required of more than two numbers, find it of two of them, and then of that common divisor and of one of the other numbers, and so on for all the given numbers. The last common divisor will be the greatest common divisor required.
- NOTE 2.—If any remainder be such a prime number as will not exactly divide the preceding divisor, or if the last remainder be 1, the numbers, being prime to each other, will have no common divisor.

#### EXAMPLES.

- 2. What is the greatest common divisor of 56 and 168?
- 3. What is the greatest common divisor of 96 and 128?
- 4. What is the greatest common measure of 57 and 285?

  Ans. 57.
  - 5. What is the greatest common measure of 169 and 175?
- 6. What is the greatest common measure of 175 and 455?

  Ans. 35.
- 7. What is the greatest common divisor of 169 and 866?

  Ans. 1.
- 8. What is the greatest common measure of 47 and 478?

  Ans. 1.
- 9. What is the greatest common measure of 84 and 1068?

  Ans. 12.
- 10. What is the greatest common divisor of 75 and 165?

  Ans. 15.
- 11. What is the greatest common measure of 78, 234, and 468?

  Ans. 78.
- 12. I have three fields; one containing 16 acres; the second, 20 acres; and the third, 24 acres. Required the largest-sized lots, containing each an exact number of acres, into which the whole can be divided.

  Ans. 4 acre lots.
- 13. A farmer has 12 bushels of oats, 18 bushels of rye, 24 bushels of corn, and 30 bushels of wheat. Required the largest bins, of uniform size, and containing an exact number of bushels, into which the whole can be put, each kind by itself, and all the bins be full.

#### LEAST COMMON MULTIPLE.

201. A common multiple of two or more numbers is a number that can be divided by each of them without a remainder; thus, 14 is a common multiple of 2 and 7.

The least common multiple of two or more numbers is the least number that can be divided by each of them without a remainder; thus, 12 is the least common multiple of 4 and 6.

- 202. A multiple of a number contains all the prime factors of that number; the common multiple of two or more numbers contains all the prime factors of each of the numbers; and the least common multiple of two or more numbers contains only each prime factor taken the greatest number of times it is found in any of the several numbers. Hence,
- 1. The least common multiple of two or more numbers must be the *least* number that will contain all the prime factors of them, and none others.
- 2. The least common multiple of two or more numbers, which are prime to each other, must equal their product.
- 3. The least common multiple of two or more numbers must equal the product of their greatest common divisor, by the factors of each number not common to all the numbers.
  - 4. The least common multiple of two or more numbers, divided by any one of them, must equal the product of those factors of the others not common to the divisor.
  - 203. To find the least common multiple of two or more numbers.
  - Ex. 1. What is the least common multiple of 8, 16, 24, 32, 44.

    Ans. 1056.

$$8 = 2 \times 2 \times 2$$

$$16 = 2 \times 2 \times 2 \times 2$$

$$24 = 2 \times 2 \times 2 \times 3$$

$$32 = 2 \times 2 \times 2 \times 2 \times 2$$

$$44 = 2 \times 2 \times 11$$

 $2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 11 = 1056$  Ans. number of times

factor in any of the given numbers is 5 times; the greatest num13 \*

Resolving the numbers into their prime factors, we find their different prime factors to be 2, 3, and 11. The greatest number of times the 2 occurs as a the greatest num-

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ber of times 3 occurs in any of the numbers is once; and the greatest number of times the 11 occurs in any of the numbers is once. Hence, 2, 2, 2, 2, 3, and 11 must be all the prime factors necessary in composing 8, 16, 24, 32, and 44; and consequently, 1056, the product of these factors, is the least common multiple required (Art. 202).

	SEC	SECOND OPERATION.				
2)8	3 16	24	32	44		
2)4	L 8	12	16	2 2		
2)2	2 4	6	8	11		
2)	. 2	3	4	11		
1	1	3	2	11		
V9V9	V9V	91/2	V11	105		

 $2\times2\times2\times2\times2\times3\times11 = 1056$  Ans.

Having arranged the numbers on a horizontal line, we divide by 2, a prime number that will divide two or more of them without a remainder, and write the quotients in a line below; and we continue to divide by a prime number as before, till

the divisor and remainders are all prime to each other. Then, these, since they include all the factors necessary to form the given numbers and no others, we multiply together for the required least common multiple, and obtain 1056, as before.

The least common multiple of two or more numbers may be found generally by a process much shorter than either of the above methods, by canceling any number that is a factor of any other of the given numbers, and also by dividing the numbers by such a composite number as may be observed to be their common or greatest common divisor.

4) 
$$\frac{816}{243244}$$
2)  $\frac{6811}{3411}$ 
4×2×3×4×11 = 1056 Ans.

Thus, in the third operation, 8 being a factor of several of the numbers, and 16 being a factor of one other number, we cancel them; and observing that 4 is the greatest common divisor of the remaining numbers, we divide them by it.

We next divide by 2, as in the second operation. The numbers in the lower line then being prime to each other, we multiply them and the divisors together, and obtain 1056 as the least common multiple.

 The fourth operation exhibits a process yet more contracted. The 8 and 16 being factors each of one or more of the other numbers, we cancel them, as in the third operation. Of the remain-

ing numbers we cut off 24 by a short vertical line from the rest as a factor of the least common multiple sought. We then strike out of the two remaining numbers the largest factor each has in common with the 24, by dividing each of them by the greatest common divisor

between it and 24, and write the result beneath. The numbers in the lower line having no factor in common, we carry the process no further. The continued product of the number cut off by the numbers in the lower line gives 1056, the least common multiple, as by the other methods. In this instance we cut off the 24, but either the 32 could have been separated from the rest, or the 44 cut off, and the needless factors striken out with like result. If, however, we had cut off the 44, the numbers placed in the second line would have contained factors common to each other, so that it would have been necessary in that line to have cut off and stricken out factors as before. The reason for this abridged process is, that by the separating off, and by the striking out of factors, we get rid, in an expeditious way, of the factors not required to form the least common multiple soughs.

Rule 1.— Resolve the given numbers into their prime factors. The product of these factors, taking each factor only the greatest number of times it occurs in any of the numbers, will be the least common multiple. Or,

Rule 2.— Having arranged the numbers on a horizontal line, cancel such of them as are factors of any of the others, and separate some convenient one from the rest. Reject from each of the numbers remaining the greatest factor common to it and that number, and write the result in a line below. Should there be in the second line numbers having factors in common, proceed as before; and so continue until the numbers written below are prime to each other. The continued product of the number or numbers separated from the others with those in the last line will be the least common multiple.

Note 1.— Some give a preference to the following rule for finding the least common multiple: Having arranged the numbers on a horizontal line, divide by such a prime number as will exactly divide two or more of them, and write the quotients and undivided numbers in a line beneath. So continue to divide until the quotients shall be prime to each other. Then the product of the divisors and the numbers of the last line will be the least common multiple.

Note 2.—The least common multiple of two or more numbers that are prime to each other is found by multiplying them together (Art. 202).

NOTE 3. — When a single number alone is prime to all the rest, it may be separated off, and used only as a factor of the least common multiple sought.

Note 4. — When the least common multiple of several numbers, and all the numbers except one, which is prime to the others, are given, to find the unknown number, divide the least common multiple given by that of the known numbers (Art. 202).

#### EXAMPLES.

- 2. What is the least common multiple of 3, 13, 37, and 91.
- 3. What is the least common multiple of 9, 14, 30, 35, and 47?

  Ans. 29610.

- 4. What is the least common multiple of 6, 8, 10, 18, 20, and 24?
- 5. What is the least common multiple of 14, 19, 38, and 57?

  Ans. 798.
- 6. What is the least common multiple of 20, 36, 48, and 50?

  Ans. 3600.
- 7. What is the least common multiple of 15, 25, 35, 45, and 100?

  Ans. 6300.
- 8. What is the least common multiple of 100, 200, 300, 400, and 575?

  Ans. 27600.
- 9. The least common multiple of 1, 2, 3, 4, 5, 6, 8, 9, and one other number prime to them, is 2520. What is that other number?

  Ans. 7.
- 10. What is the least common multiple of 18, 24, 36, 126, 20, and 48?
- 11. I have four different measures; the first contains 4 quarts, the second 6 quarts, the third 10 quarts, and the fourth 12 quarts. How large is a vessel, that may be filled by each one of these, taken a certain number of times full? Ans. 60 quarts.
- 12. What is the smallest sum of money with which I can purchase a number of oxen at \$50 each, cows at \$40 each, or horses at \$75 each?

  Ans. \$600.

# MISCELLANEOUS EXAMPLES.

- 1. How many times does 7 occur as a factor of 6174? Ans. 3 times.
- 2. Required the largest prime factor of 5775.
- 3. Required the largest composite factor of 19929.

Ans. 6643.

- 4. Required the quotients of 2338 divided by its two prime factors next larger than 1.

  Ans. 1169; 334.
- 5. Required all the prime numbers that will divide 17385 without a remainder.

6. A farmer has 3000 bushels of grain; which are the three smallest-sized bags, and the three largest-sized bins, holding an exact number of bushels, that will each measure the same without a remainder?

Ans. Bags of 1, 2, or 3 bushels each; and bins of 1500, 1000, or 750 bushels each.

- 7. A teacher having a school consisting of 152 ladies and 136 gentlemen, divided it in such a manner that each class of ladies equalled each class of gentlemen, and the classes were the largest the school would admit of, and have them all of the same size. Required the number of classes, and the number in each class.
  - Ans. 19 classes of ladies, 17 classes of gentlemen, and 8 pupils in a class.
- 8. At noon the second, minute, and hour hand of a clock are together; how long after will they be again, for the first time, in the same position?
- 9. J. Porter has a four-sided garden, the first side of which is 348 feet in length; the second, 372 feet; the third, 444 feet; and the fourth, 492 feet. Required the length of the longest rails that can be used in fencing it, allowing the end of each rail to lap by the other 9 inches, and all the panels to be of equal length; also, the number of rails, if 5 rails be allowed to each panel.

  Ans. Length 12ft. 9in.; and 690 rails.
- 10. L. Ford has 5 pieces of land, the first containing 3A. 2R. 1p.; the second, 5A. 3R. 15p.; the third, 8A. 29p.; the fourth, 12A. 3R. 17p.; and the fifth, 15A. 31p. Required the largest sized house-lots, containing each an exact number of square rods, into which the whole can be divided.

Ans. 1A. 27p. each.

- 11. What three numbers between 30 and 140 have 12 for their greatest common divisor, and 2772 for their least common multiple.

  Ans. 36, 84, and 132.
- 12. Four men, A, B, C, and D, are engaged in making regular excursions into the country, between which each stays at home just 1 day; and A is always absent exactly 3 days, B 5 days, and C and D 7 days. Provided they all start off on the same day, how many days must elapse before they can all be at home again on the same day?

  Ans. 23 days.



#### COMMON FRACTIONS.

- 204. A FRACTION is an expression denoting one or more equal parts of a unit.
- 205. A fractional unit is one of the equal parts into which the whole thing or integral unit has been divided. Thus halves, thirds, &c., being equal parts of integral units or whole things, are fractional units.
- 206. The unit of a fraction is the unit or whole thing from which its fractional parts have been derived.
- 207. A COMMON FRACTION is expressed by two numbers one above the other, with a line between them.
- 208. The number below the line is called the *denominator*. It shows into how many parts the whole number has been divided. It gives *name* to the fraction and value to the fractional unit. Thus, in the expression 3, the denominator is 7, indicating that the unit of the fraction has been divided into 7 equal parts, and that the value of the fractional unit is one seventh.

The number above the line is called the *numerator*. It shows how many parts have been taken, or *numbers* the fractional units expressed by the fraction. Thus, in the expression 2, the numerator is 2, indicating that the fractional unit, which is one seventh, has been taken 2 times.

- **209.** The *terms* of a fraction are its numerator and denominator. Thus, the terms of the fraction  $\frac{2}{3}$  are the numerator 2 and the denominator 3.
- **210.** A proper fraction is one whose numerator is less than the denominator; as  $\frac{3}{5}$ ,  $\frac{2}{3}$ ,  $\frac{7}{3}$ .
- 211. An *improper* fraction is one whose numerator is equal to, or greater than, the denominator; as 4, 14, 15.
- 212. A mixed number is a whole number with a fraction; as  $3\frac{1}{2}$ ,  $16\frac{1}{3}$ ,  $90\frac{1}{6}$ .
- 213. A simple or single fraction has but one numerator and one denominator. It may be either proper or improper; as  $\frac{7}{4}$ ,  $\frac{13}{4}$ .

- 214. A compound fraction is a fraction of a fraction, or two or more fractions connected by the word of; as  $\frac{7}{4}$  of  $\frac{7}{3}$  of  $\frac{7}{3}$  of  $\frac{7}{3}$  of  $\frac{7}{3}$ .
- 215. A complex fraction is a fraction having a fraction or a mixed number for its numerator or denominator, or both; as  $\frac{1}{3}$ ,  $\frac{7}{4\frac{1}{4}}$ ,  $\frac{3\frac{1}{6}}{13}$ ,  $\frac{2\frac{1}{3}}{4\frac{1}{4}}$ .
- **216.** A fraction is an expression of division; the numerator answering to the dividend, and the denominator to the divisor (Art. 67); and the value of a fraction is the quotient arising from the division of the numerator by the denominator (Art. 80). Thus, in the fraction  $\frac{1}{7}$ , the numerator 15 is the dividend, the denominator 7 is the divisor, and the value expressed  $2\frac{1}{7}$ , or the quotient arising from the division of the 15 by the 7.
  - 217. Since a fraction is an expression of division, it follows,
- 1. That, if the numerator be multiplied, or the denominator be divided, by any number, the fraction is multiplied by the same number (Art. 81).
- 2. That, if the numerator be divided, or the denominator multiplied, by any number, the fraction is divided by the same number (Art. 82).
- 3. That, if the numerator and denominator be both multiplied, or both divided, by the same number, the fraction will not be changed in value (Art. 83).

#### REDUCTION OF COMMON FRACTIONS.

- 218. Reduction of fractions is the process of changing their form of expression without altering their value.
- 219. A fraction is in its lowest terms, when its numerator and denominator are prime to each other (Art. 166).
  - 220. To reduce a fraction to its lowest terms.
  - Ex. 1. Reduce  $\frac{1}{4}$  to its lowest terms.

Ans. 3.

FIRST OPERATION.

4)  $\frac{16}{48} = \frac{4}{12}$ 4)  $\frac{4}{42} = \frac{1}{3}$  Ans.

By dividing both terms of the fraction by 4, a factor common to them both, it is reduced to  $\frac{4}{12}$ . Dividing both terms of  $\frac{4}{12}$  by 4, a factor common to them both, it is re-

duced to 1. Now, as 1 and 3 are prime to each other, the fraction 1 is in its lowest terms.

The same result is often more readily obtained by dividing the terms of the fraction by their greatest common divisor, as by the second operation.

Since dividing the numerator and denominator of a fraction by the same number, or cancelling equal factors in both, changes only the form of the fraction, while the volume expressed remains unchanged (Art. 217).

Rule. — Divide the numerator and denominator by any number greater than 1 that will divide them both without a remainder, and thus proceed until they are prime to each other. Or,

Divide both the numerator and denominator by their greatest common

divisor.

#### EXAMPLES.

2.	Reduce 15 to its lowest terms.	Ans. 🛧
3.	Reduce 34 to its lowest terms.	Ans. $\frac{2}{4}$ .
4.	Reduce # to its lowest terms.	Ans. 3.
5.	Reduce 108 to its lowest terms.	
6.	Reduce $\frac{15}{175}$ to its lowest terms.	Ans. $\frac{3}{35}$ .
7.	Reduce \$\frac{162}{844}\$ to its lowest terms.	Ans. $\frac{81}{422}$ .
8.	Reduce $\frac{6}{43}\frac{17}{19}$ to its lowest terms.	,
9.	Reduce \$\frac{4}{5}\$ to its lowest terms.	Ans. 476.
10.	Reduce \$11 to its lowest terms.	Ans. $\frac{811}{1116}$ .
11.	Reduce § 15 to its lowest terms:	
12.	Reduce \$16 to its lowest terms.	Ans. 488.
13.	Reduce $\frac{123}{56}$ to its lowest terms.	Ans. 152.

221. To reduce an improper fraction to an equivalent whole or mixed number.

Ex. 1. How many yards in  $\frac{1}{19}$  of a yard? Ans.  $6\frac{3}{19}$ .

 $\begin{array}{c} 1\ 9\ )\ 1\ 1\ 7\ (\ 6_{19}^{3}\ \text{Ans.} \\ \hline 1\ 1\ 4 \\ \hline 3 \end{array}$ 

Since 19 nineteenths make one yard, it is evident there will be as many yards in 117 nineteenths as 19 is contained times in 117, which is  $6\frac{3}{15}$  times. Therefore,  $6\frac{3}{15}$  yards is the answer required.

Rule. — Divide the numerator by the denominator.

NOTE. — Should there a remainder occur, write it over the denominator, and make this fraction a part of the answer.

#### EXAMPLES.

2. Reduce $\frac{167}{15}$ to a mixed number.	Ans. $11_{75}^2$ .
3. Reduce $\frac{1631}{116}$ to a mixed number.	Ans. $14\frac{7}{176}$ .
4. Reduce $\frac{131}{17}$ to a mixed number.	
5. Reduce $\frac{617}{61}$ to a mixed number.	Ans. 3134.
6. Change $\frac{1000}{9}$ to a mixed number.	Ans. $111\frac{1}{9}$ .
7. Change $\frac{4}{1}\frac{23}{5}$ to a mixed number	Ans. $91\frac{28}{45}$ .
8. Change 125 to a whole number.	Ans. 125.
9. Change $\frac{37}{37}$ to a whole number.	

222. To reduce a whole or mixed number to an improper fraction.

Ex. 1. Reduce 19 to a fraction whose denominator shall be 7.

operation. Since there are 7 sevenths in 1  $19 \times 7 = 133$ . Whole one, 19 whole ones = 133 sevenths =  $\frac{133}{2}$ , Ans.

2. Reduce  $17\frac{3}{5}$  to an improper fraction? Ans.  $\frac{9.8}{5}$ .

17 <del>3</del> 5	Since there are 5 fifths in 1 whole one, in 17 whole ones there are 85 fifths, and
8 5 3	adding 3 fifths for the fraction, we have as the equivalent of 173. Hence the
$\overline{88}$ fifths $=\frac{81}{5}$	8, Ans.

RULE. — Multiply the whole number by the given denominator, and to the product add the numerator of the fractional part, if any; and write the result over the denominator.

NOTE. — A whole number may be expressed in its simplest fractional form, by taking it for a numerator with 1 for a denominator. Thus, 4 may be written 4, and read 4 ones.

# EXAMPLES.

3. Reduce 15 to fourths.
4. Reduce 161 1/5 to sixteenths.
5. Reduce 171 5/5 to an improper fraction.
6. Change 11 to a fractional form.
Ans. 10145.
Ans. 11.

7. Change 100 to an improper fraction.

N 14

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8. Change 5 to a fraction whose denominator shall be 17.

Ans. §5.

9. Reduce 9844 to an improper fraction. Ans. 2554.

10. Reduce 11631 to an improper fraction. Ans. 7197.

11. 718  $\frac{69}{9}$  equal how many ninety-sevenths? Ans.  $\frac{69}{97}$ .

12. Reduce 100+88 to an improper fraction. Ans. 29880.

13. Reduce 7 to an improper fraction.

Reduce 19 to a fraction whose denominator shall be 13.
 Ans. 247.

15. 1161 yards equal how many fourths of a yard?

Ans. 465 fourths.

223. To reduce a compound fraction to a simple fraction.

Ex. 1. Reduce  $\frac{3}{8}$  of  $\frac{7}{8}$  to a simple fraction. Ans.  $\frac{21}{32}$ .

By multiplying the denominator of  $\frac{7}{4}$  by  $\frac{3}{4}$ ,  $\frac{7}{8} = \frac{21}{32}$ , Ans.

Ans.

By multiplying the denominator of  $\frac{7}{4}$ , it is evident, we obtain  $\frac{1}{4}$  of  $\frac{7}{4} = \frac{7}{32}$ , since the parts into which the number is divided are 4 times as many, and consequently only  $\frac{7}{4}$  as large as before; and since  $\frac{7}{4}$  of  $\frac{7}{4} = \frac{7}{34}$ ,  $\frac{7}{4}$  of  $\frac{7}{4}$  will be 3 times  $\frac{7}{32} = \frac{23}{34}$ .

RULE. — Multiply all the numerators together for a new numerator, and all the denominators for a new denominator.

Note 1. — All whole and mixed numbers in the compound fraction must be reduced to improper fractions, before multiplying.

NOTE 2. — When there are factors common to both numerator and denominator, they may be cancelled in the operation.

#### EXAMPLES.

2. Reduce \$\frac{3}{7}\$ of \$\frac{1}{2}\frac{5}{5}\$ of \$\frac{2}{3}\frac{3}{5}\$ to a simple fraction. Ans. \$\frac{1}{1}\$.

$$\frac{1}{3} \times \frac{7}{15} \times \frac{15}{29} \times \frac{29}{33} = \frac{1}{11}$$
, Ans.

3. What is  $\frac{2}{3}$  of  $\frac{5}{6}$  of  $\frac{7}{6}$  of  $\frac{11}{12}$ ? Ans.  $\frac{770}{1728} = \frac{385}{862}$ .

4. What is \$\frac{6}{7}\$ of \$\frac{1}{15}\$ of \$\frac{3}{5}\$ of \$\frac{1}{18}\$?

5. Reduce \$ of \$ of \$ of 11 to a simple fraction.

Ans.  $\frac{165}{1456}$ .

6. What is the value of  $\frac{6}{11}$  of  $\frac{6}{7}$  of  $\frac{1}{4}$  of 21?

Ans.  $\frac{756}{308} = 25$ 

- 7. What is the value of  $\frac{7}{11}$  of  $15\frac{7}{8}$  of  $5\frac{7}{10}$  of 100? Ans.  $5758\frac{13}{44}$ .
- 8. What is \( \frac{3}{4} \) of \( \frac{4}{2} \) f?
- 9. What is the value of  $\frac{7}{11}$  of  $\frac{1}{2}$  of  $\frac{2}{3}$  of  $\frac{2}{3}$ ?

Ans. \$1.75.

10. What is the value of \( \frac{4}{5} \) of \( \frac{9}{17} \) of \( \frac{17}{18} \) of 3\( \frac{3}{5} \) gallons?

Ans. 4 gal.

- 11. What part of a ship is 1 of 2 of 3?
- 12. What is the value of  $\frac{4}{7}$  of  $\frac{6}{18}$  of  $\frac{14}{18}$  of  $\frac{4}{18}$  of  $\frac{4$

#### A COMMON DENOMINATOR.

- 224. Fractions have a common denominator when all their denominators are alike.
- 225. A common denominator of two or more fractions is a common multiple of their denominators; and their least common denominator is the least common multiple of their denominators.
  - 226. To reduce fractions to a common denominator.
- Ex. 1. Reduce  $\frac{7}{8}$ ,  $\frac{5}{12}$ , and  $\frac{11}{16}$  to other fractions of equal value, having a common denominator.

#### FIRST OPERATION.

 $8 \times 12 \times 16 = 1536$  common denominator.

We first multiply the numerator of  $\frac{1}{8}$  by the denominators 12 and 16, and obtain 1344 for a new numerator. We next multiply the numerator of  $\frac{1}{12}$  by the denominators 8 and 16, and obtain 640 for a new numerator; and then we multiply the numerator of  $\frac{1}{18}$  by the denominators 8 and 12, and obtain 1056 for a new numerator. Finally, we multiply all the denominators together for a common denominator, and write it under the several numerators, as in the operation.

By this process, since the numerator and denominator of each fraction are multiplied by the same numbers, their relation to each other is not changed, and the value of the fraction remains the same. (Art. 217.)

#### - SECOND OPERATION.

48 least common denominator.

8 12 | 16 8 
$$6 \times 7 = 42$$
, new numerator.  $\frac{7}{8} = \frac{43}{48}$  Ans.  $\frac{12}{3} \times 12 \times 5 = 20$ , "  $\frac{7}{2} = \frac{23}{48}$  Ans.  $\frac{1}{16} = \frac{33}{48}$ 

 $16 \times 3 = 48$ , least common multiple, and least common denominator.

Having first obtained the least common multiple of all the denominators of the given fractions, we assume this to be their least common denominator. We then take such a part of this number, 48, as is expressed by each of the fractions separately for their respective new numerators. Thus, to get a new numerator for  $\frac{1}{6}$ , we take  $\frac{1}{6}$  of 48, the least common denominator, by dividing it by 8, and multiplying the quotient 6 by 7. We proceed in like manner with each of the fractions, and write the numerators thus obtained over the least common denominator. In this process the value of each fraction remains unchanged, as both terms are multiplied by the same number. (Art. 217.)

The method used in the second operation, it will be perceived, expresses the fractions of the result in lower terms than that used in the first. On this account it is often to be preferred to the other.

Rule. — Find the least common multiple of the denominators for the LEAST COMMON denominator.

Divide the least common denominator by the denominator of each of the given fractions, and multiply the quotients by their respective numerators. Or,

Multiply each numerator by all the denominators except its own, for the new numerators; and all the denominators together for A COMMON denominator.

Note 1. — Compound fractions must be reduced to simple ones, whole and inixed numbers to improper fractions, before finding a common denominator, and all to their lowest terms, before finding the least common denominator.

NOTE 2. — Fractions may sometimes be reduced to a common denominator most readily by multiplying both terms of one or more of them by such a number as will make all the denominators alike. Thus  $\frac{1}{2}$  and  $\frac{1}{4}$  may be brought to a common denominator simply by multiplying both terms of the  $\frac{1}{2}$  by 2, and changing in that way its form to  $\frac{2}{4}$ .

Note 3. — Fractions may often be reduced to lower terms, without destroying their common denominator, by dividing all their numerators and denominators by a common divisor.

#### Examples.

Reduce the following fractions to their least common denominator: —

- 2. Reduce  $\frac{3}{4}$ ,  $\frac{5}{8}$ ,  $\frac{7}{8}$ , and  $\frac{5}{12}$ . Ans.  $\frac{54}{72}$ ,  $\frac{49}{72}$ ,  $\frac{63}{72}$ ,  $\frac{39}{72}$ .
- 3. Reduce  $\frac{6}{11}$ ,  $\frac{8}{15}$ ,  $\frac{19}{20}$ , and  $\frac{5}{24}$ . Ans.  $\frac{720}{1320}$ ,  $\frac{704}{1320}$ ,  $\frac{1254}{1320}$ ,  $\frac{275}{1320}$ .

4. Reduce 1, 3, 3, and 28. 5. Reduce 19, 4, 11, and 1. Ans.  $\frac{32}{42}$ ,  $\frac{24}{42}$ ,  $\frac{33}{42}$ ,  $\frac{21}{42}$ . 6. Reduce 15, 3, 4, and 2. Ans. 210, 84, 176, 231. Ans. 30, 40, 45, 48. 7. Reduce  $\frac{1}{2}$ ,  $\frac{2}{3}$ ,  $\frac{3}{4}$ , and  $\frac{4}{5}$ . 8. Reduce \$, \$, \frac{1}{2}\$, and \$\frac{7}{11}\$. Ans. 594, 308, 315, 441. 9. Reduce  $\frac{3}{4}$ ,  $\frac{7}{8}$ ,  $\frac{1}{8}$ , and  $3\frac{4}{8}$ . 10. Reduce 12, 3, 4, and 43. Ans. 70, 63, 96, 798. Ans.  $\frac{24}{42}$ ,  $\frac{15}{42}$ ,  $\frac{22}{42}$ ,  $\frac{21}{42}$ . 11. Reduce 4, 15, 11, and 1. 12. Reduce  $\frac{8}{9}$ ,  $\frac{7}{12}$ ,  $\frac{1}{18}$ , and  $\frac{2}{21}$ . Ans.  $\frac{234}{252}$ ,  $\frac{147}{252}$ ,  $\frac{238}{252}$ ,  $\frac{24}{252}$ . 13. Reduce  $\frac{19}{20}$ ,  $\frac{7}{30}$ ,  $\frac{11}{40}$ , and  $\frac{1}{50}$ . Ans.  $\frac{570}{600}$ ,  $\frac{140}{600}$ ,  $\frac{165}{600}$ ,  $\frac{12}{600}$ . 14. Reduce  $\frac{7}{8}$ ,  $\frac{7}{12}$ ,  $\frac{7}{16}$ , and  $\frac{7}{20}$ . Ans. 210, 140, 105, 84. 15. Reduce \$, 7, 8, and 54. Reduce the following fractions to a common denominator: — 16. Reduce  $\frac{2}{3}$ ,  $\frac{4}{5}$ , and  $\frac{7}{8}$  to fractions having a common de-Ans.  $\frac{80}{120}$ ,  $\frac{96}{120}$ ,  $\frac{105}{120}$ . nominator. Ans. 360, 560, 189 17. Reduce 4, 8, and 3. 18. Reduce  $\frac{6}{11}$ ,  $\frac{4}{7}$ , and  $\frac{8}{13}$ . Ans. 546, 572, 616. 19. Reduce  $\frac{7}{12}$ ,  $\frac{4}{13}$ , and  $7\frac{3}{4}$ . 20. Reduce  $\frac{14}{15}$ ,  $\frac{4}{5}$ , and  $\frac{4}{15}$ . Ans. 1485, 1020, 612 21. Reduce  $\S$ ,  $\frac{4}{17}$ , and  $11\frac{7}{15}$ . Ans. 2249, 540, 26316. • 22. Reduce  $\frac{1}{2}$ ,  $\frac{2}{3}$ ,  $\frac{4}{7}$ , and 8. Ans. 21, 28, 24, 336. 23. Reduce  $\frac{4}{9}$ ,  $\frac{7}{11}$ , and  $\frac{2}{3}$  of  $7\frac{5}{9}$ . Ans. 1183, 7188, 6939.

# ADDITION OF COMMON FRACTIONS.

24. Reduce ½, ¾, ⅙, and 17.
25. Reduce ¼, ⅙ of 6, and 21½.

26. Reduce \( \frac{4}{7}, \frac{4}{13}, \frac{4}{7}, \text{ and } \frac{1}{2}. \)

27. Reduce  $\frac{27}{169}$ ,  $\frac{127}{611}$ , and  $\frac{7}{1728}$ .

227. Addition of fractions is the process of finding the value of two or more fractions in one sum.

Ans. 12012, 5096, 5390, 8008, 7007.

Ans. 28506816, 37088064, 722813

NOTE. — Only units of the same kind, whether integral or fractional, can be collected into one sum; if, therefore, the fractions to be added do not express the same fractional unit, they require to be brought to the same, by being reduced to a common or the least common denominator.

228. To add together two or more fractions.

Ans. 110, 576, 2580.

Ex. 1. Add  $\frac{3}{12}$ ,  $\frac{3}{12}$ ,  $\frac{3}{12}$ , and  $\frac{11}{12}$  together. Ans.  $\frac{25}{12}$  = 21.

These fractions  $\frac{2}{12} + \frac{2}{12} + \frac{7}{12} + \frac{1}{12} = \frac{26}{12} = \frac{12}{12} = \frac{21}{12}$ all being treelfths, that is, having 12

for a common denominator, we add their numerators together, and write their sum, 26, over the common denominator, 12. Thus we obtain 38, which, being reduced, = 21, the sum required.

2. What is the sum of  $\frac{7}{8}$ ,  $\frac{5}{12}$ ,  $\frac{11}{18}$ , and  $\frac{13}{28}$ ? Ans. 2444.

Sum of numerators, 631 Least com. denom.,  $\frac{1}{240} = \frac{2151}{240}$ , Ans.

The given fractions not expressing the same kind of fractional unit, we reduce them to their least common denominator, and thus make the fractional parts all of the same kind. The fractions now all expressing two-hundred-fortieths, we add their numerators, and write the result, 631, over the least common denominator, 240, and obtain \$\$\$ = 2\$\$\$, the answer required.

RULE. — Reduce the fractions, if necessary, to a common, or the least common denominator, and write the sum of the numerators over their common denominator.

Note 1. - Mixed numbers must be reduced to improper fractions, and compound fractions to simple fractions, and each fraction to its lowest terms, before attempting to obtain the common denominator.

Note 2. - In adding mixed numbers, the fractional parts may be added separately, and their sum added to the amount of the whole numbers.

# EXAMPLES.

- 3. Add  $\frac{5}{17}$ ,  $\frac{6}{17}$ ,  $\frac{9}{17}$ ,  $\frac{11}{17}$ ,  $\frac{14}{17}$ , and  $\frac{16}{17}$  together. Ans. 319.
- 4. Add  $\frac{4}{23}$ ,  $\frac{9}{23}$ ,  $\frac{11}{23}$ ,  $\frac{13}{23}$ , and  $\frac{19}{23}$  together. Ans. 219.
- 5. What is the sum of  $\frac{4}{87}$ ,  $\frac{1}{87}$ ,  $\frac{29}{87}$ , and  $\frac{7}{87}$ ?
- 6. What is the sum of  $\frac{11}{144}$ ,  $\frac{19}{144}$ ,  $\frac{29}{144}$ , and  $\frac{197}{144}$ ? Ans.  $1\frac{1}{72}$ .
- 7. What is the sum of  $\{\frac{1}{5}, \frac{1}{5}, \frac{1}{$ Ans. 2439.
- 8. Add 1, 7, 11, and 5 together.
- 9. Add  $\frac{7}{11}$ ,  $\frac{9}{22}$ ,  $\frac{5}{3}$ , and  $\frac{1}{6}$  together. Ans. 233.
- 10. Add 3, 78, 16, and 7 together. Ans. 2188.

	11 Add 5 3 16 and 1 tempthon	A 1
•	11. Add $\frac{5}{17}$ , $\frac{3}{34}$ , $\frac{16}{136}$ , and $\frac{1}{2}$ together.	. Ans. 1.
	12. Add $\frac{14}{15}$ , $\frac{17}{25}$ , $\frac{13}{3}$ , and $\frac{9}{10}$ together.	Ans. 318.
	13. Add $\frac{1}{2}$ , $\frac{1}{3}$ , $\frac{1}{4}$ , $\frac{1}{5}$ , $\frac{1}{6}$ , and $\frac{1}{7}$ together.	Ans. $1\frac{83}{140}$ .
	14. Add §, ¾, and 5¾ together.	Ans. 6342.
	15. Add $\frac{4}{11}$ , $\frac{7}{22}$ , and $9\frac{3}{11}$ together.	•••
	16. Add $\frac{3}{4}$ , $\frac{5}{6}$ , and $\frac{43}{4}$ together.	Ans. 63.
	17. Add 4, 71, and 81 together.	Ans. 17 26.
	18. Add 7, 31, and 53 together.	Ans. 9115.
	19. Add 63, 78, and 43 together.	Ans. 18344.
	20. What is the sum of $17\frac{2}{3}$ , $14\frac{1}{2}$ , and $13\frac{2}{3}$ ?	
	21. What is the sum of 163, 87, 93, 31, and	17?
	5.	Ans. 40-4.
	22. What is the sum of $371\frac{1}{12}$ , $614\frac{19}{20}$ , and 8	
		Ans. 106837.
	23. Add $\frac{4}{7}$ of $18\frac{3}{11}$ , and $\frac{11}{12}$ of $\frac{4}{7}$ of $6\frac{3}{11}$ toget	her.
		Ans. 12681.
	24. Add $\frac{1}{8}$ of 18, and $\frac{4}{11}$ of $\frac{11}{12}$ of $\frac{4}{11}$ togeth	ner.

229. To add any two fractions, whose numerators are alike.

Ex. 1. Add \(\frac{1}{4}\) to \(\frac{1}{4}\).

Ans.  $\frac{9}{20}$ .

Sum of the denominators,  $5+4=\frac{9}{20}$  We first find the sum of the denominators,  $4\times 5=\frac{9}{20}$  sum of the denominators, which is 9, and then their

product, which is 20; and the 9 being written as a numerator of fraction, and the 20 as its denominator, the result,  $\frac{9}{20}$ , is the answer required. The reason of the operation is, that the process reduces the fractions to a common denominator, and then adds their numerators. Hence, to add two fractions whose numerators are a unit,

Write the sum of the given denominators over their product.

2. Add 3 to 3.

Ans.  $1\frac{7}{20}$ .

Sum of the denominators X

by one of the numerators,  $\frac{(4+5)\times 3}{4\times 5} = \frac{27}{20} = 1_{20}^{7}$ , Ans.

OPERATION.

By multiplying the sum of the denominators by one of the numerators for a new numerator, and the denominators together for a new denominator, we reduce the fractions to a common denominator, and add their numerators, and thus obtain  $\frac{27}{20} = 1\frac{7}{20}$ , the answer required. Hence, to add fractions whose numerators are alike, and greater than a unit,

Write the product of the sum of the given denominators by one of the numerators over the product of the denominators.

## EXAMPLES.

- 8. Add 12 to 1, \(\frac{1}{3}\) to \(\frac{1}{2}\), \(\frac{1}{3}\) to \(\frac{1}{3}\), \(\frac{1}{3}\) to \(\frac{1}{3}\).
- 4. Add 1 to 1, 1 to 1.
- 5. Add to to 1, to 1.
- 6. Add  $\frac{1}{4}$  to  $\frac{1}{2}$ ,  $\frac{1}{4}$  to  $\frac{1}{3}$ ,  $\frac{1}{4}$  to  $\frac{1}{4}$ ,  $\frac{1}{4}$  to  $\frac{1}{6}$ ,  $\frac{1}{4}$  to  $\frac{1}{4}$ ,  $\frac{1}{4}$  to  $\frac{1}{8}$ .
- 7. Add \( \frac{1}{2} \) to \( \frac{1}{2} \), \( \frac{1}{2} \) to \( \frac{1}{2} \).
- 8. Add  $\neq$  to  $\frac{1}{2}$ ,  $\neq$  to  $\frac{1}{3}$ ,  $\neq$  to  $\frac{1}{4}$ ,  $\neq$  to  $\frac{1}{6}$ ,  $\neq$  to  $\frac{1}{6}$ ,  $\neq$  to  $\frac{1}{6}$ ,  $\neq$  to  $\frac{1}{6}$ .
- 9. Add \( \frac{1}{2} \) to \( \frac{1}{2} \), \( \frac{1}{2} \), \( \frac{1}{2} \) to \( \frac{1}{2} \), \
- 10. Add \( \frac{8}{2} \) to \( \frac{8}{11} \), \( \frac{8}{2} \) to \( \frac{8}{13} \), \( \frac{8}{2} \) to \( \frac{8}{15} \), \( \frac{8}{2} \) to \( \frac{8}{17} \), \( \frac{8}{2} \) to \( \frac{8}{19} \), \( \frac{8}{2} \) to \( \frac{21}{21} \).
- 11. Add  $\frac{4}{5}$  to  $\frac{4}{5}$ ,  $\frac{4}{5}$  to  $\frac{4}{6}$ ,  $\frac{4}{5}$  to  $\frac{2}{6}$ ,  $\frac{2}{5}$  to  $\frac{2}{6}$ ,  $\frac{2}{5}$  to  $\frac{2}{11}$ ,  $\frac{3}{5}$  to  $\frac{2}{11}$ .
- 12. Add \(\frac{3}{2}\) to \(\frac{3}{17}\), \(\frac{3}{2}\) to \(\frac{3}{2}\), \(\frac{3}{2}\) to \(\frac{5}{2}\), \(\frac{3}{2}\) to \(\frac{5}{2}\).
- 14. Add \( \frac{8}{3} \) to \( \frac{8}{17} \), \( \frac{8}{17} \) to \( \frac{8}{17} \), \( \frac{8}{17} \) to \( \frac{8}{13} \), \( \frac{8}{13} \) to \( \frac{8}{15} \), \( \frac{8}{13} \) to \( \frac{8}{17} \), \( \frac{8}{13} \) to \( \frac{8}{19} \).
- 15. Add 10 to 17, 10 to 14, 10 to 18, 10 to 17, 10 to 18.

#### SUBTRACTION OF COMMON FRACTIONS.

230. Subtraction of Fractions is the process of finding the difference between two fractions.

NOTE. — When the fractions express different fractional units, they require to be brought to those of the same kind before the subtraction can be performed.

To subtract one fraction from another.

Ex. 1. From  $\frac{1}{12}$  take  $\frac{5}{12}$ .

Ans.  $\frac{6}{12} = \frac{1}{2}$ .

The fractions both being twelfths, having 12 for a common denominator, we subtract the less numerator from the greater, and write the difference, 6, over the common denominator, 12. Thus, we have  $\frac{4}{12}$  as the difference required.

- 2. From +9 take #.

Ans. 36.
The given frac-

OPERATION.

77 common denominator.

$$\begin{array}{c|c}
1 & 7 \times 1 & 0 = 7 & 0 \\
7 & 1 & 1 \times 4 = 4 & 4 \\
\hline
2 & 6 & dif. of numerators. \\
\hline
7 & common denominator.
\end{array}$$

we reduce them to a common denominator, and thus make the fractional parts all

tions not expressing the same kind

of fractional unit.

of the same kind. We next find the difference of the new numerators, which we write over the common denominator, and obtain \$4, the answer required.

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RULE. — Reduce the fractions, if necessary, to a common, or the least common denominator. Write the difference of the numerators over their common denominator.

NOTE. — If the minuend or subtrahend, or both, are compound fractions, they must be reduced to simple ones.

## EXAMPLES.

3.	Subtract $\frac{6}{17}$ from $\frac{13}{17}$ .	Ans. $\frac{7}{17}$ .
4.	Subtract $\frac{4}{19}$ from $\frac{13}{3}$ .	Ans. 19.
5.	From $\frac{23}{37}$ take $\frac{8}{37}$ .	
6.	From \frac{12}{3} take \frac{17}{3}.	Ans. $\frac{2}{39}$ .
7.	From 23 take 18.	Ans. $\frac{5}{29}$ .
8.	From § take 14.	
9.	Subtract ½‡ from ½9.	Ans. $\frac{1}{4}$ .
10.	Subtract $\frac{16}{144}$ from $\frac{36}{144}$ .	Ans. 35.
11.	Subtract $\frac{19}{160}$ from $\frac{47}{160}$ .	Ans. $\frac{7}{40}$ .
<b>. 12.</b>	Subtract $\frac{327}{1728}$ from $\frac{596}{1728}$ .	Ans. 269.
13.	Subtract $_{1000}^{48}$ from $_{1000}^{100}$ .	Ans. $\frac{13}{250}$ .
` 14.	From $\frac{12}{7}$ take $\frac{5}{34}$ .	
<b>15.</b>	From $\frac{7}{10}$ take $\frac{5}{12}$ .	Ans. $\frac{1}{6}$ .
16.	From 17 take 75.	Ans. 🚼 🛂 🕏 .
17.	From $\frac{13}{18}$ take $\frac{3}{38}$ .	Ans. $\frac{627}{22}$ .
18.	From ½6 take 4.	Ans. $\frac{92}{231}$ .
19.	From $\frac{31}{60}$ take $\frac{3}{10}$ .	
<b>2</b> 0.	From $\frac{17}{25}$ take $\frac{7}{20}$ .	Ans. $\frac{33}{100}$ .
21.	From $\frac{10}{27}$ take $\frac{5}{18}$ .	Ans. 54.
<b>2</b> 2.	From $\frac{3}{28}$ take $\frac{3}{56}$ .	Ans. $\frac{3}{56}$ .
<b>2</b> 3.	From 73 take 5 of 9.	Ans. $1\frac{9}{28}$ .
24.	What is the value of $\frac{3}{5}$ of $8\frac{3}{4}$ — $\frac{2}{3}$ of $5$ ?	
<b>25.</b>	What is the value of $\frac{1}{4}$ of $3 - \frac{1}{3}$ of 2?	Ans. $\frac{1}{12}$ .

231. To subtract a proper fraction or a mixed number from a whole number.

Ex. 1. Fr	om 7 take 3 <del>§</del> .	<b>A</b> ns. $3\frac{3}{8}$ .
From 7 Take 35 Rem. 33	Since we have no fraction the \( \frac{1}{3} \), we must add 1, or its et and say \( \frac{1}{3} \) from \( \frac{1}{3} \) leaves \( \frac{1}{3} \), the line, and carry 1 to the 3 subtract as in subtraction of The result will be obtained, if	qual, \(\frac{8}{8}\), to the minuend, We write the \(\frac{3}{8}\) below in the subtrahend, and simple whole numbers.

Subtract the numerator from the denominator of the fraction, and under the remainder write the denominator, and adding one to the whole number in the subtrahend, subtract the sum from the minuend.

Note. — When the subtrahend is a mixed number, we may reduce it to an improper fraction, and change the whole number in the minuend to a fraction having the same denominator, and then proceed as in Art. 230.

		Exam	PLES.		
2.	3.	4.	5.	6.	7.
From 32	16	671	385	16	18
Take 5	ş 4 <u>3</u>	0 <del>41</del>	1 6 <del>17</del>	$0_{\frac{1}{2}\frac{17}{37}}$	13
Rem. 2 6	111	670 9	36833	$\overline{15\frac{129}{239}}$	161
8.	9.	10.	11.	12.	13.
From 19	27	169	711	46	81
Take 13	<del>3</del> <u>8<del>3</del></u>	$91\frac{14}{23}$	3015	158	4 9 11

232. To subtract one mixed number from another mixed number.

Ex. 1. From 83 take 45.

Ans.  $3\frac{2}{3}\frac{2}{5}$ .

FIRST OPERATION.
From  $8\frac{3}{4} = 8\frac{15}{15}$ Take  $4\frac{4}{5} = \frac{4\frac{25}{35}}{3\frac{25}{35}}$ Rem.

We first reduce the fractional parts to a common denominator, and obtain as their equivalents  $\frac{1}{10}$  find  $\frac{1}{10}$  from  $\frac{1}{10}$  from

carry 1 to the 4 in the subtrahend, and subtract from the 8 above as in subtraction of simple whole numbers.

From  $8\frac{3}{4} = \frac{59}{4} = \frac{295}{35}$ Take  $4\frac{4}{5} = \frac{24}{5} = \frac{168}{35}$ Rem.  $\frac{127}{357} = 3\frac{22}{35}$  In this operation, we reduce the mixed numbers to improper fractions, and these fractions to a common denominator. We then subtract the less fraction from the greater, and, reducing

the remainder to a mixed number, obtain 322, as before. Hence, in performing like examples, we may

Reduce the fractional parts, if necessary, to a common denominator, and subtract the fractional parts and the whole numbers separately. Increase the fractional part of the minuend, when otherwise it would be test than the subtrahend, before subtracting, by as many parts as it takes to make a unit of the fraction (Art. 208), and carry 1 to the whole number of the subtrahend before subtracting it. Or,

Reduce the mixed numbers to improper fractions, then to a common denominator, and subtract the less fraction from the greater.

			EXAMP	LES.		
	2. cwt.	3. Tuns.	<b>4.</b> ●	5. 1b.	6. oz.	7. Miles.
From	18 <del>3</del>	7 3 <del>1</del>	67 <del>1</del>	29 <del>18</del>	1 4 4 <del>3</del>	17144
Take	98	1 6 1 1	$\frac{1.6\frac{2}{3}}{}$	154	9 9 1 3	$91\frac{9}{10}$
	8.	9.	10.	11.	12.	13.
	Furlongs.	Rods.	Inches.	Feet.	Bushels.	Pecks.
From	1017	1654	$77\frac{1}{12}$	8 <del>423</del>	671 <del>/11</del>	17123
Take	934	987	1918	1513	18383	8734

- 14. From a hogshead of wine there leaked out 7 g gallons; what quantity remained?

  Ans. 55 g gal.
- 15. A man engaged to labor 30 days, but was absent  $5\frac{7}{12}$  days; how many days did he work?
- 16. From 144 pounds of sugar there were taken at one time 175 pounds, and at another  $28\frac{7}{12}$  pounds; what quantity remains?

  Ans.  $97\frac{1}{2}$ lb.
- 17. A man sells 97 yards from a piece of cloth containing 34 yards; how many yards remain?

  Ans. 241yd.
- 18. The distance from Boston to Providence is 40 miles. A, having set out from Boston, has travelled  $\frac{2}{17}$  of the distance; and B, having set out at the same time from Providence, has gone  $\frac{2}{11}$  of the distance; how far is A from B?

Ans.  $28_{187}$ m.

- 19. From 1 of a square yard take 1 of a yard square.
- 233. To subtract one fraction from another, when their numerators are alike.

Ex. 1. From 1 take 1.

3

7

le:

1

MA.

13.

15

Ans.  $\frac{4}{21}$ .

OPERATION.

7 — 3 = 4, difference of the denominators.

 $7 \times 3 = 21$ , product of the denominators.

We first find the product of the denominators, which is 21, and then their difference, which is 4, and write the former for the denominator of the required fraction, and the latter for the numerator. By this process the fractions are reduced to a common denominator, and their difference found. Hence, to subtract one fraction from another, whose numerators are a unit, we may

Write the difference of the denominators over their product.

# 2. Take 4 from 3.

Ans. 21.

OPERATION.

Difference of the denominators multiplied by one of the numerators,  $\frac{(7-3)\times 2}{3\times 7}=\frac{8}{21}$ , Ans.

We multiply the difference of the denominators by one of the numerators for a new numerator, and the denominators together for a new denominator, by which process the fractions are reduced to a common denominator, and the difference of their numerators is found. Hence, when the given fractions have their numerators alike and greater than a unit, we may

Write the product of the difference of the given denominators, by one of the numerators, over the product of the denominators.

#### EXAMPLES.

- 3. Take  $\frac{1}{9}$  from  $\frac{1}{6}$ ,  $\frac{1}{2}$ ,  $\frac{1}{3}$ ,  $\frac{1}{4}$ ,  $\frac{1}{5}$ ,  $\frac{1}{7}$ ;  $\frac{1}{19}$  from  $\frac{1}{18}$ ,  $\frac{1}{16}$ ,  $\frac{1}{15}$ .
- 4. Take  $\frac{1}{6}$  from  $\frac{1}{5}$ ,  $\frac{1}{2}$ ,  $\frac{1}{3}$ ,  $\frac{1}{4}$ ;  $\frac{1}{13}$  from  $\frac{1}{4}$ ,  $\frac{1}{3}$ ,  $\frac{1}{5}$ ,  $\frac{1}{7}$ ,  $\frac{1}{8}$ .
- 5. Take  $\frac{1}{5}$  from  $\frac{1}{2}$ ,  $\frac{1}{3}$ ,  $\frac{1}{4}$ ;  $\frac{1}{7}$  from  $\frac{1}{2}$ ,  $\frac{1}{3}$ ,  $\frac{1}{4}$ ,  $\frac{1}{5}$ .
- 6. Take 1 from 10, 1, 1, 1, 1, 1, 1, 1, 1, 1.
- 7. Take \(\frac{1}{4}\) from \(\frac{1}{2}\); \(\frac{1}{4}\) from \(\frac{1}{2}\), \(\frac{1}{3}\); \(\frac{1}{4}\) from \(\frac{1}{8}\).
- 8. Take  $\frac{1}{12}$  from  $\frac{1}{4}$ ,  $\frac{1}{2}$ ,  $\frac{1}{3}$ ,  $\frac{1}{5}$ ,  $\frac{1}{6}$ ,  $\frac{1}{7}$ ,  $\frac{1}{8}$ ,  $\frac{1}{9}$ ,  $\frac{1}{10}$ ,  $\frac{1}{11}$ .
- 9. Take  $\frac{1}{16}$  from  $\frac{1}{3}$ ,  $\frac{1}{2}$ ,  $\frac{1}{4}$ ,  $\frac{1}{5}$ ,  $\frac{1}{6}$ ,  $\frac{1}{7}$ ,  $\frac{1}{8}$ ,  $\frac{1}{9}$ ,  $\frac{1}{10}$ ,  $\frac{1}{11}$ .
- 10. Take  $\frac{1}{10}$  from  $\frac{1}{2}$ ,  $\frac{1}{3}$ ,  $\frac{1}{4}$ ,  $\frac{1}{5}$ ,  $\frac{1}{6}$ ,  $\frac{1}{7}$ ,  $\frac{1}{8}$ ,  $\frac{1}{9}$ .
- 11. From  $\frac{1}{4}$  take  $\frac{1}{9}$ ;  $\frac{1}{10}$  take  $\frac{1}{20}$ ;  $\frac{1}{12}$  take  $\frac{1}{16}$ ;  $\frac{1}{17}$  take  $\frac{1}{25}$ .
- 12. Take  $\frac{3}{5}$  from  $\frac{3}{4}$ ;  $\frac{3}{8}$  from  $\frac{3}{7}$ ;  $\frac{3}{10}$  from  $\frac{3}{4}$ ;  $\frac{3}{11}$  from  $\frac{3}{5}$ .
- 13. Take \$ from \$; A from \$; A from \$; A from \$.
- 14. Take \$\frac{5}{2}\$ from \$\frac{5}{2}\$; \$\frac{15}{17}\$ from \$\frac{5}{2}\$; \$\frac{15}{17}\$ from \$\frac{5}{2}\$.
- 15. Take  $_{11}^{6}$  from  $_{7}^{6}$ ;  $_{11}^{6}$  from  $_{8}^{6}$ ;  $_{13}^{6}$  from  $_{7}^{6}$ ;  $_{13}^{6}$  from  $_{11}^{6}$ .
- 16. Take  $\frac{4}{15}$  from  $\frac{4}{5}$ ;  $\frac{4}{15}$  from  $\frac{4}{7}$ ;  $\frac{4}{15}$  from  $\frac{4}{9}$ ;  $\frac{4}{15}$  from  $\frac{4}{17}$ .
- 17. Take  $\frac{8}{11}$  from  $\frac{8}{9}$ ;  $\frac{8}{11}$  from  $\frac{8}{7}$ ;  $\frac{8}{11}$  from  $\frac{8}{5}$ ;  $\frac{8}{11}$  from  $\frac{8}{3}$ .
- 18. Take  $\frac{2}{11}$  from  $\frac{2}{5}$ ;  $\frac{2}{11}$  from  $\frac{2}{9}$ ;  $\frac{2}{11}$  from  $\frac{2}{3}$ ;  $\frac{2}{11}$  from  $\frac{2}{7}$ .
- 19. Take 18 from 19; 18 from 19; 28 from 18; 19 from 19.
- 20. Take  $\frac{12}{5}$  from  $\frac{12}{5}$ ;  $\frac{12}{7}$  from  $\frac{12}{5}$ ;  $\frac{12}{5}$  from  $\frac{12}{5}$ ;  $\frac{12}{12}$  from  $\frac{12}{7}$ .

# MISCELLANEOUS EXAMPLES IN ADDITION AND SUBTRACTION OF FRACTIONS.

1. A benevolent man has given to one poor family  $\frac{3}{4}$  of a cord of wood, to another  $\frac{1}{2}$  of a cord, and to a third  $\frac{7}{4}$  of a cord; how much has he given to them all?

Ans.  $2\frac{1}{4}$  cords.

- 2. I have paid for a knife  $\$ \frac{1}{6}$ , for a Common School Arithmetic  $\$ \frac{1}{2}$ , for a slate  $\$ \frac{1}{3}$ , and for stationery  $\$ \frac{1}{6}$ ; what did I pay for the whole?
- 3. R. Howland travelled one day  $20\frac{7}{16}$  miles, another day  $19\frac{1}{2}$  miles, and a third day  $22\frac{1}{16}$  miles; what was the whole distance travelled?

  Ans.  $62\frac{2}{16}$  miles.
- 4. I have bought  $6\frac{1}{2}$  tons of anthracite coal,  $19\frac{4}{5}$  tons of Cumberland coal, and  $3\frac{3}{4}$  tons of cannel coal; what is the whole quantity purchased?

  Ans.  $30\frac{1}{20}$  tons.
- 5. There is a pole standing  $\frac{1}{2}$  in the mud,  $\frac{1}{3}$  in the water, and the remainder above the water; what portion of it is above the water?
- 6. F. Adams, having a lot of sheep, sold at one time  $\frac{2}{3}$  of them, and at another time  $\frac{1}{4}$  of the remainder; what portion of the original number had he then left?

  Ans.  $\frac{1}{4}$ .
- 7. From a piece of calico containing  $31\frac{1}{4}$  yards there have been sold  $11\frac{5}{8}$  yards,  $9\frac{1}{2}$  yards, and  $3\frac{3}{4}$  yards; how much remains?
- 8. From a cask of molasses containing  $84\frac{3}{8}$  gallons, there were drawn at one time  $4\frac{3}{7}$  gallons, at another time 11 gallons; at a third time  $26\frac{1}{7}$  gallons were drawn, and  $\frac{1}{7}$  of  $7\frac{1}{7}$  gallons returned to the cask; and at a fourth time  $13\frac{8}{11}$  gallons were drawn, and  $3\frac{1}{7}$  gallons of it returned to the cask. How much then remained in the cask?

  Ans.  $35\frac{5}{7}\frac{7}{7}$  gal.
- 9. A merchant had 3 pieces of cloth, containing, respectively, 19\(\frac{3}{4}\) yards, 36\(\frac{1}{3}\) yards, and 33\(\frac{5}{4}\) yards. After selling several yards from each piece, he\_found he had left in the aggregate 71\(\frac{3}{8}\) yards. How many yards had he sold?

  Ans. 18\(\frac{1}{3}\).

#### MULTIPLICATION OF COMMON FRACTIONS.

234. MULTIPLICATION of Fractions is the process of multiplying when the multiplier, or multiplicand, or both, are fractional numbers.

NOTE. — If the multiplier is less than 1, only such a part of the multiplicand is taken as the multiplier is of 1. Therefore, the product resulting from multiplying a number by a proper fraction is not larger, but less, than the multiplicand.

235. To multiply when one or both of the factors are fractions.

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Multiply  $\sqrt{g}$  by 9. Ex. 1.

Ans. §3 = 31.

FIRST OPERATION.

It is evident that the fraction  $7 \times 9 = 93 = 7 = 31$  Ans. is multiplied by 9 by multiplying its numerator by 9, since

the parts taken, 63, are 9 times as many as before, while the parts into which the unit of the fraction is divided remain the same.

SECOND OPERATION. It is evident, also, that the fraction  $J_{\rm g} \times 9 = J_{\rm g} = 81$  Ans.  $\frac{7}{18}$  is multiplied by 9 by dividing its denominator by 9, since the parts into which the unit of the fraction is divided are only as many, and consequently 9 times as large, as before, while the parts taken remain

Therefore, the same. Multiplying the numerator or dividing the denominator of a fraction by any number multiplies the fraction by that number (Art. 217).

2. Multiply 14 by 3.

Ans. 6.

FIRST OPERATION. 7)14  $2 \times 3 = 6$  Ans.

By dividing the whole number, 14, by 7, the denominator of the fraction, we obtain  $\frac{1}{2}$  of 14 = 2, which multiplied by 3, the numerator of the fraction, gives \$ of 14 = 6.

SECOND OPERATION. 14 3  $42 \div 7 = 6$  Ans.

By multiplying the whole number, 14, by 3, the numerator of the fraction, we obtain 42, a product 7 times as large as it should be, as the multiplier was not 3, a whole number, but  $\frac{2}{7}$ , or  $3 \div 7$ ; hence, we divide the 42 by 7; and thus obtain

 $\clubsuit$  of 14 = 6, as before. Therefore,

Multiplying by a fraction is taking the part of the multiplicand denoted by the multiplier.

3. Multiply 7 by 3.

Ans. 21.

To multiply  $\frac{7}{8}$  by  $\frac{3}{8}$  is to take  $\frac{3}{8}$  of the multiplicand,  $\frac{7}{8}$ . Now, to obtain  $\frac{3}{8}$  of  $\frac{7}{8}$ , we OPERATION.  $\frac{7}{4} \times \frac{3}{5} = \frac{21}{20}$  Ans. multiply the numerators together for a new numerator, and the denominators together for a new denominator (Art. 226). Therefore.

Multiplying one fraction by another is the same as reducing compound fractions to simple ones.

When either of the factors is not a fraction, as in examples first and second, it may be reduced to a fractional form, and then the operation may be like that in the last example. Hence the general

Rule. — Reduce whole or mixed numbers, if any, to improper fractions. Multiply the numerators together for a new numerator, and the denominators together for a new denominator.

Note. — When there are common factors in the numerators and denominators, the operation may be shortened by cancelling those factors.

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# EXAMPLES.

4. Multiply 7 by 11.

Ans. 1.

	OPERATIO	N.
7	11	1
11	× <del>2</del> 1 =	=
	3	

3	
5. Multiply 12 by 4.	Ans. 8\$.
6. Multiply <sup>2</sup> by 12.	•
7. Multiply \(\frac{1}{4}\) by \(\frac{17}{2}\).	Ans. 1/3.
8. Multiply $\frac{3}{7}$ by $\frac{19}{20}$ .	Ans. 157.
9. Multiply \( \frac{4}{5} \) by \( \frac{6}{11} \).	Ans. 24.
10. Multiply $\frac{7}{12}$ by $\frac{1}{16}$ .	Ans. $\frac{77}{192}$ .
11. Multiply 1 by 4.	Ans. 33.
12. Multiply \( \frac{3}{4} \) by \( \frac{15}{16} \).	
13. Multiply \(\frac{16}{17}\) by \(\frac{12}{2}\).	Ans. 👯.
14. Multiply $\frac{9}{10}$ by 14.	Ans. 123.
15. Multiply 13 by 4.	Ans. 73.
16. Multiply 16 by 3	Ans. 218.
17. Multiply 11 by #.	Ans. 67.
18. Multiply 10 by 14.	
19. Multiply § by 19.	Ans. 16§.
20. Multiply $\frac{9}{11}$ by $\frac{22}{27}$ .	Ans. <del>3</del> .
21. Multiply \(\frac{4}{5}\) by \(\frac{1}{5}\).	Ans. 3.
22. Multiply $\frac{3}{19}$ by $\frac{38}{39}$ .	Ans. $\frac{2}{13}$ .
23. Multiply $\frac{7}{9}$ by $\frac{9}{11}$ .	
24. Multiply $\frac{1}{99}$ by $\frac{99}{100}$ .	Ans. 100.
25. Multiply $\frac{3}{7}$ of $\frac{7}{11}$ of $\frac{11}{24}$ by 100.	Ans. $12\frac{1}{2}$ .
26. Multiply ½ of 3 of 7 by 11.	Ans. $3\frac{5}{24}$ .
27. What cost $\frac{7}{12}$ of a ton of hay, at \$17	
	Ans. \$911.
28. What cost $\frac{9}{20}$ of an acre of land, at \$	
29. At § of a dollar per foot, what cost 7	
30. Multiply $161\frac{1}{15}$ by $19\frac{1}{25}$ .	Ans. 313633.
31. Multiply 3 by 83.	Ans. $3\frac{3}{4}$ .
32. Multiply $\frac{9}{10}$ by $17\frac{3}{11}$ .	Ans. $15_{\frac{6}{11}}$ .
33. Multiply \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \	Ans. $63\frac{3}{4}\frac{7}{5}$ .
34. Multiply $\frac{3}{4}$ of $9\frac{1}{4}$ by $\frac{2}{3}$ of 17.	Ans. $78\frac{5}{8}$ .
35. Multiply $\frac{9}{10}$ of 7 by $\frac{11}{15}$ of $87\frac{3}{11}$ .	
36. Multiply 8 by 7.	Ans. $6\frac{2}{6}$ .

37. Multiply 12 by \$. Ans. 84. 38. Multiply 15 by \$1. Ans.  $8\frac{2}{1}$ . 39. A merchant owning 7 of a ship sells 4 of his share to A. What part is that of the whole ship? 40. Multiply 37 by 104. Ans. 3953. 41. Multiply \( \frac{2}{3} \) of 7\( \frac{1}{2} \) by \( \frac{7}{3} \) of 11\( \frac{2}{3} \). Ans. 49133. 42. Multiply 2 of 9 by 3 of 17. Ans. 26.8. 43. Multiply \$ of 8-3 by \$ of 91. Ans.  $25\frac{17}{245}$ . When one of the factors is a whole number, and the 236. other a mixed number, we may Multiply the fractional part and the whole number separately, and add together the products. .. EXAMPLES. 1. Multiply 75 by 9. 2. Multiply 12 by 33. OPERATION. OPERATION. 74 12 9 33 8 × 9 = \frac{1}{2} = 5\frac{1}{2} 3 of 12 =  $7 \times 9$  $12 \times 3 = 36$ 684 Ans. 45 Ans. 3. Multiply 83 by 7. Ans. 604. 4. Multiply 17 by 31. 5. Multiply 13 by 83. Ans. 109#. 6. Multiply 37 by 13.8. Ans. 50719. 7. Multiply 114 by 8. Ans. 944. 8. What cost  $7_{11}^{6}$ lb. of beef at 5 cents per pound? 9. What cost 23 72 bbl. of flour at \$ 6 per barrel? Ans. \$ 1414. What cost 8 3 yd. of cloth at \$5 per yard? Ans. \$41 7.

11. What cost 9 barrels of vinegar at \$63 per barrel? Ans. \$ 573.

12. What cost 12 cords of wood at \$6.371 per cord? Ans. \$76.50.

13. What cost 11cwt. of sugar at \$9\frac{2}{3} per cwt.?

14. What cost 43 bushels of rye at \$1.75 per bushel? Ans. \$ 7.65 §.

15. What cost 7 tons of hay at \$117 per ton?

Ans.  $$83\frac{1}{4}$ .

- 16. What cost 9 dozen of adzes at \$104 per dozen?
- 17. What cost 5 tons of timber at \$3\frac{1}{2} per ton?

Ans. \$154.

18. What cost 15cwt. of rice at \$7.62½ per cwt.?

Ans. \$ 114.374.

19. What cost 40 tons of coal at \$8.37½ per ton?

Ans. \$ 335.

#### DIVISION OF COMMON FRACTIONS.

237. Division of Fractions is the process of dividing when the divisor or dividend, or both, are fractional numbers.

NOTE. — If the divisor is less than 1, the quotient arising from the division will be as many times the dividend as the divisor is contained times in 1. Therefore, the quotient arising from dividing a whole or mixed number by a proper fraction will always be larger than the dividend.

- 238. The reciprocal of a fraction is the number resulting from taking its numerator as denominator, and its denominator as numerator, since any two numbers, whose product is 1, are the reciprocals of each other. Thus, the reciprocal of  $\frac{1}{9}$  is that fraction inverted, or  $\frac{1}{90}$ , since  $\frac{1}{90} \times \frac{1}{90} = 1$ .
- 239. To divide when the divisor or dividend, or both, are fractions.

Ex. 1. Divide + by 7.

Ans. 2.

It is evident that the fraction  $\frac{14}{17}$  is divided by 7 by dividing its numerator by 7, since the size of the parts, as denoted by the denominator, remains the same, while the number of parts taken is only  $\frac{1}{4}$  as large as before.

It is evident the fraction is also ditaken, as denoted by the numerator, remains the same, while the size of the parts is only  $\frac{1}{4}$  as large as before. Therefore,

Dividing the numerator or multiplying the denominator of a fraction by any number divides the fraction by that number (Art. 217).

2. Divide  $\frac{12}{3}$  by  $\frac{6}{13}$ .

Ans. 2.

OPERATION. Since the fractional units of the two fractions are of the same kind, it is evident that 12 thirteenths contain 6

thirteenths as many times as 6 is contained in 12; 12 \div 6 = 2. Ans. Therefore.

When the fractions have a common denominator, the division can be performed as in whole numbers, by dividing the numerator of the dividend by the numerator of the divisor.

3. Divide 4 by 3.

Ans. 144.

FIRST OPERATION.  $4 \div \frac{3}{8} = \frac{32}{16} \div \frac{21}{16} = 1\frac{1}{21}$  Ans. tions to a common denomi-

Having reduced the fracnator, we divide the nu-

merator 32 of the dividend by the numerator 21 of the divisor, as in working the last example, and obtain as the required result 111.

SECOND OPERATION. In the second oper- $\frac{4}{3} + \frac{3}{4} = \frac{4}{3} \times \frac{3}{3} = \frac{32}{2} = \frac{1}{2}$  Ans. ation, we invert the divisor, and then pro-

ceed as in multiplication of fractions (Art. 235). The reason of this process, which in effect reduces the fractions to a common denominator, and divides the numerator of the dividend by that of the divisor, will be seen, if we consider that the divisor, \( \frac{3}{5} \), is an expression denoting that 3 is to be divided by 8. Now, regarding 3 as a whole number, we divide the fraction 4 by it, by multiplying the denominator; thus,  $\frac{4}{3} \times 8 = \frac{4}{21}$ . But the divisor 3 is 8 times as large as it ought to be, since it was to be divided by 8, as seen in the original fraction; then the quotient,  $\frac{4}{21}$ , is  $\frac{1}{8}$  as large as it should be, and must be multiplied by 8; thus,  $\frac{1}{21} \times 8 = \frac{32}{81} = 1\frac{1}{21}$ , the answer, as before. By this operation we have multiplied the dividend by the reciprocal of the divisor, the denominator of the dividend having been multiplied by the numerator of the divisor, and the numerator of the dividend by the denominator of the divisor. Therefore,

Dividing by a fraction is the same as multiplying by its reciprocal.

When either divisor or dividend is not a fraction, it may be changed to a fractional form, and the division performed by the last method. Hence the general

Rule. — Invert the divisor, and then proceed as in multiplication of fractions.

Note 1. - When either divisor or dividend is a whole or mixed number, or a compound fraction, it must be reduced to the form of a simple fraction before dividing.

Note 2. — Factors common to both numerator and denominator should be cancelled.

Note 3. — When the given fractions have a common denominator, the answer may be obtained by dividing the numerator of the dividend by that of the divisor. Also, if the fractions have numerators alike, the answer may be obtained by dividing the denominator of the divisor by that of the dividend.

Note 4. — When the numerator of the divisor will exactly divide the numerator of the dividend, and the denominator of the divisor exactly divide the denominator of the dividend, the division can be effected in that way.

# EXAMPLES,

4. Divide  $\frac{7}{10}$  by  $\frac{7}{62}$ .

Ans. 61.

$$\frac{7}{40} \div \frac{7}{62} = \frac{7}{10} \times \frac{62}{7} = \frac{62}{10} = 6\frac{1}{5} \text{ Ans.}$$
Or  $\frac{7}{10} \div \frac{7}{62} = 62 \div 10 = 6\frac{1}{5} \text{ Ans.}$ 

5. Divide 28 by 3.

Ans. 3.

$$\frac{9}{28} \div \frac{3}{7} = \frac{\cancel{9}}{\cancel{2}\cancel{8}} \times \frac{\cancel{7}}{\cancel{8}} = \frac{8}{4} \text{ Ans.}$$

$$\text{Or } \frac{9}{28} \div \frac{3}{7} = \frac{9}{28} \div \frac{3}{7} = \frac{3}{4} \text{ Ans.}$$

- 6. Divide 7 by 18.
- 7. Divide # by 7.
- 8. Divide 18 by 7.
- 9. Divide 4 by 4.
- 10. Divide 44 by 3.
- 11. Divide 45 by 28. .
- 12. Divide 17 by 27.
- 13. Divide  $\frac{2}{15}$  by 128.
- 14. Divide ++ by 98.
- 15. Divide 14 by 19.
- 16. Divide § by 167.
- 17. Divide 18 by 49.
- 18. Divide 15 by 15.
- 19. Divide 27 by 1/7.
- 20. Divide 128 by 25.
- 21. Divide 98 by 11.
- 22. Divide 19 by 14. 23. Divide 167 by +§.
- 24. Divide 49 by 18.
- 25. Divide 15 by  $\frac{1}{15}$ .
- 26. Divide  $\frac{36}{5}$  by  $\frac{3}{17}$ .
- 27. Divide  $\frac{31}{97}$  by  $\frac{31}{301}$ . Divide & by &.
- 29. Dide 11 by 7.

Ans. 178. Ans. 33.

Ans. 4. Ans.  $\$ = 1 \downarrow$ .

Ans. 415. Ans. 960.

Ans. 1666. Ans. Alf. Ans.  $\frac{5}{1002}$ . Ans.  $\frac{16}{121}$ . Ans.  $\frac{1}{225}$ .

Ans. 960. Ans. 1515. Ans. 31-3. Ans. 2002.

Ans. 225. Ans. 4. Ans. 349. Ans. 18. Ans.  $1_{\frac{16}{105}}$ .

30. Divide 25 by 14.	
31. Divide 16 by 15.	Ans. 113.
32. Divide $\frac{9}{25}$ by $7\frac{3}{4}$ .	Ans. $\frac{36}{775}$ .
33. Divide $\frac{8}{11}$ by $16\frac{4}{3}$ .	Ans. $\frac{18}{407}$ .
34. Divide 11 <sup>3</sup> by 4.	
35. Divide 21 <sup>4</sup> by 18 <sup>4</sup> .	Ans. 1413.
36. Divide $17\frac{3}{11}$ by $28\frac{1}{26}$ .	Ans. 4948.
37. Divide $161\frac{3}{17}$ by $14\frac{3}{5}$ .	Ans. $11_{\frac{49}{1241}}$ .
38. Divide $\frac{7}{11}$ of $\frac{4}{5}$ by $\frac{3}{5}$ of $\frac{8}{11}$ .	Ans. $1\frac{1}{6}$ .
39. Divide $\frac{5}{9}$ of $7\frac{3}{11}$ by $\frac{4}{11}$ of $17\frac{3}{7}$ .	Ans. 359.
40. Divide $\frac{6}{17}$ of 15 by $\frac{7}{15}$ of 22.	

41. Bought  $\frac{7}{8}$  of a coal-mine for \$3675, and having sold  $\frac{5}{8}$  of my share, I gave  $\frac{2}{3}$  of the remainder to a charitable society, and divided the residue among 7 poor persons; what was the share of each?

Ans. \$50 for each poor person.

42. Of an estate valued at \$5000, the widow receives \( \frac{1}{3} \), the oldest son \( \frac{2}{3} \) of the remainder; the residue is equally divided among 7 daughters; what is the share of each daughter?

Ans. \\$158\frac{1}{4}\frac{1}{3}.

# 240. When the dividend is a mixed number, and the divisor a whole number, we may

Divide the integral part of the mixed number as in division of whole numbers, and the remainder divide as in Art. 239; and add together the results for the quotient required.

OPERATION.

Ex. 1. Divide 27 by 6.

. Ans. 43.

 $6)27\frac{3}{5}$ 

 $\overline{4}$ , rem.  $3\frac{2}{5}$ ;  $3\frac{2}{5} = \frac{18}{5}$ ;  $\frac{18}{5} \times 6 = \frac{3}{36} = \frac{3}{5}$ ;  $4 + \frac{3}{5} = 4\frac{2}{5}$ , Ans.

2. Divide 29 by 9. Ans. 3 pg. 3. Divide 14 pg 7. Ans. 2 pg. Ans. 2 pg.

4. Divide 13\frac{3}{8} by 8.

5. Divide 143 by 6. Ans. 233

6. Divide \$37\frac{2}{3} among 9 men. Ans. \$4\frac{1}{3}\frac{1}{3}\frac{1}{3}. Divide \$96\frac{2}{3} among 11 persons. Ans. \$8\frac{2}{3}\frac{1}\frac{1}{3}\frac{1}{3}\frac{1}{3}\frac{1}{3}\frac{1}{3}\frac{1}

8. What is  $\frac{1}{8}$  of  $167\frac{7}{11}$  cwt. of iron? Ans.  $20\frac{2}{2}$  cwt.

9. Divide 7 of a prize, valued at \$1723, equally between 12 seamen.

- 10. What will a barrel of flour cost, if 19 barrels can be purchased for \$107#? Ans. \$ 5.65 54.
- 11. If 15 pounds of raisins can be obtained for \$34, what will 1 pound cost? Ans. \$ 0.2119.
- 12. If 12 quarts of wine cost \$3.75\frac{3}{2}, what will a quart cost?
- 13. If \$19 will buy 375+1 acres of land, how much can be Ans. 19235 acres. bought for \$1?

#### REDUCTION OF COMPLEX FRACTIONS.

- A COMPLEX fraction is one having a fraction in its numerator or denominator, or in both. Thus,  $\frac{3}{1}$  and  $\frac{24}{6}$  are complex fractions.
  - To reduce complex to simple fractions.
- Ex. 1. Reduce  $\frac{6}{5}$  to a simple fraction. Ans. 18. Since the numerator of a fraction  $\frac{2}{5}$  =  $\frac{2}{5}$   $\times$   $\frac{2}{5}$  =  $\frac{16}{45}$ , Ans. is the dividend, and the denominator the divisor (Art. 216), we divide the numerator,  $\frac{2}{5}$ , by the denomination nator,  $\frac{1}{8}$ , as in division of fractions (Art. 239).
  - 2. Reduce  $\frac{7}{14}$  to a simple fraction. Ans.  $\frac{21}{5} = 4\frac{1}{5}$ .

fractions.

$$\frac{7}{1\frac{2}{3}} = \frac{7}{\frac{1}{3}} = \frac{7}{1} \times \frac{3}{5} = \frac{21}{5} = \frac{41}{5}$$
, Ans. We reduce the numerator, 7, and the denominator, 1\frac{2}{3}, to improper fractions,

We reduce the nuand then proceed as in Ex. 1. Hence, to reduce complex to simple

Consider the denominator as a divisor, and the numerator as a dividend, and proceed as in division of fractions (Art. 239).

Note. — Another and often a ready method of reducing a complex fraction is to multiply both its terms by the least common multiple of their denominators.

# EXAMPLES.

3. Reduce  $\frac{4}{61}$  to a simple fraction. Ans. -J.

$$\frac{\frac{1}{6}}{6\frac{1}{4}} = \frac{\frac{1}{4}}{\frac{1}{43}} = \frac{1}{4} \times \frac{2}{13} = \frac{2}{52} = \frac{1}{26}$$
, Ans.

Or, multiply by the least common multiple of the denominators,

$$\frac{1}{6\frac{1}{2}} \times 4 = \frac{1}{26}$$
, Ans.

4. Reduce  $\frac{4\frac{3}{7}}{\frac{2}{3}}$  to a simple fraction. Ans.  $6\frac{9}{14}$ .

5. Reduce  $\frac{\frac{3}{4}}{54}$  to a simple fraction.

6. Reduce  $\frac{7}{4\frac{2}{3}}$  to a simple fraction. Ans.  $1\frac{1}{2}$ .

7. Reduce  $\frac{7\frac{4}{11}}{8}$  to a simple fraction. Ans.  $\frac{81}{8}$ .

8. Reduce  $\frac{6\frac{2}{3}}{8\frac{2}{3}}$  to a simple fraction. Ans.  $\frac{2}{3}$ .

9. Reduce  $\frac{2}{\frac{7}{3}}$  to a simple fraction.

10. Reduce  $\frac{8}{3}$  to a whole number. Ans. 24.

11. Reduce  $\frac{7}{2}$  to a simple fraction. Ans. 7.

12. Reduce  $\frac{5\frac{1}{3}}{\frac{3}{7}}$  to a mixed number. Ans. 124.

13. Reduce  $\frac{\frac{1}{4}}{6\frac{1}{2}}$  to a simple fraction.

14. Reduce  $\frac{3}{2\frac{1}{2}}$  to a mixed number. Ans.  $1\frac{1}{6}$ .

15. Reduce  $\frac{3\frac{1}{4}}{9}$  to a simple fraction. Ans.  $\frac{18}{38}$ .

16. Reduce  $\frac{11\frac{2}{3}}{12\frac{2}{3}}$  to a simple fraction. Ans.  $\frac{25}{3}$ .

17. If 7 were to be the denominator of the fraction whose numerator is  $\frac{7\frac{7}{5}}{11\frac{4}{5}}$ , what would be its value?

18. If  $\frac{3}{5}$  is the numerator of the fraction whose denominator is  $\frac{3}{5}$ , what is its value?

Ans.  $6\frac{13}{27}$ .

243. Complex fractions, after being reduced to simple ones, may be added, subtracted, multiplied, and divided, according to the respective rules for simple fractions.

# EXAMPLES.

1. Add 
$$\frac{1}{3}$$
 of  $\frac{2}{4}$  of  $28\frac{3\frac{6}{4}}{4\frac{7}{4}}$  to  $3\frac{39\frac{1}{2}}{105}$ . Ans.  $6\frac{9}{70}$ .

2. Add 
$$\frac{1}{9}$$
,  $2\frac{5}{9}$ ,  $\frac{45}{94\frac{7}{11}}$ , and  $\frac{47\frac{5}{9}}{314\frac{2}{3}}$  together. Ans.  $3\frac{1425799}{32299932}$ .

3. What is the difference between 
$$\frac{49\frac{5}{97}}{97}$$
 and  $\frac{34\frac{5}{5}}{145\frac{3}{11}}$ ?

Ans.  $\frac{84765}{3100}$ ?

- 4. What is the continued product of the following numbers:  $\frac{27}{37\frac{4}{5}}$ ,  $\frac{87\frac{2}{5}}{98\frac{1}{5}}$ ,  $\frac{7}{2\frac{1}{3}}$  and  $\frac{81\frac{5}{11}}{128}$ ?
  - 5. Divide  $\frac{2}{3}$  of  $7\frac{3}{4}$  by  $\frac{4}{5}$  of  $11\frac{4}{11}$ . Ans.  $\frac{34}{600}$ .
  - 6. Divide \$ of 91 by \$\frac{9}{10}\$ of 87. Ans. \$\frac{3649}{2}\$.

# MISCELLANEOUS EXAMPLES IN MULTIPLICATION AND DIVISION OF FRACTIONS.

- 1. At 23 bushels to an acre, how many bushels of wheat will be required to sow 74 acres?

  Ans. 174.
- 2. Bought 81 bushels of apples for \$4.684; what did they cost per bushel?

  Ans. \$0.571.
- 3. Bought a bale of cloth for \$96 $\frac{3}{8}$ ; I dispose of it for  $\frac{5}{8}$  of the cost, and by so doing I lose \$2 on a yard; required the number of yards in the bale.

  Ans.  $18_{\frac{3}{12}\frac{3}{2}}$ yd.
- 4. If a dividend be 18½ times ½ and a quotient 6½ times ¾, what was the divisor?
- 5. By what number must  $1\frac{1}{3}$  be multiplied, that the product shall just equal 1?

  Ans. 5.
- 6. Bought a horse and chaise for \$250, and paid for the harness  $\frac{7}{1}$  of what I paid for the horse. The chaise cost  $\frac{11}{2}$  the value of the horse. What was the price of each?

Ans. Horse, \$130 $\frac{1}{2}$ ; chaise, \$119 $\frac{1}{2}$ ; harness, \$83 $\frac{1}{2}$ 3.

7. S. Walker has engaged to work at yearly wages of \$200 and a suit of clothes. At the end of 9 months, falling sick, and being unable to labor longer, he receives the suit of clothes and \$144, as the amount justly due. What was the cost of the clothes?

Ans. \$24.

8. What will be the result if  $\frac{1}{5}$  of  $\frac{3}{4}$  of  $3\frac{1}{6}$  be multiplied by  $\frac{1}{2}$ 

of itself, and the product divided by 1?

9. Bought 13% acres of land at \$25% per acre, and paid for it in wheat at \$2% per bushel. How many bushels did it require?

Ans. 13747% bushels.

10. How long will it take a man to travel 553 miles, provided he travels 3½ miles per hour, and 9½ hours per day?

- 11. If  $$^9_{10}$  per cord is paid E. Holmes for sawing into three pieces wood that is 4 feet long, how much more should he receive per cord for sawing into pieces of the same length wood that is 8 feet long?

  Ans.  $$0.22\frac{1}{2}$ .
- 12. A steamboat leaves New Orleans, January 1st, bound up the river to a place distant 2317½ miles. Her forward motion is at the rate of 9½ miles per hour for 16½ hours each day, and she lies at anchor in the night for fear of running upon a snag. But having lost her anchor on the fifth day, she each succeeding night drifts backward, at the rate of 2 miles per hour. On what day of January will she reach her point of destination?

  Ans. 15th day.

# A PROPOSED NUMERATOR, OR DENOMINATOR.

244. To reduce one fraction to another of equal value, having a proposed numerator, or denominator.

Ex. 1. Reduce 5 to an equivalent fraction having 4 for a

Ans.  $\frac{4}{5\frac{3}{8}}$ .

$$\frac{\frac{4}{\cancel{5}} \times \cancel{5}}{\frac{4}{5} \times 7} = \frac{4}{5\frac{3}{5}} \text{ Ans.}$$

The proposed numerator, 4, is such a part of the given numerator as 5 divided by 4, or 4. Now, as the numerator proposed is only 4 as large as the given numerator, in order that the value of the two fractions be the same, the denominator of the proposed fraction should be only 4 as large as the denominator of the

only ‡ as large as the denominator of the given fraction. Taking ‡ of the given denominator, 7, we obtain

 $5\frac{3}{6}$ , which, written under the proposed numerator, gives  $\frac{4}{5\frac{3}{6}}$  as the fraction required.

2. Reduce § to a fraction of equal value having 12 for a denominator.

$$\frac{\frac{12}{9} \times 8}{\frac{12}{\cancel{9}} \times \cancel{9}} = \frac{10^{\frac{2}{3}}}{12} \text{ Ans.}$$

Since the proposed denominator, 12, is  $\frac{1}{2}$  of the given denominator, 9, we find  $\frac{1}{2}$  of the given numerator, 8, for numerator of the proposed fraction;  $\frac{1}{2}$  of  $8 = 10\frac{2}{3}$ , which, written over the proposed denominator, gives  $\frac{10\frac{2}{3}}{12}$  as the fraction required.

RULE. — Take of both terms of the given fraction such a fractional part as the proposed numerator, or denominator, is of the given numerator, or denominator, and the result will be the required fraction.

#### EXAMPLES.

- 3. Change  $\frac{17}{20}$  to a fraction whose numerator shall be 34. Ans.  $\frac{34}{40}$ .
- 4. Change 3\\(\frac{1}{2}\) to a fraction whose numerator shall be 9.

Ans.  $\frac{9}{2\frac{1}{3}}$ .

- 5. Reduce 4 to a fraction whose numerator shall be 5.
- 6. Reduce 15 to a fraction having 12 for its denominator.
- 7. Change  $\frac{3}{5}$  to fifteenths. Ans.  $\frac{9}{15}$ .
- 8. Reduce  $\frac{7}{3}$  to halves.

9. Reduce  $\frac{19}{20}$  to thirty-fifths.

Ans.  $\frac{33\frac{1}{4}}{35}$ .

10. J. Holton owns  $\frac{19}{57}$  of a wood-lot, and his brother  $\frac{67}{201}$  of the same lot; what fraction whose denominator shall be 12 will express the part each owns?

Ans.  $\frac{4}{12}$ .

#### A COMMON NUMERATOR.

- 245. A COMMON numerator of two or more fractions is a common multiple of their numerators.
  - 246. To reduce fractions to a common numerator.

N 16

Ex. 1. Change  $\frac{3}{4}$ ,  $\frac{4}{5}$ ,  $\frac{6}{7}$ , and  $\frac{9}{10}$  to other fractions of the same value, having a common numerator. Ans.  $\frac{3}{4}$ ,  $\frac{3}{4}$ ,  $\frac{3}{4}$ ,  $\frac{3}{4}$ ,  $\frac{3}{4}$ .

#### OPERATION.

36, least common multiple of the numerators, = common nu-

 $\frac{36}{16}$  of 4 = 48, new denominator.  $\frac{3}{4} = \frac{36}{16}$   $\frac{36}{16}$  of 5 = 45, new denominator.  $\frac{4}{5} = \frac{36}{16}$  Ans.  $\frac{36}{10}$  of 7 = 42, new denominator.  $\frac{6}{7} = \frac{36}{10}$  Ans.

We find the least common multiple of all the numerators, which is 36, for the common numerator; and to obtain the several new denominators we take such a part of the given denominators, respectively, as the common numerator, 36, is of each given numerator. Thus, both terms of each fraction being proportionably increased, its value is not changed.

RULE. — Find the least common multiple of the given numerators for a common numerator. Take, for the new denominator of each fraction, respectively, such a part of its given denominator as the common numerator is of its given numerator.

Note. — Compound fractions, or whole and mixed numbers, must be reduced to simple fractions, and all to their lowest terms, before finding the common numerator.

#### EXAMPLES.

- 2. Reduce \(\frac{3}{5}\), \(\frac{1}{2}\), and \(\frac{7}{5}\) to other fractions of equal value having a common numerator.

  Ans. \(\frac{2}{2}\), \(\frac{2}{4}\), \(\frac{2}{4}\), \(\frac{2}{4}\), \(\frac{2}{4}\), \(\frac{2}{4}\), \(\frac{2}{4}\), \(\frac{2}{4}\).
- 3. Change \$, 2½, and 1¾ to fractions having a common numerator.
- 4. A can travel round a certain island, which is 50 miles in circumference, in  $4\frac{\pi}{15}$  days, B in  $6\frac{\pi}{2}$  days, and C in  $6\frac{\pi}{2}$  days. If they all set out from the same point, and travel round the island the same way, in how many days will they all meet at the point from which they started, and how many times will each have gone round the island?

Ans. They will meet in 320 days; A will have gone round the island 75 times; B, 50 times; and C, 48 times.

#### GREATEST COMMON DIVISOR OF FRACTIONS.

- 247. The greatest common divisor of two or more fractions is the greatest number that will divide each of them, and give a whole number for the quotient.
- 248. To find the greatest common divisor of two or more fractions.

Ex. 1. What is the greatest common divisor of  $\frac{4}{15}$ ,  $2\frac{2}{8}$ , and  $5\frac{1}{3}$ . Ans.  $\frac{4}{5}$ .

OPERATION.

 $\frac{4}{15}$ ,  $2\frac{2}{9}$ ,  $5\frac{1}{3} = \frac{4}{15}$ ,  $\frac{20}{9}$ ,  $\frac{16}{3} = \frac{12}{45}$ ,  $\frac{100}{45}$ ,  $\frac{240}{45}$ .

Greatest common divisor of the numerators  $=\frac{4}{45}$  Greatest common denominator of the fractions  $=\frac{4}{45}$  Greatest common divisor of the fractions  $=\frac{4}{45}$ 

Having reduced the fractions to equivalent fractions having the least common denominator, we find the greatest common divisor of the numerators 12, 100, and 240 to be 4. Now, since the 12, 100, and 240 represent forty-fifths, their greatest common divisor is not 4, a whole number, but 4 forty-fifths; therefore we write the 4 over the least common denominator, 45, and have  $\frac{4}{45}$  as the answer.

Rule. — Reduce the fractions, if necessary, to their least common denominator. The greatest common divisor of the numerators, written over the least common denominator, will give the greatest common divisor required.

#### EXAMPLES.

- 2. What is the greatest common divisor of  $\frac{4}{5}$ ,  $\frac{4}{5}$ , and  $\frac{1}{2}$ ?

  Ans.  $\frac{4}{3}$ 
  - 3. What is the greatest common divisor of 12\\\\ 9\\\\ 2, and 8\\\\\ 2?
  - 4. What is the greatest common divisor of  $\frac{1}{2}$ ,  $\frac{3}{4}$ ,  $\frac{4}{5}$ , and  $\frac{5}{8}$ ?

    Ans.  $\frac{1}{8}$ n.
  - 5. What is the greatest common divisor of  $3\frac{4}{5}$ ,  $5\frac{7}{10}$ , and  $2\frac{4}{15}$ ?
- 6. A farmer has 33½ bushels of corn, 67½ bushels of rye, 70½ bushels of wheat. He wishes to put this grain, without mixing, into the largest bags, each of which shall contain the same quantity. Required the number of bags and the quantity each will contain.

Ans. The capacity of each bag, 33 bushels; and the number of bags, 51.

7. I have three fields; the first contains  $73_{11}^{7}$  acres, the second  $88_{11}^{4}$  acres, the third  $139_{11}^{4}$  acres. Required the largest-sized house-lots of the same extent into which the three fields can be divided, and also the number of lots.

Ans. Size of each lot, 7 acres; number of lots, 41.

# LEAST COMMON MULTIPLE OF FRACTIONS.

249. The least common multiple of two or more fractions is the least number that can be divided by each of them, and give a whole number for the quotient.

250. To find the least common multiple of two or more fractions.

Ex. 1. What is the least common multiple of  $\frac{3}{4}$ ,  $\frac{6}{16}$ , and  $2\frac{1}{16}$ ?

Ans.  $8\frac{1}{4}$ .

 $\frac{3}{4}, \frac{6}{16}, \frac{2}{16} = \frac{3}{4}, \frac{3}{8}, \frac{33}{16}$ 

Least common multiple of the numerators =  $\frac{33}{4}$  =  $8\frac{1}{4}$  { Least common multiple of denominators =  $\frac{33}{4}$  =  $8\frac{1}{4}$  { Least common multiple required.

Having reduced the fractions to their simplest form, we find the least common multiple of the numerators, 3, 3, and 33, to be 33. Now, since the 3, 3, and 33 are, from the nature of a fraction, dividends, of which their respective denominators, 4, 8, and 16, are the divisors (Art. 216), the least common multiple of the fractions is not 33, a whole number, but so many fractional parts of the greatest common divisor of the denominators. This common divisor we find to be 4, which, written as the denominator of the 33, gives  $\frac{3}{4} = 8\frac{1}{4}$  as the least number that can be exactly divided by the given fractions.

RULE.—Reduce the fractions, if necessary, to their lowest terms. Then find the least common multiple of the numerators, which, written over the greatest common divisor of the denominators, will give the least common multiple required.

Note. — The least whole number that will contain two or more fractions an exact whole number of times, is the least common multiple of their numerators. Thus, 4, the least common multiple of the numerators of  $\frac{2}{3}$  and  $\frac{4}{5}$ , is the least whole that can be divided by those fractions, and give a whole number for a quotient;  $4 \div \frac{2}{3} = 6$ ;  $4 \div \frac{4}{5} = 5$ .

# EXAMPLES.

2. What is the least common multiple of  $\frac{4}{5}$ ,  $\frac{8}{5}$ , and  $\frac{5}{7}$ ?

Ans.  $\frac{2}{3}$ 4 = 24.

3. Find the least number that  $3\frac{1}{16}$ ,  $7\frac{1}{8}$ , and  $5\frac{1}{4}$  will divide without a remainder.

Ans.  $15\frac{3}{4}$ .

4. What is the least common multiple of  $\frac{3}{8}$ ,  $\frac{6}{7}$ , and  $\frac{9}{10}$ ?

5. What is the smallest sum of money with which I could purchase a number of sheep at \$2\frac{1}{4}\$ each, a number of calves at \$4\frac{1}{2}\$ each, and a number of yearlings at \$9\frac{2}{3}\$ each? and how many of each could I purchase with this money?

Ans.  $$112\frac{1}{2}$ ; 50 sheep; 25 calves; 12 yearlings.

6. There is a certain island 80 miles in circumference. A, B, and C agree to travel round it. A can walk  $3\frac{1}{2}$  miles in an hour, B  $4\frac{2}{3}$  miles, and C  $5\frac{1}{4}$  miles. They start from the same point and travel round the same way, and continue

their travelling 8 hours a day, until they shall all meet at the point from which they started. In how many days will they all meet, and how far will each have travelled?

Ans. In 174 days; A 480m., B 640m., and C 720m.

7. How many times the least common multiple of 31, 43, and 51, is the least whole number that 31, 43, and 51 will exactly divide.

#### DENOMINATE FRACTION.

251. A DENOMINATE Fraction is one in which the unit of the fraction is a denomination of a compound number; as, 3 of a pound, 4 of a mile, and 7 of a gallon.

#### REDUCTION OF DENOMINATE FRACTIONS.

- 252. REDUCTION of denominate fractions is the process of changing fractions from the unit of one denomination to that of another, without altering their value.
- To reduce a denominate fraction from a higher denomination to a lower.
  - Ex. 1. Reduce  $\frac{1}{6}$  of a pound to a fraction of a penny. Ans. # d.

$$\frac{1}{640} \times 20 = \frac{20}{640}$$
s.;  $\frac{20}{640} \times 12 = \frac{240}{640}$ d. =  $\frac{3}{8}$ d. Ans.

Or, 
$$\frac{1}{640} \times 20 \times 12 = \frac{3}{8}$$
 d. Ans. Since 20s. make a pound. there will be 20 times as many shillings as pounds, or  $\frac{20}{8}$  s and since 12d.

many shillings as pounds, or  $\frac{20}{640}$ s.; and since 12d.

make a shilling, there will be 12 times as many pence as shillings, or #18d. — #d.

Rule. - Multiply the given fraction by the same numbers that would be employed in the reduction of whole numbers to the lower denomination required.

# EXAMPLES.

- 2. Reduce  $\frac{1}{1200}$  of a pound to the fraction of a farthing.
- 3. Reduce  $\frac{1}{9600}$  of a pound troy to the fraction of a grain.
- 4. Reduce  $\frac{5}{2592}$  of a pound, apothecaries' weight, to the fraction of a scruple.
  - 5. Reduce  $\frac{3}{8800}$  of a cwt. to the fraction of an ounce.

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- 6. Reduce  $\frac{3}{80000}$  of a ton to the fraction of a pound.
- 7. What part of an inch is 215 of an ell English?
- 8. What part of an inch is TYOBRO of a mile?
- 9. Reduce TRATE of a league to the fraction of an inch.
- 10. Reduce 250 30 550 of an acre to the fraction of an inch.
- 11. Reduce TIB2 of a tun of wine measure to the fraction of a quart.
  - 12. What part of a pint is  $\frac{3}{3}$  of a bushel?
  - 13. What part of a minute is x20 + 680 of a year?
- 14. Reduce 3200 of a hundred-weight to a fraction of an ounce.
- 254. To reduce a denominate fraction from a lower denomination to a higher.
  - Ex. 1. Reduce \( \frac{3}{8} \) of a penny to a fraction of a pound.

$$\frac{3}{8} \times 12 = \frac{3}{96}$$
s.;  $\frac{3}{96} \times 20 = \frac{3}{1920}$ £.  $= \frac{1}{640}$ £. Ans.

Or, 
$$\frac{8}{8} \times 12 \times 20 = \frac{1}{640} £$$
. Ans. Since 12 pence make a shilling, there will be  $\frac{1}{12}$  as many shillings as

1 as many shillings as pence, or  $\frac{8}{96}$ s.; and

since 20s. make a pound, there will be  $\frac{1}{20}$  as many pounds as shillings, or 1 £. Ans.

RULE. - Divide the fraction by the same numbers that would be employed in the reduction of whole numbers to a higher denomination.

# EXAMPLES.

- 2. What part of a pound is 4 of a farthing?
- 3. What part of a pound is  $\frac{3}{5}$  of a grain troy?
- 4. Reduce # of a scruple to the fraction of a pound.
- 5. Reduce 5 of an ounce to the fraction of a hundredweight.
  - 6. Reduce 3 of a pound to the fraction of a ton.
  - 7. Reduce 1 of an inch to the fraction of an ell English.
  - 8. Reduce \$ of an inch to the fraction of a mile.
  - 9. Reduce 1 of an inch to the fraction of a league.
  - 10. Reduce \( \frac{3}{2} \) of an inch to the fraction of an acre.

- 11. Reduce 7 of a quart to the fraction of a tun, wine measure.
- 12. Reduce \(\frac{3}{5}\) of a pint to the fraction of a bushel.
- 13. Reduce † of a minute to the fraction of a year (365† days).
  - 14. What part of a hundred-weight is 3 of an ounce?

Ans. 3200.

255. To find the value of a fraction in whole numbers of lower denominations.

Ex. 1. What is the value of  $\frac{3}{11}$  of a £. Ans. 5s. 5d.  $1\frac{9}{11}$  far.

$$\begin{array}{r}
3 £. \\
2 0 \\
1 1) 6 0 s. (5 s. \\
\underline{5 5} \\
5 s. \\
1 2
\end{array}$$

$$\begin{array}{r}
11 \\
50 \\
\underline{55} \\
5 \\
4
\end{array}$$

11) 
$$20 \text{ far.} (1\frac{9}{11} \text{ far.})$$

Since 1£. = 20s.,  $\frac{2}{11}$  of a £. is  $\frac{2}{11}$  of 20s. =  $\frac{2}{11}$ 8s. = 5 $\frac{2}{11}$ 8s.; and since 1s. = 12d.,  $\frac{2}{11}$  of a shilling is  $\frac{2}{11}$ 7 of 12d. =  $\frac{2}{11}$ 1 of a penny =  $\frac{2}{11}$ 7 of 4far. =  $\frac{2}{11}$ 1 of a penny =  $\frac{2}{11}$ 7 of 4far. =  $\frac{2}{11}$ 1 far. Therefore,  $\frac{2}{11}$ 1 £. = 5s. 5d.  $\frac{2}{11}$ 1 far. This is equivalent to multiplying the numerator of the fraction by the numbers required to reduce it to successive lower denominations, beginning with the highest, and dividing each product by the denominator, as in the operation.

Ans. 5s. 5d. 1- far.

RULE. — Multiply the numerator of the given fraction by the number required to reduce it to the next lower denomination, and divide the product by the denominator.

Then, if there is a remainder, proceed as before, until it is reduced

to the denomination required.

# EXAMPLES.

- 2. What is the value of  $\frac{7}{24}$  of a shilling? Ans.  $3\frac{1}{2}$ d.
- What is the value of \( \frac{7}{4} \) of a guinea, at 28 shillings?
   Ans. 21s. 9d. 14far.
- 4. What is the value of 7 of a cwt.?
- 5. What is the value of \$ of a lb. avoirdupois?

Ans. 7oz. 17dr.

6. What is the value of § of a lb. troy?

Ans. 10oz. 13pwt. 8gr.

- 7. What is the value of  $\frac{4}{13}$  of a lb. apothecaries' weight? Ans. 33 53 19 12 13gr.
- 8. What is the value of § of an ell English?

Ans. 2gr. 3na. 01in.

9. What is the value of +4 of a mile?

Ans. 6fur. 30rd. 12ft. 8 43 in.

10. What is the value of \ of a furlong?

Ans. 35rd. 9ft. 2in.

11. What is the value of  $\frac{1}{\sqrt{2}}$  of an acre?

Ans. 2R. 6rd. 4yd. 5ft. 127-5-in.

- 12. What is the value of  $\frac{9}{17}$  of a rod?
- 13. What is the value of 13 of a cord?

Ans. 9ft. 1462 3in.

14. What is the value of  $\frac{2}{3}$  of a hhd. of wine?

Ans. 6gal. 2qt. 1pt. 04ggi.

- 15. What is the value of 7 of a hhd. of beer? Ans. 42gal.
- 16. What is the value of  $\frac{1}{2}$  of a year (3651 days)? Ans. 174d. 16h. 26m. 5-5-sec.
- 17. What is the value of  $7\frac{3_{11}^{3}}{42}$  of a dollar?
- To find the value of whole numbers in a fraction of a higher denomination.
  - Ex. 1. What part of a £. are 5s. 5d. 1- far.?

5s. 5d.  $1\frac{9}{11}$  far. =  $\frac{2880}{10560} = \frac{3}{11}$ £. Ans. 5d.  $1\frac{9}{11}$  far. to elevenths of a farthing, the low-

We reduce the 3s. est denomination in the

question, for the numerator of the required fraction, and 1 £. to the same denomination for the denominator. We then have the fraction  $\frac{2880}{10860}$ £. =  $\frac{8}{11}$ £. as the answer.

Rule. - Reduce the given numbers to the lowest denomination mentioned in either of them. Then write the number which is the fractional part for the numerator, and the other number for the denominator, of the required fraction.

# Examples.

- 2. Reduce 31d. to the fraction of a shilling. Ans. 🚜.
- 3. Reduce 21s. 9d. 14 far. to the fraction of a guinea.

Ans. 4.

- 4. Reduce 2qr. 13lb. 10oz. 211dr. to the fraction of a cwt.

  Ans. 7.
- 5. What part of a pound avoirdupois are 7oz. 17dr.?
- 6. What part of a pound troy are 10oz. 13pwt. 8gr.?

Ans. §.

- Express 1 gallon liquid measure as a fraction of a gallon dry measure.
   Ans. 1344.
  - 8. What part of a yard are 2qr. 0na.  $1_{13}^{5}$  in.? Ans.  $\frac{7}{13}$ .
  - 9. What part of an ell English are 2qr. 3na. 01in.?

Ans. §.

- 10. What part of a mile are 6 fur. 30rd. 12ft.  $8\frac{4}{13}$ in.?
- 11. Reduce 35rd. 9ft. 2in. to the fraction of a furlong.

Ans. §.

- 12. What part of an acre are 2R. 6rd. 4yd. 5ft.  $127_{15}^{-5}$  in. Ans.  $7_{7}$ .
- 13. What part of a square rod are 144ft. 19<sub>17</sub>in.?

Ans. 3.

- 14. What part of a cord are 9ft.  $1462_{13}$  in.?
- 15. What part of a hogshead of wine are 6gal. 2qt. 1pt.  $0_{19}^{4}$ gi.? Ans.  $1_{9}^{2}$ .
  - 16. What part of a pound avoirdupois is 1 pound troy?

    Ans. 144.

17. What part of a year  $(365\frac{1}{4})$  days are 174d. 16h. 26m.  $5\frac{5}{23}$  sec.? Ans.  $\frac{1}{23}$ .

### ADDITION OF DENOMINATE FRACTIONS.

257. To add denominate fractions.

Ex. 1. Add 73 of a £. to 37 of a £.

Ans. 1£. 7s. 1d.  $2_{143}^{54}$  far.

 $_{13}^{7}$  of a £. = 10s. 9d. 0 $_{13}^{2}$ far.  $_{19}^{9}$  of a £. = 16s. 4d.  $_{13}^{5}$ far.

Ans. 1£. 7s. 1d. 254 far.

We find the value of each fraction separately, and add the two values together, according to the rule for adding compound numbers. (Art. 145.)

second operation.

 $\frac{7}{13}\pounds$ .  $+\frac{9}{11}\pounds$ .  $=\frac{194}{14}\pounds$ .  $=1\pounds$ . 7s. 1d.  $2\frac{54}{143}$ far. Ans.

By the second operation, we first add the two fractions together, and then find the value of their sum. (Art. 255.)

### EXAMPLES.

2. Add together  $\frac{7}{11}$  of a ton and  $\frac{17}{22}$  of a cwt.

Ans. 13cwt. 2qr.

- 3. Add together \( \frac{2}{7} \) of a yard, \( \frac{1}{7} \) of an ell English, and \( \frac{5}{7} \) of a qr.
- 4. Add together  $\frac{7}{11}$  of a mile,  $\frac{4}{13}$  of a furlong, and  $\frac{9}{22}$  of a yard.

  Ans. 5 fur. 1 6 rd. 0 ft.  $3\frac{9}{143}$  in.
- 5. A has three house-lots; the first contains  $\frac{1}{7}$  of an acre, the second  $\frac{2}{3}$  of an acre, and the third  $\frac{1}{12}$  of an acre. How many acres do they all contain?

Ans. 2A. 1R. 9p. 142ft. 873in.

- 6. A man travelled 183 miles the first day, 2311 miles the second day, and 1921 miles the third day. How far did he travel in the three days? Ans. 61m. 2fur. 3rd. 13ft. 42in.
  - 7. Add  $\frac{11}{12}$  of a gallon of wine to  $\frac{1}{12}$  of a hhd.
  - 8. Add  $\frac{5}{16}$  of a week to  $\frac{1}{5}$  of a day. Ans. 2d. 9h. 18m.
  - 9. Add 3 of a square foot to 1 a foot square. Ans. 1 foot.
  - 10. Add 6 inches to 11rd. 16ft. 5in. Ans. 12rd. 0ft. 5in.

### SUBTRACTION OF DENOMINATE FRACTIONS.

258. To subtract one denominate fraction from another.

Ex. 1. From  $\frac{7}{11}$  of a £. take  $\frac{2}{9}$  of a £.

Ans. 8s. 3d. 13 far.

 $_{1}^{T_{1}}$  of a £. = 12s.  $8_{1}^{B_{1}}$ d.  $_{2}^{2}$  of a £. =  $\frac{4s. \ 5_{3}^{1}$ d. Ans.  $\frac{4s. \ 5_{3}^{1}}$ d.

We find the value of each fraction separately, and subtract one from the other, according to the rule for subtracting compound numbers. (Art. 146.)

 $7_{1}$ £. —  $\frac{2}{3}$ £. =  $\frac{4}{3}$ £. = 8s. 3d.  $1\frac{1}{3}$ far. Ans. Subtract the less fraction from the greater, and then find the value of their difference. (Art. 255.)

# EXAMPLES.

2. From 7 of an ell English take 7 of a yard.

Ans. 3qr. 0na. 256 in.

3. Take  $\frac{7}{11}$  of a furlong from  $\frac{2}{3}$  of a mile.

Ans. 1fur. 5rd. 10ft. 10in.

4. Take \(\frac{3}{4}\) of a mile from \(\frac{5}{4}\) of a degree.

Ans. 48m. 6fur. 17rd. 8ft. 75in.

5. From  $\frac{4}{11}$  of an acre take  $\frac{4}{9}$  of a rod.

Ans. 1R. 17p. 22yd. 2ft. 108in.

- 6. From  $\frac{9}{10}$  of a cord take  $\frac{2}{11}$  of a cord.
- 7. From  $\frac{7}{13}$  of a hogshead of wine there leaked out  $\frac{4}{5}$  of it; what remained?

  Ans. 6gal. 3qt. 0pt.  $1\frac{7}{65}$ gi.
- 8. From Boston to Concord, N. H., the distance is 72 miles; 4 of this distance having been travelled, how much remains?

  Ans. 30m. 6fur. 34rd. 4ft. 84in.
  - 9. From 4 of a year take 4 of a week.

Ans. 101da. 5h. 54m. 174sec.

10. From  $\frac{4}{11}$  of an acre take  $\frac{1}{2}$  of a foot.

#### MISCELLANEOUS EXAMPLES IN FRACTIONS.

1. How far will a man walk in  $17\frac{3}{11}$  hours, provided he goes at the rate of  $4\frac{7}{4}$  miles an hour?

Ans. 82m. 4fur. 8rd. 1ft. 4in.

- 2. How much land is there in a field which is  $29\frac{7}{13}$  rods square? Ans. 5A. 1R. 32p. 141ft.  $109\frac{1}{16}\frac{5}{16}$  in.
- 3. How much wood in a pile which is  $17\frac{3}{4}$  feet long,  $7\frac{1}{11}$  feet high, and  $4\frac{3}{6}$  feet wide?

  Ans. 4C.  $66\frac{1}{11}$  feet.
- 4. What is the value of 197 barrels of flour, at \$63 a barrel?
- 5. What is the value of 37611 acres of land, at \$753 per acre?

  Ans. \$28387.061.
- 6. What cost  $17\frac{19}{112}$  quintals of fish, at \$4.75 per quintal?

  Ans. \$81.55 45.
- 7. What cost  $1670_{13}^{7}$  pounds of coffee, at  $12\frac{3}{4}$  cents per pound? Ans.  $$212.99\frac{1}{6}$ .
- 8. What cost  $28\frac{4}{17}$  tons of Lackawana coal, at \$11\frac{3}{4}\$ a ton?

  Ans. \$333.27\frac{3}{17}\$.
- 9. Bought  $37\frac{1}{63}$  hogsheads of molasses, at \$17.62\frac{1}{2}\$ a hogshead; what was the whole cost? Ans. \$655.20\frac{5}{27}\$.
  - 10. What cost 7 of a cord of wood, at \$5.75 a cord?
- 11. What are the contents of a field which is 1394 rods long, and 384 rods wide?

  Ans. 33A. 3R. 1513p.
  - 12. Bought 15 loads of wood, each containing 113 feet, cord

- measure. I divide it equally between 9 persons; what does each receive?

  Ans. 194ft.
- 13. If the transportation of  $18\frac{3}{8}$  tons of iron costs \$48.15 $\frac{3}{4}$ , what is it per ton?

  Ans. \$2.62 $\frac{4}{8}$ .
- 14. If a hogshead of wine costs \$987, what is the price of one gallon?
- 15. If 5 bushels of wheat cost \$8\frac{2}{3}\$, what will a bushel be worth?

  Ans. \$1.64\frac{4}{3}\$.
- 16. What will 11 hogsheads and 17½ gallons of wine cost, at 19½ cents a gallon?

  Ans. \$140.32%.
- 17. How many bottles, each containing 12 pints, are sufficient for bottling a hogshead of cider?

  Ans. 288.
- 18. I have a shed which is  $18\frac{7}{12}$  feet long,  $10\frac{5}{12}$  feet wide, and  $7\frac{1}{12}$  feet high; how many cords of wood will it contain?

  Ans. 11C.  $124\frac{829}{1728}$ ft.
- 19. What will  $6\frac{7}{8}$  pounds of tea cost, at  $65\frac{3}{4}$  cents per pound?

  Ans.  $$4.52\frac{1}{32}$ .
- 20. How many cubic feet does a box contain, that is  $8\frac{3}{4}$  feet long,  $5\frac{7}{12}$  feet wide, and 3 feet high? Ans.  $146\frac{3}{15}$  ft.
- 21. How many feet of boards will it take to cover a side of a house which is  $46\frac{1}{12}$  feet long and  $17\frac{1}{2}$  feet high?
- 22. Required the number of square feet on the surface of 7 boxes, each of which is  $5\frac{1}{2}$  feet long,  $2\frac{5}{12}$  feet high, and  $3\frac{1}{12}$  feet wide; required also the number of cubic feet they would occupy?

  Ans.  $527\frac{5}{2}\frac{5}{2}$ t.;  $286\frac{2}{2}\frac{5}{2}\frac{3}{2}$  cubic feet.
- 23. A certain room is 12 feet long, 11½ feet wide, and 7½ feet high; how much will it cost to plaster it, at 2¾ cents per square foot?

  Ans. \$13.48₹.
- 24. A man has a garden that is 14½ rods long, and 10¼ rods wide; he wishes to have a ditch dug around it, that shall be 3 feet wide and 4½ feet deep; what will be the expense, if he give 2 cents per cubic foot?

  Ans. \$ 223.76¼.
- 25. How many bushels of grain will a box contain which is  $14\frac{7}{12}$  feet long,  $5\frac{1}{12}$  feet deep, and  $4\frac{1}{12}$  feet wide, there being  $2150\frac{2}{5}$  cubic inches in a bushel?

  Ans.  $294\frac{2}{3}\frac{1}{5}\frac{2}{5}\frac{1}{5}$  bu.
- 26. Which will contain the most, and by how much, a box that is 10 feet long, 8 feet wide, and 6 feet deep, or a cubical one, each of whose sides measures 8 feet?

Ans. The last contains 32 cubic feet the most.

27. Which will contain the most gallons, a cistern that is  $7\frac{1}{2}$  feet long, 6 feet wide, and  $5\frac{1}{2}$  feet deep, or one that is  $9\frac{1}{2}$  feet long,  $4\frac{1}{2}$  feet wide, and  $5\frac{1}{2}$  feet deep?

Ans. The first cistern contains 92‡ gallons most.

28. My field has four sides. The first side is 31 rods  $13\frac{3}{10}$  feet in length; the second, 41 rods  $1\frac{9}{10}$  feet; the third, 38 rods  $0\frac{1}{5}$  feet; and the fourth, 45 rods  $12\frac{7}{10}$  feet. I wish to enclose this field with a rail-fence four rails high, using rails of equal length. Required the length of the longest rails that can be used, allowing that the rails lap by each other  $\frac{7}{10}$  of a foot; also the number of rails it will take to fence it.

Ans. Length, 13½ feet; number, 808.

29. Required the least number of yards of velvet, expressed by a whole number, that can be cut up without waste, into vest patterns of §, ¾, or § yards each?

Ans. 30 yards.

- 30. If a company whose capital stock is divided into 100 equal shares should conclude to divide the same stock into only 30 shares, how much larger would a share of the latter be than one of the former size?
- 31. D. Ripley's farm contains 31A. 3R. 6p., and J. Ford's farm contains 39A. 2R. 37½p. What is the fraction, in its simplest form, that will express their comparative size? Ans. ½.
- 32. Bought 68 barrels of flour, at \$7\frac{1}{2}\$ per barrel; what was the amount of the whole?

  Ans. \$5384.
  - 33. What cost 83 acres of land, at \$423 per acre?

Ans. \$369.20.

- 34. How shall four 3's be arranged, that their value shall be nothing?
- 35. I have a room 20 feet long, 15 feet wide, and 8½ feet high. This room contains 4 windows, each of which is 5½ feet in height and 3½ feet in width. There are two doors 7 feet high and 3 feet wide. The mop-boards are ¾ of a foot wide. A mason has agreed to plaster this room at 6½ cents per square yard; a painter is to lay on the paper at 9 cents per square yard; the paper which I wish to have laid on is 2½ feet wide, for which I pay 5 cents per yard. What is the amount of my bill for plastering, for papering, and for paper?

Ans. For plastering, \$5.11 $\frac{29}{36}$ ; for papering, \$4.37; for paper, \$2.80 $\frac{5}{38}$ .

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#### EXAMPLES TO BE PERFORMED BY ANALYSIS.

1. If  $\frac{1}{4}$  of a bushel of corn cost 63 cents, what cost a bushel? What cost 15 bushels?

Ans. \$ 10.80.

ILLUSTRATION. — If 7 eighths of a bushel cost 63 cents, 1 eighth will cost 1 seventh of 63 cents — 9 cents; and 8 eighths will cost 8 times 9 cents — 72 cents, and 15 bushels will cost 15 times 72 cents — \$10.80.

2. If 4\forall lb. of pepper cost \$2.15, what cost 1 pound? what cost 30lb.?

Ans. \$13.50.

ILLUSTRATION. — In 47 lb. there are  $^45$  lb. Then, if 43 ninths lb. cost \$2.15, 1 ninth will cost 1 forty-third of \$2.15 = \$0.05, and 9 ninths or 1lb. will cost 9 times \$0.05 = \$0.45, and 30 lb. will cost 30 times \$0.45 = \$13.50.

- . 3. When \$1728 are paid for  $30\frac{4}{15}$  tons of iron, what cost 1 ton? what cost  $7\frac{1}{3}$  tons? Ans. \$432.
- 4. When \$432 are paid for  $7\frac{17}{37}$  tons of iron, what quantity should be received for \$1728?
- 5. For  $7\frac{1}{37}$  tons of iron there were paid \$432; what sum will it require to pay for  $30\frac{4}{15}$  tons?
- 6. For 30<sub>4</sub>5 tons of iron \$1728 were paid; what quantity should be received for \$432?
- 7. Gave  $7\frac{7}{10}$  bushels of rye for a barrel of flour; how much rye will it then require to purchase  $6\frac{4}{5}$  barrels of flour?

Ans. 4928 bushels.

- 8. Divide \$1728 among 17 boys and 15 girls, and give each boy  $_{7}^{7}$  as much as a girl; what sum will each receive?
  - Ans. Each girl, \$ 6699; each boy \$ 4247.
  - 9. If  $\frac{7}{6}$  of a ton of hay cost \$14.49, what cost  $\frac{43}{7}$  tons?
- 10. If 43 tons of hay cost \$82.503, what part of a ton will \$14.49 buy?
- 11. If \$14.49 will buy  $\frac{7}{6}$  of a ton of hay, how much hay can be obtained for \$82.50?
- 12. When \$82.503 are paid for 43 tons of hay, what will be the cost of  $\frac{7}{4}$  of a ton?
- 13. When 147 tons of copperas are sold for \$500, what is the value of 1 ton? what is the value of 911 tons?

- 14. When 911 tons of copperas are sold for \$333.331, what is the value of 147 tons?
- 15. Gave \$333.33 $\frac{1}{3}$  for  $9\frac{1}{12}$  tons of copperas; what quantity of copperas should be received for \$500?
- 16. For  $14\frac{7}{8}$  tons of copperas \$500 were paid; how much might be purchased for \$333.33 $\frac{1}{3}$ ?
- 17. Purchased 97% gallons of molasses for \$31.32; what cost 1 gallon? what cost 763% gallons?
- 18. Sold 763 gallons of molasses for \$244.36; what should I receive for 97 gallons?
- 19. If \$244.36 will buy 7635 gallons of molasses, what quantity can be obtained for \$31.32?
- 20. Gave 1975lb. of flax for 40 barrels of flour; how many pounds were given for 1 barrel? how many pounds would it require to buy 144 barrels?

  Ans. 7110lb.
- 21. If 17 bushels of rye cost \$15.75, what cost 1 bushel? what cost 9\frac{1}{2} bushels? Ans. \$8.56\frac{6}{6}\frac{7}{6}.
- 22. If 9 barrels of flour cost \$50\\(\frac{4}{7}\), what cost 1 barrel? what cost 87\\(\frac{3}{5}\) barrels?

  Ans. \$492\(\frac{3}{35}\).
- 23. If 13 boarders consume a barrel of pork in 78 days, how long would it last, if 7 more boarders were added to their number?

  Ans.  $50\frac{7}{10}$  days.
- 24. If a man by laboring 10 hours a day can, in 9 days, perform a certain piece of work, how many days would it require to do the same work, were he to labor 15 hours a day?
- 25. If a man, by laboring 15 hours a day, in 6 days can perform a certain piece of work, how many days would it require to do the same work by laboring 10 hours a day?
- 26. If a man, by laboring 10 hours a day, can in 9 days perform a certain piece of work, how many hours must he labor each day to perform the same work in 6 days?
- 27. Sold  $17\frac{3}{11}$  bushels of corn for \$5\frac{2}{7}\$; what was received for 1 bushel? what should I have charged for  $97\frac{4}{5}$  bushels?

  Ans. \$30\frac{2}{3}\frac{1}{5}\$.
- 28. Bought  $9\frac{3}{5}$  tons of hay at \$19 $\frac{7}{5}$  per ton; for what must it be sold per cwt. to gain \$7 on my bargain? Ans. \$ $1\frac{7}{2}\frac{3}{80}$ .
- 29. If I sell hay at \$12 per cwt., what should I give for 92 tons, that I may make \$7 on my bargain?

  Ans. \$329.
  - 30. How many bushels of corn at \$0.75 per bushel will

- it require to purchase  $47\frac{3}{11}$  bushels of wheat at \$2\frac{3}{2}\$ per bushel?

  Ans.  $168\frac{3}{11}$ bu.
- 31. If 15 cords of wood cost \$  $57_{11}^{9}$ , what cost 1 cord? what cost 19 $\xi$  cords?
- 32. If 197 cords of wood cost \$76 $_{110}^{67}$ , how many cords may be obtained for \$57 $_{11}^{9}$ ?
  - 33. At  $7_{10}^{3}$  shillings per yard, what cost  $47_{\frac{1}{3}}$  yards?

Ans. 17£. 5s.  $6\frac{2}{5}$ d.

- 34. When  $172\pounds$ . 15s. 02d. are paid for  $47\frac{1}{3}$  yards of broadcloth, what is the value of 1 yard? Ans.  $3\pounds$ . 12s.  $11\frac{3}{3}\frac{2}{3}\frac{2}{3}$ d.
- 35. If 1lb. of sugar cost  $\frac{7}{13}$  of a dollar, what is the value of 43\$\frac{1}{15}\$lb.?

  Ans. \$\frac{2}{3}61\frac{7}{3}\$.
  - 36. If  $17\frac{3}{8}$ lb. of sugar cost  $2\frac{7}{11}$ , what cost 50lb.

- 37. Bought 87 $\frac{3}{7}$  yards of broadcloth for \$612; what was the value of  $14\frac{7}{70}$  yards?

  Ans. \$102.90.
  - 38. If 7 of an acre of land cost \$43.75, what cost 10 acres?
- 39. When \$500 are paid for 10 acres of land, how much might be obtained for \$43.75?
- 40. If 9 hogsheads of sugar cost \$71.87, what cost \$ of a hogshead?
- 41. Paid \$4.5623 for 4 of a hogshead of sugar; what ought to be given for 9 hogsheads?
- 42. If 19 men can grade a certain road in 111 days, how long would it require 47 men to perform the same labor?
- 43. When 47 men can grade a certain road in 44‡‡ days, how long would it require 19 men to perform the same labor?
  - 44. If  $\frac{4}{1}$  of a ton of hay cost \$9.20, what cost 17 tons?
- 45. When \$430.10 are paid for 17 tons of hay, what cost 4 of a ton?
  - 46. If  $\frac{7}{16}$  of a tub of butter cost \$7.15, what cost 7 tubs?
- 47. When \$114.40 are paid for 7 tubs of butter, what cost  $T_6$  of a tub?
- 48. If a horse eat 193 bushels of oats in 873 days, how many will 7 horses eat in 60 days? Ans. 933 bushels.
- 49. Henry Smith can reap a field in 10 days, by laboring 8 hours a day. His son John can reap the same field in 9 days, by laboring 12 hours a day. How long would it take both to reap the field, provided they labored 8 hours a day?

Ans. 534 days.

#### DECIMAL FRACTIONS.

259. A DECIMAL FRACTION is a fraction whose denominator is some power of ten.

It, therefore, originates from dividing a unit, first into 10 equal parts, and then each of these parts into 10 other equal parts, and so on indefinitely, so that its fractional units are tenths, hundredths, thousandths, or some like order of parts.

260. Decimal fractions, their denominators being obvious, are commonly expressed by writing the numerator only, with the decimal point (.) before it, care being taken to put a cipher in any decimal place not requiring a digit; thus,

9 may	be written	.9	and	be	read 9	tenths.
100	<b>"</b>	.13	•	"	13	hundredths.
1000	"	.005		"	5	thousandths.
10000	"	.0105		"	105	ten-thousandth

- **261.** By examining the foregoing fractions, it will be seen that, —
- 1. The denominator of a decimal fraction is 1 with as many ciphers annexed as the numerator has places of figures.
- 2. In writing a decimal fraction without its denominator, every decimal place not having a significant figure must be filled by a cipher.
- 3. The first figure or place of a decimal fraction on the right of the decimal point is tenths; the second, hundredths; the third, thousandths; the fourth, ten-thousandths; &c.
- 4. Each figure in the expression of decimal fractions, as in whole numbers, represents value, according to its distance from the place of units.
- 262. A whole number and a decimal fraction, in a single expression, constitute a mixed number. Thus, 17.63 is a mixed number, and is read seventeen, and decimal sixty-three hundredths; 150.302, read one hundred and fifty, and decimal three hundred and two thousandths.

Note. — For the sake of brevity, especially in reading mixed numbers, as in the instance just given, a decimal fraction is commonly called simply a decimal.

- 263. If ciphers are placed on the left of decimal figures, between them and the decimal point, those figures change their places, each cipher removing them one place to the right, and thus diminishing the value represented tenfold. Thus,  $.9 = \frac{9}{100}$ , but  $.09 = \frac{9}{100}$ , and  $.009 = \frac{9}{1000}$ .
- **264.** If ciphers are placed on the right of decimal figures, or are taken away, since their places remain the same, the value represented is not changed. Thus,  $.7 = \frac{7}{10}$ , and  $.70 = \frac{7}{10} = \frac{7}{10}$
- 265. By regarding the dollar as a unit, we may consider cents and mills of United States money as fractional parts of a decimal character. Thus, 3 dollars and 25 cents, is 3 dollars and 25 hundredths of a dollar, or \$3.25; also, 10 dollars 12 cents and 5 mills, is 10 dollars and 125 thousandths of a dollar, or \$10.125.

#### NOTATION AND NUMERATION OF DECIMALS.

266. The relation of decimals to whole numbers and to each other, and also the names of their different orders and places, are shown by the following

							T	ABI	Æ.							
2 Millions.	a Hundr. of Thousands.	cy Tens of Thousands.	F Thousands.	⇔ Hundreds.	∾ Tens.	r Units.	· Decimal point.	∾ Tenths.	w Hundredths.	Thousandths.	c. Ten-Thousandths.	e Hundred-Thousandths.	~ Millionths.	∞ Ten-Millionths.	w Hundred-Millionths.	Billionths.
$\int$ 7th order or place,	6th order or place,	5th order or place,	4th order or place,	3d order or place,	2d order or place,	list order or place,		1st order or place,	2d order or place,	3d order or place,	4th order or place,	{ 5th order or place,	6th order or place,	7th order or place,	8th order or place,	9th order or place,
	Wh	ole	Nun	aber	8.						De	cime	ıls.			

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Of the mixed number expressed in the table, the part on the left of the decimal point is the whole number, and that on the right the decimal. The decimal part is numerated from the left to the right, and the value represented is expressed in words thus: Two hundred thirty-four million five hundred sixty-seven thousand eight hundred ninety-two billionths. And the mixed number thus: Seven million six hundred fifty-four thousand three hundred twenty-one, and decimal two hundred thirty-four million five hundred sixty-seven thousand eight hundred ninety-two billionths.

# 267. From the table we deduce the following rules:

- 1. Read a decimal as though it were a whole number, adding the name of the right-hand order.
- 2. WRITE a decimal as though it were a whole number, supplying with ciphers such places as have no significant figures.

#### EXAMPLES.

Express orally, or write in words the following numbers: -

1.	.056	6.	1.631	11.	1.000007
2.	.1003	7.	48.07	12.	5.101016
3.	.2786	8.	1.315	13.	1.000327
4.	.16302	9.	5.6001	14.	0.000001
5.	.97500	10.	87.0006	15.	16.000000007

# Express in the decimal form by figures: -

16.	13	19.	100000	22.	$7\frac{17}{10000}$
17.	1000 ·	20.	1000000	23.	$333\frac{3}{1000}$
18.	10000	21.	1031 1000000	24.	$1_{\text{TOO}}$

- 25. Three hundred twenty-five, and seven tenths.
- 26. Four hundred sixty-five, and fourteen hundredths.
- 27. Ninety-three, and seven hundredths.
- 28. Twenty-four, and nine millionths.
- 29. Two hundred twenty-one, and nine hundred-thousandths.
- 30. Forty-nine thousand, and forty-nine thousandths.
- 31. Seventy-nine million two thousand, and one hundred five thousandths.
- 32. Sixty-nine thousand fifteen, and fifteen hundred-thousandths.

- 33. Eighty thousand, and eighty-three ten-thousandths.
- 34. Nine billion nineteen thousand nineteen, and nineteen hundredths.
- 35. Twenty-seven, and nine hundred twenty-seven thousandths.
  - 36. Forty-nine trillion, and one trillionth.
  - 87. Twenty-one, and one ten-thousandth.
  - 38. Eighty-seven thousand, and eighty-seven millionths.
- 39. Ninety-nine thousand ninety-nine, and nine thousand nine billionths.
  - 40. Seventeen, and one hundred seventeen ten-thousandths.
  - 41. Thirty-three, and thirty-three hundredths.
  - 42. Forty-seven thousand, and twenty-nine ten-millionths.
  - 43. Fifteen, and four thousand seven hundred-thousandths.
  - 44. Eleven thousand, and eleven hundredths.
  - 45. Seventeen, and eighty-one quadrillionths.
  - 46. Nine, and fifty-seven trillionths.
- 47. Sixty-nine thousand, and three hundred forty-nine thousandths.
- 268. Decimals, since they increase from right to left, and decrease from left to right, by the scale of ten, as do simple whole numbers, may be added, subtracted, multiplied, and divided in the same manner.

#### ADDITION OF DECIMALS.

**269.** Ex. 1. Add together 23.61, 161.5, 2.6789, and 61.111. Ans. 248.8999.

	<b>2</b>	3.6	1		
1	6	1.5			
		2.6	7	8	9
	6	1.1	1	1	
$\overline{2}$	4	8.8	9	9	9

OPERATION.

We write the numbers so that figures of the same decimal place shall stand in the same column, and then, beginning at the right hand, add them as whole numbers are added, and place the decimal point in the result directly under those above.

Rule. — Write the numbers so that figures of the same decimal place shall stand in the same column.

Add as in whole numbers, and point off in the sum, from the right hand, as many places for decimals as equal the greatest number of decimal places in any of the numbers added.

*Proof.*—The proof is the same as in addition of whole numbers.

#### EXAMPLES.

- 2. Add together the following numbers: 81.61356, 6716.31, 413.1678956, 35.14671, 3.1671, 314.6. Ans. 7564.0052656.
- 3. What is the sum of the following numbers: 1121.6116, 61.87, 46.67, 165.13, 676.167895?

  Ans. 2071.449495.
  - 4. Add 7.61, 637.1, 6516.14, 67:1234, 6.1234 together.

    Ans. 7234.0968.
  - 5. Add 21.611, 6888.32, 3.6167 together.

Ans. 6913.5477.

- 6. Add together \$15.06, \$107.09, \$1.625, and \$93.765.
- 7. I have bought a horse for \$137.50, a wagon for \$55.63, a whip for \$1.375, and a halter for \$0.87½; what did they all cost?

  Ans. \$195.38.
- 8. What is the sum of twenty-three million ten; one thousand, and five hundred-thousandths; twenty-seven, and nineteen millionths; seven, and five tenths? Ans. 23001044.500069.
- 9. Add the following numbers: fifty-nine, and fifty-nine thousandths; twenty-five thousand, and twenty-five ten-thousandths; five, and five millionths; two hundred five, and five hundredths.

  Ans. 25269.111505.
- 10. What is the sum of the following numbers: twenty-five, and seven millionths; one hundred forty-five, and six hundred forty-three thousandths; one hundred seventy-five, and eightynine hundredths; seventeen, and three hundred forty-eight hundred-thousandths?

  Ans. 363.536487.
- 11. A farmer has sold at one time 3 tons and 75 hundredths of a ton of hay, at another time 11 tons and 7 tenths of a ton, and at a third time 16 tons and 125 thousandths of a ton. How much has he sold in all?

  Ans. 31.575.
- 12. Add together 73 and 29 hundredths, 87 and 47 thousandths, 3005 and 116 ten-thousandths, 28 and 3 hundredths, 29000 and 5 thousandths.
- 13. Add together two hundred nine thousand, and forty-six millionths; ninety-eight thousand two hundred seven, and fifteen ten-thousandths; fifteen, and eight hundredths; and forty-nine ten-thousandths.

  Ans. 307222.086446.

#### SUBTRACTION OF DECIMALS.

270. Ex. 1. From 61.9634 take 9.182. Ans. 52.7814.

Having written the less number under the greater, so that figures of the same decimal place stand in the same column, we subtract as in whole numbers, and place the decimal point in the result, as in addition of decimals.

RULE. — Write the less number under the greater, so that figures of the same decimal place shall stand in the same column.

Subtract as in whole numbers, and point off the remainder as in

addition of decimals.

*Proof.* — The proof is the same as in subtraction of whole numbers.

	Exa	MPLES.	
2.	3.	· <b>4.</b>	<b>5.</b>
3 9.3	5.	6.1	4 1.7
1.6789	1.678	1.99999	2 1.9 7 6 7
$\overline{37.6211}$	3.3 2 2	$\overline{\textbf{4.1}\ \textbf{0}\ \textbf{0}\ \textbf{0}\ \textbf{1}}$	$\begin{array}{c} \bullet \\ \hline 1\ 9.7\ 2\ 3\ 3 \end{array}$

- 6. From 29.167 take 19.66711. Ans. 9.49989.
- 7. From 91.61 take 2.6671. Ans. 88.9429.
- 8. From 96.71 take 96.709.
- 9. Take twenty-seven, and twenty-eight thousandths from ninety-seven, and seven tenths.

  Ans. 70.672.
- 10. Take one hundred fifteen, and seven hundredths from three hundred fifteen, and twenty-seven ten-thousandths.

Ans. 199.9327.

11. From twenty-nine million four thousand and five take twenty-nine thousand, and three hundred forty-nine thousand two hundred, and twenty-four hundred-thousandths.

Ans. 28625804.99976.

12. From one million take one millionth.

Ans. 999999.999999.

- 13. From \$19 take \$1.375. Ans. \$17.625.
- 14. A merchant bought flour to the amount of \$316.87\frac{1}{2}, and sold it for \$400; how much did he gain by the sale?
  - 15. From 19 million take 19 billionths.

Ans. 18999999.99999981.

16. Charles Washburne has in one farm 93.45 acres, in another 124 acres, in a third 244.285 acres, and in wood-lots 216.136 acres; how many acres more would he require to have exactly 1000 acres?

#### MULTIPLICATION OF DECIMALS.

Ex. 1. Multiply 76.81 by 3.2. Ans. 245.792.

OPERATION. We multiply as in whole numbers, and point off 7 6.8 1 on the right of the product as many figures for deci-3.2 mals as there are decimal figures in the multiplicand and multiplier counted together. The reason for 15362 pointing off the decimals in the product, as in the 23043 operation, will be seen, if we convert the multiplicand and multiplier into common fractions, and mul-2 4 5.7 9 2 tiply them together. Thus,  $76.81 = 76\frac{81}{100} = \frac{7681}{100}$ ; Then  $\frac{7681}{1000} \times \frac{82}{10000} = \frac{245792}{10000} = 245\frac{793}{10000} =$ 

and  $3.2 = 3\frac{2}{10} = \frac{32}{10}$ . 245.792, Ans., the same as in the operation.

2. Multiply .1234 by .0046.

Ans. .00056764.

.1234 .0046 7404 4936 .00056764

OPERATION.

Since the number of figures in the product is not equal to the number of decimals in the multiplicand and multiplier, we supply the deficiency by placing ciphers on the left hand. The reason of this process will appear, if we perform the operation thus:  $1234 = \frac{1284}{10000}$ ; and  $.0046 = \frac{16}{10000}$ . Then  $\frac{1284}{10000} \times \frac{10000}{10000} =$  $\frac{10000}{10000000} = .00056764$ , Ans., the same as in the operation.

Rule. - Multiply as in whole numbers, and point off as many figures for decimals, in the product, as there are decimal figures in the multiplicand and multiplier.

If there be not so many figures in the product as there are decimal figures in the multiplicand and multiplier, supply the deficiency by prefixing ciphers.

**Proof.**— The proof is the same as in multiplication of whole numbers.

### EXAMPLES.

3. Multiply 61.76 by .0071.	Ans438496.
4. Multiply .0716 by 1.326.	Ans0949416.
5. Multiply .61001 by .061.	• .
6. Multiply 71.61 by 365.	Ans. 26137.65.
7. Multiply .1234 by 1234.	Ans. 152.2756.

8. Multiply 6.711 by 6543.

Ans. 43910.073.

9. Multiply .0009 by .0009.

Ans. .00000081.

- 10. What is the product of one thousand and twenty-five, multiplied by three hundred and twenty-seven ten-thousandths?

  Ans. 33.5175.
- 11. What is the product of seventy-eight million two hundred five thousand and two, multiplied by fifty-three hundredths?

  Ans. 41448651.06.
- 12. Multiply one hundred and fifty-three thousandths by one hundred twenty-nine millionths.

  Ans. .000019737.
  - What will 26.7 yards of cloth cost, at \$5.75 a yard?
     Ans. \$153.525.
- 14. What will 14.75 bushels of wheat cost, at \$1.25 a bushel?

  Ans. \$18.4375.
- 15. What will 375.6 pounds of sugar cost, at \$0.125 per pound?
- 16. What will 26.58 cords of wood cost, at \$5.625 a cord?

  Ans. \$149.512\frac{1}{2}.
- 17. What will 28.75 tons of potash cost, at \$125.78 per ton?

  Ans. \$3616.175.

### CONTRACTIONS IN MULTIPLICATION OF DECIMALS.

272. To multiply a decimal by 10, 100, 1000, &c. Remove the decimal point as many places to the right as there are ciphers in the multiplier, annexing ciphers if required. Thus,  $1.25 \times 10 = 12.5$ ; and  $1.6 \times 100 = 160$ .

# EXAMPLES.

1. Multiply 131.634 by 1000.

Ans. 131634.

2. Multiply 3478.9 by 100.

Ans. 347890.

- 3. Multiply one thousandth by one thousand.
- 4. What is the profit on one million yards of cotton cloth, at \$0.007 per yard. Ans. \$7000.
- 273. When it is not necessary that all the decimal places of the product should be retained, tedious multiplications may often be obviated, by contracting the work as follows:—

Write the units' place of the multiplier under that figure of the multiplicand whose place it is proposed to retain in the product, and dispose of all the other figures of the multiplier in an order contrary to the usual one. Then, in multiplying, begin, for each partial product, with that figure of the multiplicand which stands above the multiplying figure, observing to add to the product the number nearest to that which would have been carried if the places at the right had not been rejected. Write down the several partial products, so that the right-hand figure of each shall be in the same column, and their sum will be the product required.

#### EXAMPLES.

1. Multiply 3.141592 by 52.7438, retaining only four places for decimals in the product.

Ans. 165,6995

r	IRST OPERATION.	SECOND OPERATION.
3.1 4 1 5 9 2	2 = Multiplicand	3.1 4 1 5 9 2
	= Multiplier reversed.	5 2.7 4 3 8
$\overline{1570796}$	= Product by $5, +1$	25 132736
$6\ 2\ 8\ 3\ 2$	= Product by 2, $+$ 2	94 24776
21991	= Product by 7, $+4$	$1\ 2\ 5\ 6   6\ 3\ 6\ 8$
1257	= Product by 4, $+1$	21991144
94	= Product by 3, $+1$	62831 84
2 5	= Product by $8, +1$	15707960
165.6995	= Product sought.	165.6995 001296

By comparison of the two methods of solution, it will be seen that the common one, as shown in the second operation, gives ten places of decimals, or six more than are required by the question, thus rendering unnecessary the several figures on the right of the vertical line. By the contrasted way, the multiplier, for convenience, has its figures reversed, or placed contrary to the usual order, so that the product of each figure by the one of the multiplicand above it, must be of the order of ten-thousandths. The first figure, at the right, of each partial product, being of the order of ten-thousandths, is written in the same column. To the product by 5 we add 1, since, if the 2 in the multiplicand had not been rejected, there would have been 1 to carry to the product of the 9 by the 5; to the product by 2 we add 2, since the product of the rejected figures, 92, by 2, approximates to 2 hundred, which would require 2 to be carried; to the product by 7 we add 4, since the product of the two rejected figures, 59, by 7, would require 4 to be carried; to the pro-

duct by 4 we add 1, since the product of the two rejected figures, 15, by 4, approximates to 1 hundred, which would require 1 to be carried; and so on, it being sufficient to increase the partial product only by such a number as approximates most nearly to that which would have been carried, provided the two rejected figures next to the figure of the multiplicand had been retained.

2. Multiply 325.701428 by .7218393, retaining only three places of decimals in the product.

Ans. 235.104.

3. Multiply 56.7534916 by 5.376928, retaining only five places of decimals in the product.

4. Multiply 843.7527 by 8634.175, retaining only the integers in the product.

Ans. 7285109.

#### DIVISION OF DECIMALS.

# 274. Ex. 1. Divide 1.728 by 1.2.

Ans. 1.44.

1.2 ) 1.7 : 1 2		1.4 4	Ans
5 :	$\overline{2}$		
4	8		
	48		
	48		

We divide as in whole numbers, and, since the divisor and quotient are the two factors, which, being multiplied together, produce the dividend, we point off two decimal figures in the quotient, to make the number in the two factors equal to the number in the product or dividend.

The reason for pointing off will also be seen by performing the ex-

ample with the decimals in the form of common fractions. Thus,  $1.728 = 1\frac{728}{1000} = \frac{1}{1000}$ ; and  $1.2 = 1\frac{1}{10} = \frac{1}{10}$ . Then  $\frac{1}{1000} = \frac{1}{10} = \frac{1}{10}$ . Then  $\frac{1}{1000} = \frac{1}{1000} = \frac{1}$ 

2. Divide 86.6947 by 589.

Ans. .0623.

		0	PE	RA'	TIC	ĸ.						
)	3	6	.6	9	4	7	(	.0	6	2	3	Ans
′							`					
	_	1	3	5	4	'						
		1	1	7	8							
			1	7	6	7						
			1	7	6	7						
	)		) 3 6 8 5 1	) 3 6.6 8 5 8 1 3 1 1	$\begin{array}{r} 3 \ 6.6 \ 9 \\ 8 \ 5 \ 8 \ 4 \\ \hline \hline 1 \ 3 \ 5 \\ 1 \ 1 \ 7 \\ \hline \end{array}$	$ \begin{array}{r} 3 & 6.6 & 9 & 4 \\ 8 & 5 & 8 & 4 \\ \hline 1 & 3 & 5 & 4 \\ 1 & 1 & 7 & 8 \\ \hline 1 & 7 & 6 \end{array} $		$\begin{array}{c} 36.6947 \\ 8584 \\ \hline 1354 \\ 1178 \\ \hline \hline 1767 \end{array}$	$\begin{array}{c} 36.6947 (.0\\ 8584 \\ \hline 1354 \\ 1178 \\ \hline 1767 \end{array}$	$\begin{array}{c} 3 & 6.6 & 9 & 4.7 \\ 8 & 5 & 8 & 4 \\ \hline & 1 & 3 & 5 & 4 \\ \hline & 1 & 1 & 7 & 8 \\ \hline & 1 & 7 & 6 & 7 \end{array}$	$\begin{array}{c} 36.6947 \ (.062 \\ \underline{8584} \\ \hline 1354 \\ \underline{1178} \\ \hline 1767 \end{array}$	$\begin{array}{c} 3 & 6.6 & 9 & 47 \\ 8 & 5 & 8 & 4 \\ \hline & 1 & 3 & 5 & 4 \\ \hline & 1 & 1 & 7 & 8 \\ \hline & 1 & 7 & 6 & 7 \end{array}$

We divide as in whole numbers, and since we have but three figures in the quotient, we place a cipher before them, and thus make the decimal places in the divisor and quotient equal to those of the dividend.

The reason for prefixing the cipher will appear more

obvious by solving the example in the form of common fractions. Thus,  $86.6947 = 86.6947 = \frac{86.6947}{100007} = \frac{86.6947}{100007}$ ; and  $589 = \frac{58}{10000}$ . Then  $\frac{86.6947}{100007} \div \frac{818}{100007} = \frac{86.6947}{100007} = \frac{82.8}{100007} = \frac{8$ 

Rule. — Divide as in whole numbers, and point off as many figures in the quotient as the number of decimal places in the dividend exceeds the number in the divisor; but if there are not as many, supply the deficiency by prefixing ciphers.

NOTE 1. - When the decimal places in the divisor exceed those in the dividend, make them equal by annexing ciphers to the dividend, and the quotient will be a whole number.

NOTE 2. — When there is a remainder after dividing the dividend, ciphers may be annexed, and the division continued; the ciphers thus annexed being regarded as decimals of the dividend; and to indicate in any case that the division does not terminate, the sign plus (+) can be used.

*Proof.* — The proof is the same as in division of whole numbers.

	Examples.	
3.	Divide 780.516 by 2.43.	Ans. 321.2.
4.	Divide 7.25406 by 9.57.	Ans758.
5.	Divide .21318 by .38.	
6.	Divide 7.2091365 by .5201.	Ans. 13.861+.
7.	Divide 56.8554756 by .0759.	Ans. 749.084.
	Divide 119109094.835 by 38123.45.	Ans. 3124.3.
	Divide 1191090.94835 by 3812345.	
	Divide 11910909483.5 by 38.12345.	
	Divide 11.9109094835 by 381234.5.	
12.	Divide 1191.09094835 by 3.812345.	
13.	Divide 11910909483.5 by .3812345.	
14.	Divide 1.19109094835 by 3.812345.	
15.	Divide .119109094835 by .3812345.	
16.	Divide 30614.4 by .9567.	Ans. 32000.
17.	Divide .306144 by 9567.	Ans000032.
18.	Divide four thousand three hundred to	venty-two,and four

thousand, and nine thousandths. Ans. .5403 + .19. How many yards of calico at \$0.0775 per yard can be purchased for \$10.85?

thousand five hundred seventy-three ten-thousandths by eight

20. What costs 1 acre of woodland when 19.65 acres are Ans. \$ 50. sold for \$ 982.50?

21. Divide three hundred twenty-three thousand seven hun-Ans. 64753000000. dred sixty-five by five millionths.

#### CONTRACTIONS IN DIVISION OF DECIMALS.

275. To divide a decimal by 10, 100, 1000, &c.

Remove the decimal point as many places to the left as there are ciphers in the divisor, and if there be not figures enough in the number, prefix ciphers. Thus,  $2.15 \div 10 = .215$ ; and  $1.9 \div 100 = .019$ .

#### EXAMPLES.

- 1. Divide 31.675 by 10.
- 2. Divide 916.05 by 100.
- 3. Divide 7.0461 by 100000.
- 4. Divide 70.461 by 100000.
- 5. Divide 704.61 by 100000.
- 6. Divide 7046.1 by 100000.
- 7. Divide 70460 by 100000.
- 8. Divide .70460 by 100000.
- 9. Divide 196.5 by 1000000.
- 10. If \$3500 are paid for 1000 yards of broadcloth, what is it a yard?

  Ans. \$3.50.
- 11. When \$1025 are paid for 40 boxes of sugar, each containing 250 pounds, what is the cost of 1 pound?

Ans.  $$0.10\frac{1}{4}$ .

Ans. .0001965.

276. When the divisor contains many decimal places, and only a certain number of decimals are required to be retained in the quotient, the work may be contracted as follows:—

First consider how many figures, in all, it is necessary for the quotient to contain. Then, by using the same number of figures from the left of the divisor, find the first figure of the quotient, and, instead of bringing down a new figure from the dividend, or annexing a cipher to the remainder, reject a figure on the right of the divisor at each successive division, and make the other figures a divisor. In multiplying such a divisor by the quotient figure, observe to add to the product the number nearest to that which would have been carried if no figures had been rejected.

# EXAMPLES.

1. Divide 695.57270875 by 52.35775, and retain in the quotient three places of decimals.

Ans. 13.285.

FIRST OPERATION. 52.35775)695.57270875(13.285. 52358 = product by 1, + 1.17199 15707 = product by 3, + 2.1492 1047 = product by 2, + 1.445 419 = product by 8, + 3.26 = product by 5, + 1.SECOND OPERATION. **5 2.3** 5 7 7 5 ) 6 9 5.5 7 2 7 0 8 7 5 (1 8.2 8 5. 5235775 17199520 15707325 14921958 1047 | 1550 44504087 41886200 26178875 26178875

By inspection. it is evident that the first quotient figure will be of the order of tens, and therefore the quotient will contain two places of whole numbers; and as there are to be three places of decimals, it must contain five figures. Hence, we divide at first by five figures of the given visor, counting them from the left toward the right, thus using the 52.357 and rejecting the figures, 75, on the right. multiplying each contracted divisor by its quotient figure we increase the pro-

duct by having regard to rejected figures, as in contracted multiplication of decimals (Art. 273).

The nature and extent of the contraction will be seen by comparison with the common method as shown in the second operation, in which the vertical line cuts off the figures not required.

NOTE. — When the given divisor does not contain as many figures as are required in the quotient, we must begin the division in the usual way and continue till the deficiency is made up, after which begin the contraction.

- 2. Divide 4327.56284563 by 873.469, and retain five decimal places in the quotient.

  Ans. 4.95445.
- 3. Divide 252070.520751 by 591.57, and terminate the operation with four decimal places in the quotient. Ans. 426.1043.

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- 4. Divide 70.23 by 7.9863, and retain in the answer four decimals.
- 5 Divide 12193263.1112635269 by 1234.56789, and let the quotient contain as many decimal places, plus one, as there will be integers in it.

  Ans. 9876.54321.

### REDUCTION OF DECIMALS.

- 277. To reduce a decimal to a common fraction.
- · Ex. 1. Reduce .125 to its equivalent common fraction.

Ans. 1/8.

OPERATION.

$$.125 = \frac{125}{1000} = \frac{25}{200} = \frac{5}{40} = \frac{1}{8}$$
 Ans.

Erasing the decimal point and supplying the denominator, which is understood, we have  $\frac{125}{1000}$ , which reduced to its lowest terms equals  $\frac{1}{8}$ , the answer required.

RULE. — Erase the decimal point, and write under the numerator its decimal denominator, and reduce the fraction to its lowest terms.

### EXAMPLES.

2. Reduce .875 to a common fraction. Ans. 7.

3. Reduce .9375 to a common fraction.

Ans. \(\frac{15}{16}\).

4. What common fraction is equivalent to .08125?

Ans. 183.

5. Change .00075 to the form of a common fraction.

6. Express 31.75 by an integer and a common fraction.

Ans. 31%.

7. Express 96.024 by an integer and a common fraction.

Ans.  $96_{1\frac{3}{2}5}$ .

- 8. Express 163.04 by an integer and a common fraction.
- 9. Express 1001.4375 by an integer and a common fraction. Ans.  $1001_{75}$ .
  - 10. Express 1457.222 by an integer and a common fraction.
  - 11. Express 19678.36 by an integer and a common fraction.
- 12. Express 9163.8755 by an integer and a common fraction.

  Ans. 9163\frac{1}{2}75\frac{1}{6}1.
  - 278. To reduce a common fraction to a decimal.
  - Ex. 1. Reduce \( \frac{3}{3} \) to a decimal. Ans. .375.

OPERATION.

8) 3.0 (3 tenths.

24

8)  $\frac{6}{5}$  0 (7 hundredths.

56

8) 40 (5 thousandths. 40

Ans. .375.

Or thus:

8) 
$$\frac{3.000}{.375}$$
 Ans.

Since we cannot divide the numerator, 3, by 8, we reduce it to tenths by annexing a cipher, and then dividing, we obtain 3 tenths and a remainder of 6 tenths. Reducing this remainder to hundredths by annexing a cipher, and dividing, we obtain 7 hundredths; which being reduced to thousandths by annexing a cipher, and then divided, gives a quotient of 5 thousandths. The sum of the several quotients, 375, is the answer.

To prove that .375 is equal to  $\frac{3}{8}$ , we change it to the form of a common fraction, by writing its denomi-

nator, and reducing it to its lowest terms. Thus,  $.375 = \frac{875}{1000} = \frac{8}{8}$ .

RULE. — Annex ciphers to the numerator, and divide by the denominator. Point off in the quotient as many decimal places as there have been ciphers annexed.

. Note. — It is not usually necessary that the decimals should be carried to more than six places. When a decimal does not terminate, the sign plus (+) is generally annexed. Thus, in the expression .333+, the sign annexed indicates that the division could be carried further.

### EXAMPLES.

2. Reduce § to a decimal.

Ans. .625.

3. Reduce ½ to a decimal.

4. Change  $\frac{3}{32}$  to a decimal.

Ans. .09375.

5. Change 13 to a decimal.

Ans. .076923+.

6. Reduce 191 to an equivalent decimal expression.

7. Reduce \$3157 to an equivalent decimal expression.

Ans. \$315.875.

8. Reduce \$ 1163\frac{3}{4} to an equivalent decimal expression.

Ans. 1163.75.

Note. — A decimal with a common fraction annexed constitutes what is called a *complex* decimal; as,  $.87\frac{1}{2}$ ,  $.83\frac{1}{2}$ , and  $.18\frac{3}{2}$ . In such expressions, instead of the common fraction, its equivalent decimal, with the decimal point omitted, may be substituted. Thus,  $.4\frac{1}{25} = .404$ .

9. Reduce .62½ to a simple decimal.

Ans. .625.

10. Reduce  $.37\frac{1}{16}$  to a simple decimal. Ans. .370625.

11. Reduce \$4.311 to a simple decimal expression.

Ans. \$ 4.3125.

12. Reduce \$60.18\frac{2}{3} to a simple decimal expression.

Ans. \$60.1875.

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- 13. What decimal expression is equivalent to  $\frac{1}{2}$  of  $\frac{2\frac{1}{2}}{5}$  of 2.04?
- 14. What decimal expression is equivalent to  $2\frac{1}{8}$ , + 0.37½, + ½ of  $\frac{1}{8}$  of  $\frac{1}{8}$ , 1.05? Ans. 2.9875.
- 279. To reduce a simple or compound number to a decimal of a higher denomination.
  - Ex. 1. Reduce 15s. 9d. 3far. to the decimal of a £.

    Ans. .790625.

We commence with the 3far., which we reduce to hundredths by annexing two ciphers; and then, to reduce these to the decimal of a penny, we divide by 4, since there will be 1 as many hundredths of a penny as of a farthing, and obtain .75d. Annexing this to the 9d., we divide by 12, since there will be 1 as

many shillings as pence; and then, the 15s. and this quotient by 20, since there will be  $\frac{1}{20}$  as many pounds as shillings, and obtain .790625£. for the answer. Hence the following

RULE. — Divide the lowest denomination, annexing ciphers if necessary, by that number which will reduce it to one of the next higher denomination. Then divide as before, and so continue dividing till the decimal is of the denomination required?

NOTE 1.— The given numbers may also be first reduced to a common fraction (Art. 256), and then the fraction changed to a decimal. Thus, if it be required to reduce 15s. 6d. to a decimal of a £.: 15s. 6d. = 186d.; 1£. = 280d.;  $\frac{1}{250}$ £. =  $\frac{1}{20}$ £. = .775 £. Answer.

Note 2. — Shillings, pence, and farthings may be readily reduced to a decimal of three places, by inspection, thus: Call half of the greatest even number of shillings tenths, and, if there be an odd shilling, call it 5 Hundredths; reduce the pence and farthings to farthings, and increase them by 1, if they amount to 24 or more, for thousandths. Thus, if it be required to reduce, by inspection, 19s. 10d. 2 far. to the decimal of a £.; half of 18s. = 9s., which denote a value of  $.9\pounds$ .; the 1s. denotes a value of  $.05\pounds$ .; and 10d. 2 far. = 42 far., which increased by 1 far. = 43 far., which denote a value of  $.043\pounds$ .;  $.9\pounds$ . +  $.05\pounds$ . +  $.043\pounds$ . =  $.993\pounds$ . Answer.

The reason for this process is, that 2s. equal a tenth of a £.; 1 shilling equals 5 hundredths of a £., and 1 farthing equals  $\frac{1}{160}$ £., or so nearly a thousandth of a £. that 24 farthings exactly equal 25 thousandths of a £.; and therefore farthings require to be increased only by 1 when they amount to 24 or more, to denote with sufficient accuracy their value in thousandths of a £.

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### EXAMPLES.

- 2. Reduce 9s. to the fraction of a pound. Ans. .45.
- 3. Reduce 15cwt. 3qr. 14lb. to the decimal of a ton.
- Reduce 2qr. 21lb. 8oz. 12dr. to the decimal of a cwt.
   Ans. .71546875.

5. Reduce 1gr. 3na. to the decimal of a yard.

Ans. .4375.

- Reduce 5fur. 35rd. 2yd. 2ft. 9in. to the decimal of a mile.
   Ans. .73603219+.
- 7. Reduce 3gal. 2qt. 1pt. of wine to the decimal of a hogshead.

  Ans. .0575396+.
  - 8. Reduce 1pt. to the decimal of a bushel. Ans. .015625.
  - 9. Reduce 2R. 16p. to the decimal of an acre. Ans. .6.
  - Reduce 175 cubic feet to the decimal of a ton of timber.
     Ans. 4.375.
  - Reduce 3.755 pecks to the decimal of a bushel.
     Ans. .93875.
  - 12. What decimal part of a degree is 25' 34".6?
- 13. Reduce 12T. 3cwt. 2qr. 20lb. to hundred-weight and the decimal of a hundred-weight.

  Ans. 243.7.
- 14. Reduce 2hhd. 30gal. 2qt. 1½pt. to gallons and the decimal of a gallon.

  Ans. 156.6875.
- 15. Reduce to the decimal of a pound, 19s.  $11\frac{2}{4}$ d., 16s.  $9\frac{1}{4}$ d., and 17s.  $5\frac{1}{2}$ d., and find their sum.

  Ans. 2.710416+.
- 280. To find the value of a decimal in whole numbers of lower denominations.

Ex. 1. What is the value of .790625 £.?

**Ans.** 15s. 9d. 3far.

There will be 20 times as many millionths of a shilling as of a pound; therefore, we multiply the decimal, .790625, by 20, and reduce the improper fraction to a mixed number by pointing off six figures on the right, which is dividing by its denominator, 1000000. The figures on the left of the point are shillings, and those on the right, the decimal of a shilling. This decimal of a shilling we multiply by 12, and, pointing off as before, obtain 9d., and a decimal of a penny. The decimal

of a penny we multiply by 4, and pointing off have 3 farthings, which, taken with the other denominations obtained, gives 15s. 9d. 3far. for the answer.

Rule. — Multiply the decimal by that number which will reduce it to the next lower denomination, and point off as in multiplication of decimals.

Then, multiply the decimal part of the product, and point off as before. So continue till the decimal is reduced to the denominations required.

The several whole numbers of the successive products will be the answer.

NOTE. — When there is a decimal in the last product, it may be changed to a common fraction.

### EXAMPLES.

- 2. What is the value of .625 of a shilling? Ans. 71d.
- 3. What is the value of .6725 of a cwt.?

Ans. 2qr. 17lb. 4oz.

- 4. What is the value of .9375 of a yard?
- 5. What is the value of .7895 of a mile?

Ans. 6fur. 12rd. 10ft. 648in.

6. What is the value of .9378 of an acre?

Ans. 3R. 30p. 13ft.  $9\frac{99}{125}$ in.

- 7. Reduce .5615 of a hogshead of wine to its value in gallons, &c.

  Ans. 35gal. 1qt. 0pt. 3433gi.
  - 8. Reduce .367 of a year to its value in days, &c.

Ans. 134d. 1h. 7m. 194sec.

9. What is the value of .6923828125 of a cwt.?

Ans. 2qr. 19lb. 3oz. 13d.

- 10. What is the value of .015625 of a bushel?
- 11. What is the value of .55 of an ell English?

Ans. 2qr. 3na.

12. What is the value of .6 of an acre? Ans. 2R. 16p.

# MISCELLANEOUS EXAMPLES.

- 1. What is the value of 7cwt. 2qr. 18lb. of sugar, at \$11.75 per cwt.?

  Ans. \$90.24.
  - 2. What cost 19cwt. 3qr. 14lb. of iron, at \$9.25 per cwt.?

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- 3. What cost 39A. 2R. 15p. of land, at \$87.375 per acre?

  Ans. \$3459.50333.
- 4. What would be the expense of making a turnpike 87m. 3fur. 15rd., at \$578.75 per mile? Ans. \$50595.41 $\frac{1}{64}$ .
- 5. What is the cost of a board 18ft. 9in. long, and 2ft. 3½in. wide, at \$.053 per foot?

  Ans. \$2.277½.
- 6. Goliath of Gath was 6½ cubits high; what was his height in feet, the cubit being 1st. 7.168in.? Ans. 10st. 4.592in.
- 7. If a man travel 4.316 miles in an hour, how long would he be in travelling from Bradford to Boston, the distance being 29½ miles?

  Ans. 6h. 50m. 6sec. +
- 8. What is the cost of 5yd. 1qr. 2na. of broadcloth, at \$5.62½ per yard?

  Ans. \$30.234§.
- 9. Bought 17 bags of hops, each weighing 4cwt. 3qr. 7lb., at \$5.87. per cwt.; what was the cost?
- 10. Purchased a farm, containing 176A. 3R. 25rd., at \$75.37\frac{1}{2} per acre; what did it cost? Ans. \$13334.308\frac{1}{3}\frac{1}{3}.
- 11. What cost 17625 feet of boards, at \$12.75 per thousand?

  Ans. \$224.7183.
- 12. How many square feet in a floor 19ft. 3in. long, and 15ft. 9in. wide?

  Ans. 303ft. 27in.
- 13. How many square yards of paper will it take to cover a room 14ft. 6in. long, 12ft. 6in. wide, and 8ft. 9in. high?
- 14. How many solid feet in a pile of wood 10ft. 7in. long, 4ft. wide, and 5ft. 10in. high?

  Ans. 246+3ft.
- 15. How many garments, each containing 4yd. 2qr. 3na., can be made from 112yd. 2qr. of cloth?
- 16. Bought 1gal. 2qt. 1pt. of wine for \$1.82; what would be the price of a hogshead?

  Ans. \$70.56.
- 17. Bought 125 byd. of lace for \$15.06; what was the price of 1 yard?

  Ans. \$0.12.
- 18. What cost 17cwt. 3qr. of wool, at \$35.75 per hundredweight?

  Ans. \$634.562\fmu.
- 19. What cost 7hhd. 47gal. of wine, at \$87.25 per hogshead?

  Ans. \$675.8483.
- 20. How many solid feet in a stick of timber 34ft. 9in. long, 1ft. 3in. wide, and 1ft. 6in. deep?

  Ans. 65.15625ft.
- 21. If 18yd. 1qr. of cloth cost \$ 36.50, what is the price of 1 yard?

  Ans. \$ 2.00.

- 22. If \$477.72 be equally divided among 9 men, what will be each man's share? Ans. \$ 53.08.
- 23. A man bought a barrel of flour for \$5.375, 7gal of molasses for \$1.78, 9gal. of vinegar for \$1.1875, 1gal. of wine for \$1.125, 14lb. of sugar for \$1.275, and 5lb. of tea for \$2.625; what did the whole amount to? Ans. \$13.3671.
- 24. A man purchased 3 loads of hay; the first contained 23 tons, the second 37 tons, and the third 1,15 tons; what was the value of the whole, at \$17.625 a ton? Ans. \$128.88213.
- 25. How many hogsheads of water will it take to fill a cistern which is 15.25 feet long, 8.4 feet wide, and 10 feet Ans. 152hhd. 6 gal. deep?
- 26. At \$13.625 per cwt., what cost 3cwt. 2qr. 7lb. of Ans. \$ 48.6411. sugar?
  - 27. At \$125.75 per acre, what cost 37A. 3R. 35rd.?

Ans. \$ 4774.570 5.

- 28. At \$11.25 per cwt., what cost 17cwt. 2qr. 21lb. of Ans. \$ 199.2371. rice?
- 29. What cost 73 bales of cotton, each weighing 3.37cwt., at \$ 9.37\frac{1}{2} per cwt.?
- 30. What cost 7hhd. 49gal. of wine, at \$97.625 per hogshead? Ans. \$759.30524.
- 31. What cost 7yd. 3qr. 3na. of cloth, at \$4.75 per Ans. \$37.7031.
- 32. What cost 27T. 15cwt. 1qr. 3½lb. of hemp, at \$183.62 Ans. \$ 5098.071 117. per ton?
- 33. What is the cost of constructing a railroad 17m. 3fur. Ans. \$ 30067.97833. 15rd., at \$ 1725.875 per mile?
- 34. When \$624.53125 are paid for 17A. 3R. 15p. of land, what is the cost of one acre?
- 35. Paid \$494.53125 for 19T. 15cwt. 2qr. 14lb. of hay; Ans. \$ 24.999 $\frac{17.2}{32.97}$ . what was the cost per ton?
- 36. How much land, at \$40 per acre, can be obtained for \$1004.75? Ans. 25A. 0R. 19p.
- 37. How many cords of wood can be put into a space 20.5 feet long, 12.75 feet wide, and 7.6 feet high?
  - Ans. 15 cords 66.9 cubic feet.
- 38. How many bushels of corn at \$0.621 per bushel must a farmer exchange for 31 yards of sheeting at \$0.081 per yard, and  $7\frac{1}{2}$  yards of broadcloth at \$2.75 per yard? Ans.  $37\frac{23}{250}$ .

- 39. I have expended \$42.875 for a quantity of grain,  $\frac{3}{10}$  of it being corn, at \$0.75 a bushel;  $\frac{2}{20}$  of it wheat, at \$2 a bushel; and the balance oats, at \$0.40 a bushel, to the amount of \$3.50. Required the number of bushels of each kind purchased.
- 40. If a mason, in constructing a drain 250.35 feet long, begin with a width of 8 inches, and increase  $\frac{5}{10}$  of an inch in every foot of length, how many times the width of the beginning of the drain will its end be?

  Ans. 16.646875.
- 41. A gentleman gave ½ of his property to his son James; ½ of it to his son William; ½ of the remainder to his daughter Mary; and the balance to his wife. It appeared that Mary received \$2243.26 less than James. What was the amount divided, and how much did each receive?

Ans. Amount, \$13459.56; James, \$3364.89; William, \$4486.52; Mary, 1121.63; wife, \$4486.52.

# CIRCULATING DECIMALS.

281. A CIRCULATING DECIMAL is a decimal in which one or more figures are continually repeated in the same order. Thus, in reducing  $\frac{1}{3}$  to an equivalent decimal, on annexing ciphers and dividing by the denominator, the result obtained, .333+, is a circulating decimal; for, however far the division might be carried, the same figure would continue to be repeated without the decimal terminating.

Such decimals are sometimes called infinite, or repeating; and, for sake of distinguishing, those decimals that terminate are sometimes termed *finite*.

- 282. A repetend is a figure, or a series of figures, continually repeated. To mark a repetend, a point (.) is placed over a single repeating figure, or over the first and last of a series of repeating figures. Thus, in .3, the point denotes that the 3 is a repetend; and in .72, that the 72 is a repetend.
  - 283. A single repetend is one in which only one figure is

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- repeated; as in .1111+ denoted by  $\dot{i}$ ; and 2222+, denoted by  $\dot{z}$ .
- 284. A compound repetend is one in which the same set of figures is repeated; as in .135135+, denoted by .135, and .30363036+, denoted by 3036.
- 285. A pure repetend is one which contains only the figures of the repetend; as, .3, .02, and .123.
- 286. A mixed repetend is one in which a repetend is preceded in the same fraction by one or more figures. The figures preceding the repetend are called the *finite part*. Thus, 416 is a mixed repetend, of which the figure 6 is the repetend, and the figures 41 the finite part; also, 1.728 is a mixed repetend, of which the figures 28 are the repetend, and the figures 1.7 the finite part.
- 287. A perfect repetend is a pure repetend containing the same number of figures as there are units in its denominator less one. Thus, † reduced to a decimal gives .142857, which, as it contains as many figures as there are units in the denominator, 7, less one, is a perfect repetend.
- 288. Similar repetends are those which begin at the same distance from the decimal point; as .3 and .6; or 5.123 and 3.478.
- 289. Dissimilar repetends are those which begin at different distances from the decimal point; as .986 and .4625; or .5925 and .0423436.
- 290. Conterminous repetends are those which terminate at the same distance from the decimal point; as .631 and .465, or .0753 and .4752.
- 291. Similar and conterminous repetends are those which both begin and end at the same distance from the decimal point; as  $.3\dot{5}\dot{4}$  and  $.4\dot{2}\dot{5}$ ; or  $.5\dot{7}\dot{5}\dot{7}$  and  $5\dot{7}\dot{2}\dot{3}$ .
- 292. Repetends always arise from common fractions, which, when in their lowest terms, contain in their denominator other factors than 2 and 5. For when a common fraction is in its lowest terms, its numerator and denominator are prime to each

other (Art. 219), and the annexing of one or more ciphers to the numerator makes the same a multiple of 10, but does not render it divisible by any factor, except 2 and 5, the factors of 10. Therefore, when the denominator of a fraction, in its lowest terms, contains other factors than those of 10, the decimal resulting from dividing the numerator with ciphers annexed, will not terminate, but will contain one or more figures that constantly repeat.

- 293. A pure repetend is always equivalent to a common fraction whose numerator is the repeating figure or figures, and whose denominator as many places of nines as there are repeating figures. For, by reducing  $\frac{1}{5}$  to a decimal, we obtain as its equivalent the repetend .1; and since .1 is equivalent to  $\frac{1}{5}$ , .2 will be equivalent to  $\frac{2}{5}$ , .3 to  $\frac{2}{5}$ , and so on, till .9 is equal to  $\frac{2}{5}$  or 1. Again,  $\frac{1}{5}$ , and  $\frac{1}{5}$ , being reduced, give .01, and .001; that is,  $\frac{1}{5}$  = .01, and  $\frac{1}{5}$  = .001; therefore,  $\frac{2}{5}$  = .02, and  $\frac{1}{5}$  = .02, and so on; the same principle holding true in all like cases.
- **294.** A mixed repetend is equivalent to a complex decimal (Art. 278), or to a complex fraction. Thus, the mixed repetend .2412 is equivalent to the mixed decimal .2412, which is equal to the complex fraction  $\frac{24\frac{1}{9}}{100}$ .
- 295. Repetends are of the same denomination only when they are similar and conterminous. For then alone, by having a common denominator, do they express fractional parts of the same unit.

# REDUCTION OF REPETENDS.

- 296. To reduce a repetend to an equivalent common fraction.
  - Ex. 1. Reduce .123 to an equivalent common fraction.

Ans. 413.

OPERATION. We write the figures of the given repetend with the decimal point omitted for the numerator, and as many nines as places in the repetend for the denominator, of a common fraction (Art. 293), and obtain  $\frac{1}{208}$ , which, reduced to its lowest terms,  $=\frac{4}{288}$ , the answer required.

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2. Reduce .138 to an equivalent common fraction.

Ans. 35.

.1 3  $\dot{8} = \frac{13\frac{8}{9}}{100} = \frac{125}{900} = \frac{5}{36}$  Ans.

The mixed repetend .138 is equivalent to the mixed decimal .138 (Art. 294), which we readily change to the form of a complex frac-

tion by erasing the decimal point and writing the denominator, 100, which is understood; and thus obtain  $\frac{13\frac{3}{6}}{100}$ , which, reduced to its simplest form, gives  $\frac{3}{6}$ , the answer required. Hence,

If the given repetend be SIMPLE, make the repeating figure or figures the numerator, and take as many nines as the repetend has figures for the denominator.

If the given repetend be MIXED, change it to an equivalent complex fraction, and that fraction to its simplest form.

Note. — Any circulating decimal may be transformed into another decimal, having a repetend of the same number of figures; as, .78 = .787, and .581 = .5845 = .58453. Thus, when such expressions as 12.5 or 17.56 occur, they may be also transformed; as 12.5 = 12.52, and 17.56 = 17.567 = 17.5675, &c.

## EXAMPLES.

- 3. Required the common fraction equal to  $.\dot{6}$ . Ans.  $\frac{6}{3} = \frac{2}{3}$ .
- 4. Reduce 1.62 to its equivalent mixed number.

Ans. 133.

- 5. Change .53 to an equivalent common fraction.
- 6. What common fraction is equivalent to .769230?

Ans. 18.

- 7. What common fraction is equivalent to .5925?
- 8. Change 31.62 to an equivalent mixed number.

Ans. 3133.

9. Reduce .008497133 to an equivalent common fraction.

Ans. 9838.

- 297. To determine the kind of decimal to which a given common fraction can be reduced.
- Ex. 1. Required to find whether the decimal equal to  $\frac{210}{1120}$  be finite or circulating; and if finite, of how many places the decimal will consist.

  Ans. Finite, of 4 places.

$$rac{210}{1120} = rac{3}{16} = rac{3}{2 \times 2 \times 2 \times 2} = .1875.$$

We reduce the given fraction to its lowest terms, and then resolve the de-

nominator, 16, of the fraction obtained,  $\frac{3}{16}$ , into its prime factors, which we find to be  $2 \times 2 \times 2 \times 2$ . Now, since the denominator contains no prime factor other than 2 or 5, it is evident that, by annexing ciphers to the numerator, 3, and dividing by the denominator, 16, the decimal arising will terminate, and thus be finite (Art. 292).

Since, in reducing a common fraction to its equivalent decimal, we annex ciphers to the numerator and divide by the denominator (Art. 278), every 10, or 2 and 5, that enter into the denominator as factors must produce one decimal place, and no more, and therefore every other factor 2 or 5 must give one, and only one, decimal place. The denominator, 16, contains only the factor 2 taken 4 times, or  $2^4$ ; and the exponent of the 2 indicates that the decimal equivalent to  $\frac{3}{16}$  must contain exactly 4 decimal places, which we verify by reducing the  $\frac{3}{16}$  to its equivalent decimal, .1875.

2. Find whether the decimal equal to 1750 be finite or circulating; and if circulating, of how many places the finite part, if any, and the circulating part, will each consist.

Ans. Circulating: the finite part, 2 places; the repetend, 6 places.

$$\frac{235}{1750} = \frac{47}{350} = \frac{47}{2 \times 5 \times 5 \times 7} = .13\dot{4}2857\dot{1}.$$
 We reduce the given fraction to its lowest terms, and

obtain  $\frac{47_0}{350}$ . The denominator, 350, = 2 × 5 × 5 × 7, contains a prime factor, 7, other than 2 and 5; therefore the decimal equivalent to  $\frac{47_0}{47_0}$  will contain a repetend; and as, of the factors 2 and 5, the higher exponent of either, that of 5, is 2, the decimal will have 2 finite places before the repetend commences. This we verify by reducing  $\frac{47_0}{310}$  to its equivalent decimal, .13428571. Hence, to determine whether the decimal to which a given common fraction can be reduced is finite or circulating, and the number of finite decimal places, if any,

Having reduced the given common fraction to its lowest terms, resolve the denominator into its prime factors. If these factors be not other than 2 or 5, the decimal will be FINITE; if other prime factors occur with 2 or 5, the decimal will be a MIXED REPETEND; and if neither 2 nor 5 occurs as factor, the decimal will be a PURE REPETEND.

Whichever factor 2 or 5 occurs in the denominator with the higher exponent will by its exponent denote the number of finite decimal places.

Note. — The number of figures of which a repetend will consist may be discovered by dividing 1 with ciphers annexed by the factors other than 2 or 5 of the denominator, until there is a remainder 1. Thus, if it be required to discover the number of figures in the repeating part of the decimal equivalent to  $\frac{47}{50}$ , we divide 1 with ciphers annexed by 7, the only prime factor in the denominator other than 2 or 5, until there is a remainder of 1, which occurs after the sixth division, thereby indicating that the repeating part will consist of six figures. We have seen that these must be preceded by two places of finite decimals, so that the mixed repetend equal to  $\frac{47}{250}$  must consist of eight places in all.

### EXAMPLES.

3. To what kind of a decimal can 1 be reduced?

Ans. A pure repetend, of 2 places.

4. How many places of decimals, finite and repeating, will be required to express  $\frac{39}{1320}$ ?

Ans. 5 places; 3 finite and 2 repeating.

- 5. To what kind of a decimal can 197 be reduced?
- 6. Reduce  $13\frac{17}{45}$  to a mixed repetend. Ans. 13.37
- 7. Change  $\frac{166}{18538}$  to a mixed repetend. Ans. .008497133.
- 8. Of how many figures will the repetend consist that corresponds to  $\frac{2}{29}$ ? Ans. 28 figures.

#### TRANSFORMATION OF REPETENDS.

- **298.** Any finite decimal may be considered as a mixed repetend by making ciphers continually recur; thus,  $.42 = .42\dot{0} = .42\dot{0}\dot{0} = .42\dot{0}\dot{0}$ , &c.
- **299.** Any circulating decimal may be transformed into another having the same number of repeating figures; thus,  $.1\dot{2}\dot{7}$  =  $.1\dot{2}\dot{7}\dot{2}$  =  $.1\dot{2}\dot{7}\dot{2}\dot{7}$ , &c.
- 300. Any circulating decimal having as repetend any number of figures may be transformed to another having twice or thrice that number of figures, or any multiple thereof; thus, .5925, having a repetend of three figures, may be transformed to one having 6, 9, 12, &c. places; therefore .5925 = .5925925925 = 5925925925925925925, &c.
- 301. The value of a decimal is not changed by any of the above transformations, as may be seen by reducing the given

repetends to their equivalent common fractions (Art. 296) and comparing them together. Hence, they can be used in making dissimilar repetends similar and conterminous.

- 302. To make any number of dissimilar repetends similar and conterminous.
- Ex. 1. Make similar and conterminous 9.167, 14.6, 3.165, 12.432, 8.181, and 1.307.

OPERATION.					
Dissimilar.	Similar.	Similar and Conterminous.			
$9.1 \div 7$	$= 9.167\dot{6}$	= 9.16767676			
1 4.6	= 14.600	= 14.60000000			
3.1 6 5	$=$ 3.1 6 $\dot{5}$	= 3.1 6 5 5 5 5 5 5			
$1\ 2.\dot{4}\ 3\ \dot{2}$		$3 = 12.43\dot{2}4324\dot{3}$ Ans.			
$8.1 \stackrel{.}{8} \stackrel{.}{1}$		$= 8.18\dot{1}8181\dot{8}$			
$1.\dot{3}\ 0\ \dot{7}$	= 1.30730	0 = 1.30730730			

We make the finite mixed decimal, 14.6, a mixed repetend by annexing recurring ciphers, and make it and all the given repetends similar, by extending the figures to the right, so that the circulating part of each may begin at the same distance from the decimal point as does that repetend which is preceded by the most finite decimal places. Then, to make conterminous the repetends that have thus been rendered similar, as some of them consist of 1, some of 2, and the others of 3 places, we extend the repeating figures of each repetend till those of each occupy as many places as there are units in the least common multiple of 1, 2, and 3, which is 6. Hence, to make dissimilar repetends similar and conterminous,

Transform the given repetends so that the circulating parts shall commence at the same distance from the decimal point, and shall consist of as many circulating places as there are units in the least common multiple of the number of repeating figures found in the given decimals.

#### EXAMPLES.

- 2. Make 3.671, 1.0071, 8.52, and 7.616325 similar and conterminous.
- Make 1.52, 8.7156, 3.567, and 1.378 similar and conterminous.
  - 4. Make .0007, .141414, and 887.1 similar and conterminous.
  - 5. Make .3123, 3.27, and 5.02 similar and conterminous.
- Make 17.0884, 1563.0929, and 15.12345 similar and conterminous.

# ADDITION OF CIRCULATING DECIMALS.

**803.** Ex. 1. Add 2.765, 7.16674, 3.671, .7, and .1728 together.

Ans. 14.55436.

OPERATION  Dissimilar. Similar and Contermine						
Dissimilar.	8	imilar ab	1 Conten	mino		
$2.7\dot{6}\dot{5}$	-	$2.7 \dot{6}$	$56\dot{5}$			
$7.1\dot{6}67\dot{4}$	-	$7.1 \ \dot{6}$	$67\dot{4}$			
3.67 i	_	$3.6\dot{7}$	$13\dot{6}$			
. <del>7</del>	_	.7 7	777			
.i 7 2 <b>š</b>	-	.1 7	28 i			
An	s. ]	4.5 5	436			

Having made the given repetends similar and conterminous (Art. 302), we add as in addition of whole numbers, and obtain 14.55433. The right-hand figure of this result we increase by such a number as would have been carried, if the repetends had been continued farther to

the right. In that case we should have had to carry 3 after finding the amount of the first left-hand column of the repetends continued. We therefore increase the sum as first found, and thus have the true amount as in the operation, 14.55436.

RULE. — Make the given repetends, when dissimilar, similar and conterminous. Add as in addition of finite decimals, observing to increase the repetend of the amount by the number, if any, to be carried from the left-hand column of the repetends.

## Examples.

- 2. Add 3.5, 7.651, 1.765, 6.173, 51.7, 3.7, 27.631, and 1.003 together.

  Ans. 103.2591227.
  - 3. Reduce  $\frac{1}{3}$ ,  $\frac{1}{7}$ , and  $\frac{1}{9}$  to decimals, and find their sum.
- 4. Find the sum of 27.56, 5.632, 6.7, 16.356, .71, and 6.1234.

  Ans. 63.1690670868888.
  - 5. Add together .165002, 31.64, 1.06, .34634, and 13.
  - 6. Add together .87, .8, and .876. Ans. 2.644553.
- 7. Required the value of .3 + .45 + .45 + .351 + .6468 + .6468 + .6468, and .6468. Ans. 4.1766345618.
- 8. Find the value of 1.25 + 3.4 + .637 + 7.885 + 7.875 + 7.875 + 11.1.

  Ans. 40.079360724.
  - 9. Add together 131.613, 15.001, 67.134, and 1000.63.
  - 10. Find the value of  $5.\dot{1}634\dot{5} + 8.\dot{6}38\dot{1} + 3.\dot{7}\dot{5}$ .

    Ans.  $17.\dot{5}591912084737409030\dot{2}$ .

### SUBTRACTION OF CIRCULATING DECIMALS.

# **304.** Ex. 1. From $87.16\dot{4}\dot{5}$ take $19.47\dot{9}16\dot{7}$ .

Ans. 67.685377.

Dissimilar. Similar and Conterminous.  $87.1 \ 6 \ 4 \ 5 = 87.1 \ 6 \ 4 \ 5 \ 4 \ 5$   $19.47 \ 9 \ 16 \ 7 = 19.47 \ 9 \ 16 \ 7$ Ans.  $67.68 \ 5 \ 37 \ 7$ 

Having made the repetends similar and conterminous, we subtract as in whole numbers, regarding, however, the right-hand figure of the subtrahend as increased by 1, since 1 would have been carried

to it in subtracting, if the repetends had been continued farther to the right, as is evident from the circulating part of the subtrahend being greater than that of the minuend.

Rule. — Make the repetends, when dissimilar, similar and conterminous. Subtract as in subtraction of finite decimals; observing to regard the repetend of the subtrahend as increased by 1, when it exceeds that of the minuend.

## EXAMPLES.

2.	From 7.1 take 5.02.	Ans. 2.08.
3.	From 315.87 take 78.0378.	Ans. 237.838072095497.
4.	Subtract + from 2.	Ans079365.
<b>5.</b>	From 16.1347 take 11.0884.	Ans. 5.0462.
6.	From 18.1678 take 3.27.	Ans. 14.8951.
7.	From 3.123 take 0.71.	Ans. 2.405951.
8.	From <sup>3</sup> take <sup>2</sup> / <sub>11</sub> .	Ans246753.
9.	From 4 take 4.	Ans158730.
10.	From $\frac{9}{17}$ take $\frac{6}{17}$ .	Ans1764705882352941.
11.	From 5.12345 take 2.352345	i <b>ė.</b>

# MULTIPLICATION OF CIRCULATING DECIMALS.

Ans. 2.7711055821666927777988888599994.

305. Ex. 1. Multiply .36 by 25.  $.36 = \frac{36}{99} = \frac{4}{11}; .25 = \frac{2\frac{5}{3}}{10} = \frac{23}{90}.$   $4 \times \frac{23}{30} = \frac{920}{920} = \frac{48}{485} = .0929 \text{ Ans.}$  Ans. 0929.

We change the given numbers to their equivalent common fractions, and, multiplying, obtain  $\frac{92}{980} = \frac{46}{195}$ , which,

reduced to its equivalent decimal, gives .0925, the answer required.

RULE. - Change the given numbers to their equivalent common frac-Multiply them together, and reduce the product to its equivalent decimal.

## EXAMPLES.

2. Multiply 87.32586 by 4.37.

Ans. 381.6140338.

3. Multiply 582.347 by .03.

4. Multiply 3.145 by 4.297.

Ans. 13.5169533.

5. What is the value of .285714 of a guinea?

6. What is the value of .461607142857 of a ton?

Ans. 9cwt. 0qr. 23-lb.

7. What is the value of .284931506 of a year?

Ans. 104d.

### DIVISION OF CIRCULATING DECIMALS.

**306.** Ex. 1. Divide .54 by .15.

Ans. 3.506493.

 $.5\dot{4} = \frac{54}{99} = \frac{6}{11}; .1\dot{5} = \frac{15}{10} = \frac{14}{90} = \frac{7}{45}.$  $\frac{6}{11} \div \frac{7}{45} = \frac{279}{77} = 3\frac{39}{7} = 3.506493$  Ans.

We change the given numbers to their equivalent common fractions.

and. dividing, obtain 270, which, reduced to its equivalent decimal, gives 3.506493, the answer required.

RULE. — Change the given numbers to their equivalent common frac-Divide, and reduce the quotient to its equivalent decimal.

# EXAMPLES.

2. Divide 345.8 by .6.

Ans. 518.83.

3. Divide 234.6 by .7.

4. Divide 13.5169533 by 3.145.

Ans. 4.297.

5. Divide 381.6140338 by 4.37. Ans. 87.32586.

6. Divide .428571 by .625.

7. Find the value of  $2.370 \div 4.923076$ .

Ans. .481.

8. Find the value of  $.09 \div .230769$ .

Ans. .39.

9. Find the value of  $316.31015 \div .3$ .

10. Find the value of  $100006 \div .6$ .

11. Divide .36 by .25. Ans. 1.4229249011857707509881.

### CONTINUED FRACTIONS.

A CONTINUED FRACTION is a fraction having for its numerator 1, and for its denominator a whole number plus a fraction whose numerator is 1, and whose denominator is a whole number plus a fraction, and so on. Thus,

$$\frac{1}{3+1}$$

$$2+1$$

$$5+1$$

$$4+\frac{1}{6}$$
, is a continued fraction.

artial fractions composing the parts of a continue called its terms. Thus, in the fraction given a continue its terms.

The partial fractions composing the parts of a continued fraction are called its terms. Thus, in the fraction given above, 1, 1, 5, &c. are its terms.

- 308. Continued fractions are used in obtaining, in smaller numbers, the approximate values of fractions whose terms, when reduced to their simplest forms, are expressed in numbers inconveniently large.
- 309. To transform a common fraction into a continued fraction, and to find, in smaller numbers, its approximate values.
- Ex. 1. Transform  $\frac{19}{60}$  into a continued fraction, and find its several approximate values.

Dividing both terms of 19 by the numerator, which operation will not change the value expressed (Art. 217), the fraction becomes the denominator of which being between 3 and 4, the value of the given fraction must be between 1 and 1; and neglecting the fraction 3, for the present, in the denominator, we have 1 for the first approximate value. This approximation, however, is greater than the true value, since the denominator, 3, is less than the true denominator 3.8. We therefore divide both terms of the 3 by its numerator, and it becomes  $\frac{1}{61}$ , which is between  $\frac{1}{6}$  and  $\frac{1}{7}$ . By neglecting the fraction 1 in the denominator, and taking the 1 instead of the  $\frac{8}{19}$ , we have  $\frac{1}{3+\frac{1}{6}} = \frac{1}{3\frac{1}{6}} = \frac{6}{19}$  for the second approximate value of the given fraction; which approximation is too small, since in the denominator, instead of 30, we used 1, which is greater than the 30. If we now include in the calculation the remaining partial fraction  $\frac{1}{8}$ , we have  $\frac{1}{3\frac{1}{6}+1} = \frac{1}{3\frac{3}{19}} = \frac{19}{60}$ , the original fraction.

By the processes of the operation it will be seen that the first approximate value sought was obtained by disregarding all the partial fractions after the first, the second approximate value by disregarding all the partial fractions after the second, &c.

Rule. — Divide the greater term of the given fraction by the less, and the divisor by the remainder, and so on, as in finding the greatest common divisor. The quotients thus found will be the denominators of the several terms of the continued fraction, and the numerator of each will be 1.

For the FIRST approximation, take the first terms of the continued fraction.

For the SECOND approximation, multiply the terms of the first approximate fraction by the denominator of the SECOND term of the continued fraction, adding 1 to the product of the denominators.

For each succeeding approximation, multiply the terms of the approximation last found by the denominator of the NEXT term of the continued fraction, and add the corresponding terms of the preceding approximation.

Note 1. — When the fraction given is improper, the true approximations will be the reciprocals of the fractions found by the rule.

Note 2. - In a series of approximations the first is smaller, the second larger, and so on, every odd fraction being smaller, and every even one larger, than the given fraction. Each successive approximate fraction, however, approaches more nearly than the one preceding it to the value of the given fraction. When the continued fraction indicates many approximations, it is generally sufficient for ordinary purposes to find only from three to six of them.

Note 3. - A continued fraction may, for convenience, be expressed by writing its terms one directly after another, with the sign plus (+) between the denominators; thus, the continued fraction equivalent to 1% may be written 3+1+3.

### EXAMPLES.

2. Transform 18 into a continued fraction.

Ans. 
$$\frac{1}{2+\frac{1}{3+\frac{1}{4}}}$$
.

- 3. Transform 261 into a continued fraction.
- 4. Find the approximate values of 22. Ans.  $\frac{1}{2}$ ,  $\frac{1}{3}$ ,  $\frac{2}{5}$ ,  $\frac{3}{8}$ ,  $\frac{29}{77}$ .
- 5. Find the first five approximate values of  $\frac{1327}{1631}$ .

Ans. 
$$\frac{2}{1}$$
,  $\frac{5}{2}$ ,  $\frac{7}{3}$ , or  $2$ ,  $2\frac{1}{2}$ ,  $2\frac{1}{3}$ .

7. Find the first six approximate values of  $\frac{1}{13.569}$ .

Ans.  $\frac{1}{13}$ ,  $\frac{1}{14}$ ,  $\frac{2}{27}$ ,  $\frac{7}{95}$ ,  $\frac{37}{502}$ ,  $\frac{44}{597}$ . 8. What are the first four approximate values of 1.27? Ans.  $\frac{1}{4}$ ,  $\frac{4}{3}$ ,  $\frac{5}{4}$ ,  $\frac{14}{26}$ , or 1,  $1\frac{1}{3}$ ,  $1\frac{1}{4}$ ,  $1\frac{3}{11}$ ,  $1\frac{7}{26}$ .

## RATIO.

- 310. RATIO is the relation, in respect to magnitude or value, which one quantity or number bears to another of the same kind.
- 311. The comparison by ratio is made by considering how often one number contains, or is contained in, another. the ratio of 10 to 5 is expressed by 2, the quotient arising from the division of the first number by the second, or it may be expressed by  $\frac{5}{10} = \frac{1}{2}$ , the quotient arising from the division of the second number by the first, as the second or the first number shall be regarded as the unit or standard of comparison. In general, of the two methods, the first is regarded as the more simple and philosophical, and therefore has the preference in this work.

Note. - Which of the two methods is to be preferred, is not a question of so much importance as has been by some supposed, since the connection in which ratio is used is usually such as to readily determine its interpretation.

The two numbers necessary to form a ratio are called N 20

the terms of the ratio. The first term is called the antecedent, and the last, the consequent. The two terms taken together are called a couplet; and the quotient of the two terms, the index or exponent of the ratio.

- 313. The ratio of one number to another may be expressed either by two dots (:) between the terms; or in the form of a fraction, by making the antecedent the numerator and the consequent the denominator. Thus, the ratio 6 miles to 2 miles may be expressed as 6:2, or as \frac{5}{2}.
- 314. The terms of a ratio must be of the same kind, or such as may be reduced to the same denomination. Thus, cents have a ratio to cents, and cents to dollars, &c.; but cents have not a ratio to yards, nor yards to gallons.
- **315.** A *simple* ratio is that of two whole numbers; as, 3:4,8:16,9:36, &c.
- **316.** A complex ratio is that of two numbers, of which one or both are fractional; as,  $6:4\frac{1}{2}, \frac{3}{5}:\frac{1\frac{1}{2}}{5}, 4\frac{1}{2}:2\frac{1}{4}$ , &c.
- 317. A compound ratio is the product of two or more ratios. Thus, the ratio compounded of 4:2 and 6:3 is  $\frac{4}{5} \times \frac{6}{3} = \frac{26}{6} = 4$ , or  $4 \times 6:2 \times 3 = 4$ .

A compound ratio is generally expressed by writing the ratios composing it, in a column, with the antecedents in one vertical line, and the consequents in another; thus,  $\frac{4:2}{6:3}$  expresses a compound ratio.

Note. — If a ratio be compounded of two equal ratios, it is called a duplicate ratio; of three ratios, a triplicate ratio, &c.

318. A ratio is either direct or inverse. A direct ratio is the quotient of the antecedent by the consequent; an inverse ratio, or reciprocal ratio, as it is sometimes called, is the quotient of the consequent by the antecedent, or the reciprocal of the direct ratio. Thus the direct ratio of 6 to 2 is § or 3; and the inverse or reciprocal ratio of 6 to 2 is § or ½, which is the same as the reciprocal of 3, the direct ratio of 6 to 2.

NOTE 1. — One quantity is said to vary directly as another, when both increase or decrease together in the same ratio; one quantity is said to vary in-

versely as another, when the one increases in the same ratio as the other decreases.

NOTE 2. — The word ratio, when used alone, means the direct ratio.

- **319.** When the antecedent and consequent of a ratio are equal, the ratio equals 1, and is called that of equality. Thus, the ratio of  $6:6=\frac{6}{6}=1$ , and the ratio of  $6\times 4:8\times 3=\frac{24}{2}=1$ , are ratios of equality. But if the antecedent is larger than the consequent, the ratio is that of greater inequality, and if the antecedent is smaller than the consequent, the ratio is that of less inequality. Thus, the ratio of  $15:5=\frac{1}{16}=3$ , is a ratio of greater inequality; and the ratio of  $7:14=\frac{7}{14}=\frac{1}{2}$ , is a ratio of less inequality.
- **320.** The ratio of two fractions having a common numerator is the same as the inverse ratio of their denominators. Thus, the ratio of  $\frac{3}{4}:\frac{3}{8}$  is  $\frac{3}{4}\div\frac{3}{8}=2$ , which is the inverse ratio of the denominator 4 to the denominator 8.
- **321.** The ratio of two fractions having a common denominator is the same as the ratio of their numerators. Thus, the ratio of  $\frac{5}{7}:\frac{3}{7}:\frac{5}{7}:\frac{3}{7}:=2$ , which is the ratio of the numerator 6 to the numerator 3.
- 322. The inverse or reciprocal ratio of two numbers denotes what part or multiple the antecedent or dividend is of the consequent or divisor. Thus, inquiring what part of 4 is 3, or what part 3 is of 4, is the same as inquiring the inverse or reciprocal ratio of 4:3. The inverse ratio of 4:3 is  $\frac{3}{4}$ , and 3 is  $\frac{3}{4}$  of 4.
- 323. In order to compare one number with another, by ratio, it is necessary that they should not only be of the same kind, but of the same denomination. Thus, to compare 2 days with 12 hours, it is necessary that the days be reduced to hours, before we can indicate the ratio, which is 48 hours: 12 hours.
- **324.** If the antecedent of a ratio be multiplied, or the consequent divided, the ratio is multiplied. Thus, the ratio of 6:3 is 2, but  $6 \times 2:3$  is 4; or  $6:3 \div 2$  is 4.
- 325. If the antecedent of a ratio be divided, or the consequent multiplied, the ratio is divided. Thus, the ratio of 18:6 is 4, but  $18 \div 3:6$  is 1; or  $18:6 \times 3=1$ .



If both the antecedent and consequent of a ratio be multiplied or divided by the same number, the ratio is not altered. Thus, the ratio of 8:2 is 4; of  $8\times 2:2\times 2$  is 4; and of 8 + 2 : 2 + 2 = 4

### REDUCTION AND COMPARISON OF RATIOS.

- 227. Ratios, being of the nature of fractions, may be reduced, compared, and otherwise operated upon like them.
  - To reduce a ratio to its lowest terms.
  - Ex. 1. Reduce 18:9 to its lowest terms. Ans. 2:1.

OPBRATION. We cancel in the two terms the  $18:9=\frac{18}{9}=\frac{2}{9}=2:1$ . common factor 9, and obtain  $\frac{2}{9}=\frac{1}{9}$ 2:1, the answer. Hence

Cancel in the given ratio all factors common to its terms.

## Examples.

- 2. Reduce to its lowest terms 63:72. . Ans.  $\overline{I}$ .
- 3. Reduce to its lowest terms 66:24.
- 4. Reduce to its lowest terms  $4 \times 6 \times 3:8 \times 9 \times 2$ . Ans. 4.

- 5. What are the lowest terms of  $19 \times 5 \times 2 \times 3:15 \times$  $12 \times 38$ ?
- To reduce a complex or a compound ratio to a simple one.
  - Reduce  $5\frac{1}{4}:\frac{3}{4}$  to a simple ratio. Ans. 22:3. Ex. 1.

We express the  $5\frac{1}{2}: \frac{3}{4} = \frac{5\frac{1}{2}}{\frac{3}{2}} = \frac{44}{6} = \frac{22}{3} = 22:3$  Ans. given ratio in the form plex fraction, which,

changed to a simple fraction (Art. 242), and reduced to its lowest terms, gives 22 = 22:3, the answer required.

2. Reduce  $8:5 \atop 7:24$  to a simple ratio. Ans. 7:15.

 ${8:5 \atop 7:24}$  =  ${8 \atop 5}$  of  ${7 \atop 24}$  =  ${8 \atop 5}$   $\times {7 \atop 24}$  =  ${7 \atop 15}$  = 7:15 Ans.

We express the given ratio in the form of a compound fraction,

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which, reduced to a simple one (Art. 329), gives  $\frac{7}{15} = 7:15$ , the answer required. Hence, to reduce a complex or a compound ratio to a simple one,

Proceed as in like operations with fractions.

### EXAMPLES.

3. Reduce 5: 4 to a simple ratio.	Ans. 35: 24.
4. Reduce $13\frac{1}{2}$ : 27 to a simple ratio.	Ans. $1:2$ .
5. Reduce 6.25: 3.125 to a simple ratio.	Ans. $2:1$ .
6. Reduce $25:10$ to a simple ratio.	Ans. 5:8.
7. Reduce $\begin{pmatrix} 3 : 6 \\ 9 : 27 \\ 108 : 12 \end{pmatrix}$ to a simple ratio.	Ans. 3:2.
8. Reduce $\begin{cases} 12\frac{1}{2} : 6\frac{1}{4} \\ 76.5 : 25.5 \end{cases}$ to a simple ratio.	Ans. 6:1.

330. To find the ratio of one number to another.

Ex. 1. Required the direct ratio of 108 to 9. Ans. 12.

Since 9 is the unit or standard of comparison, we make it the consequent (Art. 111) and the 108 the antecedent of the ratio, and obtain  $\frac{168}{9} = 12$  Ans.

2. Required the inverse ratio of 72 to 8. Ans. \(\frac{1}{4}\).

The reciprocal of the direct ratio of 72: 8 inverted  $=\frac{8}{72}=\frac{1}{9}$  Ans.

We divide the consequent 8 by the antecedent 72, or, which is the same thing, find the reciprocal of the direct ratio of 72: 8 (Art. 318), by inverting its terms, and thus obtain  $\frac{8}{12}=\frac{1}{4}$  Ans. Hence,

The direct ratio is found by dividing the antecedent by the consequent, and the inverse ratio by dividing the consequent by the antecedent.

NOTE 1.—Ratios expressed by fractions having different denominators must be reduced to a common denominator, in order to be compared; and then they are to each other as their numerators (Art. 323).

NOTE 2. — When a ratio is expressed in terms inconveniently large and prime to each other, we may find the approximate values of the ratio expressed in smaller numbers, as in other fractional expressions (Art. 309).

#### EXAMPLES.

3. What is the ratio of 39 to 13?
4. What is the ratio of 2 yards 2 quarters to 9 yards?

- 5. What is the ratio of 21 gallons to 1 of a hogshead?

  Ans. 1.
- 6. What is the ratio of  $\frac{1}{2}$  of  $\frac{1}{$

Áns. 4.

- 7. What is the inverse ratio of 24:6?
- Ans.  $\frac{1}{4}$ .

8. What part of 36 is 4?

- Ans. 1.
- 9. What part of a farm of 94A. 2R. 16rd. is 11A. 3R.?
- 10. Which is the greater, the ratio of 17 to 9, or of 39 to 19?

  Ans. 39: 19.
- 11. By how much does the ratio of  $36 \times 4 \times 3:12 \times 16 \times 2$  exceed that of  $60 \div (3 \times 5):20 \times 2 \div 8$ ? Ans. 13.
  - 12. What is the inverse ratio of  $.0\dot{2}:2.\dot{5}0\dot{3}$ ?
- 13. Which is the greater, the ratio of  $\frac{1}{2}$  of  $\frac{1}{5}$ :  $\frac{1}{3}$  of  $\frac{1}{4}$ , or that of 5:4?
- 14. The height of Bunker Hill Monument is 220 feet, and that of the great pyramid, Egypt, 500 feet; what is the ratio of the height of the former to that of the latter?

  Ans. 11.
- 15. A certain farm contains 180 acres, and the township of which it forms a part is 36 square miles in extent. What is the ratio of the latter to the former?
  - Find approximate values for the ratio of 4900 to 11283.
     Ans. 1, 3, 10, 33, 76, 76, &c.
- 17. The ratio of the circumference of a circle to its diameter is 3.141592. Required approximate values for this ratio.

Ans. 3, 22, 333, 355, &c., or 3, 31, 315, 316, &c.

#### ANALYSIS BY RATIO.

331. Operations by analysis may often be much abridged by ratio. Thus, frequently, it is more convenient to multiply or divide by the ratio a number bears to a unit of the same kind, than to multiply or divide by the number itself.

This form of analysis is much used by business men; and, like that by aliquot parts (Art. 114), is sometimes called *Practice*.

### EXAMPLES.

1. What cost 14 tons 15cwt. 3qr. 20lb. of iron, at \$60 a ton?

Ans. \$887.85.

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OPERATION.
                                                 $60.00 = \cos t \text{ of 1 ton.}
                                                   14
                                               $840.00 =
                                                                         14 tons.
(10 \text{cwt.}: 1 \text{ ton} = \frac{1}{4}); \frac{1}{4} \text{ of } $60 = 30.00 =
                                                                         10cwt.
(5 \text{cwt.}: 10 \text{cwt.} = \frac{1}{4}): \frac{1}{4} \text{ of } $30 = 15.00 =
                                                                          5cwt.
(2qr.: 5cwt. = \frac{1}{10}); \frac{1}{10} \text{ of } $15 = 1.50 =
                                                                          2qr.
(1qr. : 2cwt. = \frac{1}{2}); \frac{1}{2} \text{ of } 1.50 = 0.75 =
                                                                          1qr.
(151b.: 3qr. = \frac{1}{5}); \frac{1}{5} \text{ of } 2.25 = 0.45 =
                                                                        15lb.
(5lb.: 15lb. = \frac{1}{3}); \frac{1}{3} of 0.45 = 0.15 =
                                                                          5lb.
                                   Ans.
                                             $887.85 =
                                                                        14T. 3gr. 20lb.
```

Since 1 ton costs \$60, 14 tons will cost 14 times \$60, or \$840. 15cwt. = 10cwt. + 5cwt. Since the ratio of 10cwt. to 1 ton or 20cwt. =  $\frac{1}{2}$ , 10cwt. will cost  $\frac{1}{2}$  as much as 1 ton, or \$30; and as the ratio of 5cwt. to 10cwt. =  $\frac{1}{2}$ , 5cwt. will cost  $\frac{1}{2}$  as much as 10cwt. or \$15. 3qr. = 2qr. + 1qr. Since the ratio of 2qr. to 5cwt. or 20qr. =  $\frac{1}{10}$ , 2qr. will cost  $\frac{1}{10}$  as much as 5cwt., or \$1.50; and as the ratio of 1qr. to 2qr. =  $\frac{1}{2}$ , 1qr. will cost  $\frac{1}{2}$  as much as 2qr., or \$0.75. 20lb. = 15lb. +5lb. Since the ratio of 15lb. to 3qr. or 75lb. =  $\frac{1}{2}$ , 15lb. will cost  $\frac{1}{2}$  as much as 3qr., or \$0.45; and as the ratio of 5lb. to 15lb. =  $\frac{1}{3}$ , 5lb. will cost  $\frac{1}{3}$  as much as 15lb., or \$0.15. The cost of the several parts equals the cost of the whole, or \$887.85, Ans.

- 2. What is the value of 17 acres 3 roods 35 rods of land, at \$80 per acre?

  Ans. \$1437.50.
  - 3. What cost 16cwt. 3qr. 10lb. of guano, at \$2.50 per cwt.?
- 4. What cost 27cwt. 1qr. 20lb. of coffee, at \$14 per cwt.?

  Ans. \$384.30.
- 5. If 1 yard of cloth cost \$5.60, what will 7yd. 3qr. 2na. cost?

  Ans. \$44.10.
  - 6. What cost 7 tons 13cwt. 2qr. of hay, at \$20 per ton?
- 7. What cost 99bu. 1pk. 4qt. of wheat, at \$1.92 per bushel?

  Ans. \$191.80.

The quantity being nearly 100 bushels, we find the cost of 100 bushels by annexing two ciphers to \$1.92, the cost of 1 bushel, and obtain \$192, from which we subtract the cost of 2pk. 4qt., the difference of quantity between that given and 100 bushels; the cost of 2pk. = \$0.96; and that of 4qt. = \$0.24; \$192 - \$0.96 + \$0.24 = \$191.80 Ans.

- 8. What cost 19yd. 3qr. 2na. of cloth, at \$ 4.40 per yard?
  Ans. \$ 87.45.
- 9. How much must be paid for 24A. 3R. 20p. of land, at \$32 per acre?
- 10. How much must be paid for 199lb. 12oz. of butter, at \$ 0.30 per lb.?

  Ans. \$ 59.925.
- 11. What cost 714 yards of broadcloth, at 15s. 6d. per yard?

  Ans. 553£. 7s.
- 12. How much must be paid for the services of a man 2y. 9mo. 15da, at \$450 yer year? Ans. \$1256.25.
- 13. If 1 acre of land cost \$80.50, what will 25 acres 2 roods 35 rods cost?

  Ans. \$2070.35-.
  - 14. What cost 498lb. of tea, at 2s. 6d. per lb.?
- 15. If 1cwt. 2qr. 12lb. of alum can be purchased for \$4.05, how much can be purchased for \$28.35?

Ans. 11cwt. 1gr. 9lb.

### OPERATION.

# $$28.35 \div $4.05 = 7;$

1cwt. 2qr. 12lb.  $\times$  7 = 11cwt. 1qr. 9lb. Ans.

Since the ratio of \$28.35 to \$4.05 = 7, \$28.35 will purchase 7 times as much as \$4.05. By multiplying what the latter will purchase by the ratio, we have the answer required.

- 16. If 11gal. 3qt. 1pt. of molasses cost \$5.83\frac{3}{4}\$, what will 35gal. 2qt. 1pt. cost?

  Ans. \$17.51\frac{1}{4}\$.
- 17. If 24yd. 3qr. of cloth cost \$ 49.50, what will 12yd. 1qr. 2na. cost?
- 18. If 17bu. 2pk. 4qt. of oats be paid for 14bu. 3pk. of salt, what quantity of oats must be paid for 73bu. 3pk. of salt?
  - Ans. 88bu. 0pk. 4qt.
- 19. If \$ 9.75 will purchase 1T. 2ewt. 2qr. 15lb. of coal, how much will \$ 3.25 purchase?
- 20. If a train of cars move at the average velocity of 27m. 3fur. 20rd. per 1h. 20m., how far will it move in 4h.?

Ans. 82m. 2fur. 20rd.

### PROPORTION.

**332.** A PROPORTION is an equality of ratios. Any four numbers are in proportion, when the ratio of the first to the second is the same as that of the third to the fourth. Thus, the ratios 9:3 and 6:2, being equal to each other, when written, 9:3=6:2, or  $\frac{9}{3}=\frac{9}{2}$ , form a proportion.

Proportion is written with the sign of equality (=), or, as is more common, with four dots (::), between the ratios. Thus, 9:3 = 6:2, or 9:3::6:2, expresses a proportion, and is read, The ratio of 9 to 3 is equal to the ratio of 6 to 2, or 9 is to 3 as 6 is to 2.

- 333. The terms of a proportion are the four numbers which form the proportion. These numbers are also called proportionals. The first and third terms, or proportionals, are called antecedents, the second and fourth are called consequents; the first and last are called the extremes, the second and third the means; the first and second compose the first couplet, the third and fourth compose the second; and when the ratio of the first of three terms is to the second as the ratio of the second is to the third, the second term is called a mean proportional to the other two terms.
- **334.** A direct proportion is an equality between two direct ratios; an inverse or reciprocal proportion is an equality between a direct and an inverse or reciprocal ratio. Thus, the numbers 4, 2, 6, 3 are, as they stand, in direct proportion, denoting 4:2::6:3; but in the order 4, 2, 3, 6, are in inverse proportion, denoting that  $4:2::\frac{1}{3}:\frac{1}{6}$ , or the direct ratio of 4 to 2 is equal to the inverse ratio of 3 to 6.

Note. - The term proportion, used alone, always means direct proportion.

**335.** In any proportion, if the antecedents or consequents, or both, are divided, or multiplied, by the same number, they are still proportionals. Thus, dividing the antecedents of the proportion 4:8::10:20 by 2, we have 2:8::5:20; dividing the consequents by 2, we have 4:4::10:10; and dividing both the antecedents and consequents by 2, we have 2:4::5:10; each of which results is a proportion, since if we divide

the second term of each by the first, and the fourth by the third, the two quotients will be equal. The effect is the same when the terms are multiplied by the same number.

- 336. In every proportion the product of the two extremes is equal to the product of the two means. Thus, the proportion 16:8::20:10 may be expressed  $\frac{16}{8}=\frac{20}{10}$ . Now, if we reduce these fractions to a common denominator, we have  $\frac{160}{80}=\frac{160}{10}$ ; but in this operation we multiplied together the two extremes of the proportion, 16 and 10, and the two means, 8 and 20; thus,  $16 \times 10 = 8 \times 20$ . Hence,
- 1. If the extremes and one of the means are given, the other mean may be found by dividing the product of the extremes by the given mean; or,
- 2. If the means and one of the extremes are given, the other extreme may be found by dividing the product of the means by the given extreme.

#### SIMPLE PROPORTION.

337. Simple Proportion is an equality between two simple ratios.

NOTE. — Simple Proportion is sometimes called the Rule of Three, and formerly was termed by arithmeticians the Golden Rule.

- 338. The object of that part of simple proportion which is usually included in arithmetics, is to find a fourth proportional to three given numbers, or, in other words, to find the fourth term of a proportion, when the other three terms are given.
- Ex. 1. If a man travel 243 miles in 9 days, how far will he travel in 24 days?

  Ans. 648 miles.

Extreme. Mean. Mean. Extreme. 9 da.: 2 4 da.:: 2 4 3 m.: — m.

2 4

9 7 2

4 8 6

9 ) 5 8 3 2

Ans. 648 m. Extreme.

Since 9 days have the same ratio to 24 days as 243 miles, the distance of travel in 9 days, have to the distance of travel in 24 days, we have the first three terms of a proportion given, namely, the two means and one of the extremes, from which to find the required extreme. Now, to arrange the given numbers in the order of

a proportion, or state the question, we make the 243 miles the third term, because it is of the same kind as the required fourth term, and as from the nature of the question the latter must be greater than the third term, we make the greater of the other two numbers the second term, and the less the first; and, then, the product of the means divided by the given extreme gives the required extreme (Art. 336).

BY ANALYSIS. — If 'a man travel 243 miles in 9 days, he will in 1 day travel  $\frac{1}{3}$  of 243 miles = 27 miles; then, if he travel 27 miles in 1 day, in 24 days he will travel 24 times 27 miles = 648 miles, the answer, as before.

By Ratio.  $-9: 24 = \frac{9}{24} = \frac{8}{8}$ ; 243 miles  $\div \frac{8}{8} = 648$  miles, Ans.

2. If 15 yards of cloth cost \$48.90, what will 5 yards cost?

Ans. \$16.30.

$$15:5::$48.90:$ 5 \times 48.90 = 16.30.$$

 $\frac{\cancel{5} \times 48.90}{\cancel{15}} = 16.30.$ 3 Ans. \$16.30.

We state the question by making \$48.90 the third term, because it is of the same kind as the required term. Then, since the answer must be less than \$48.90, because 5 yards will cost less than 15 yards, we make 5 yards, the less of the two numbers, the second term, and 15 yards the first; and pro-

ceed as in the first example, except that we abridge the work by cancellation.

By Analysis. — If 15 yards cost \$48.90, 1 yard will cost  $\frac{1}{15}$  of \$48.90 = \$3.26; then, if 1 yard cost \$3.26, 5 yards will cost 5 times \$3.26 = \$16.30.

Rule. — Write the given number that is of the same kind as the required fourth term, or answer, for the third term of the proportion.

Of the other two numbers write the larger for the second term, and the less for the first, when the answer should exceed the third term; but write the less for the second term, and the larger for the first, when the answer should be less than the third term.

Multiply the second and third terms together, and divide their product by the first; or divide the third term by the ratio of the first term to the second.

Note 1.— When the first and second terms are of different denominations, they must be reduced to the same denomination; and when the third term is a compound number, it must be reduced to the lowest denomination mentioned in it. The answer will be of the same denomination as the third term.

Note 2.—To shorten the operations, factors common to the dividend and divisor may be cancelled.

Note 3. — The pupil should perform these questions by analysis, as well as by proportion, and introduce cancellation when it will abbreviate the operation.

# EXAMPLES.

- 3. If 16 acres of land cost \$720, what will 197 acres cost?

  Ans. \$8865.
- 4. If \$8865 buy 197 acres, how many acres may be bought for \$720?
- 5. What will 84hhd. of molasses cost, if 15hhd. can be purchased for \$175.95?

  Ans. \$985.32.
- 6. If \$ 100 gain \$ 6 in 12 months, how much would it gain in 40 months?

  Ans. \$ 20.
- 7. If a certain vessel has provisions sufficient to last a crew of 10 men 45 days, how long would the provisions last if the vessel were to ship 5 new hands?

  Ans. 30 days.
- 8. If 7 and 9 were 12, what, on the same supposition, would 8 and 4 be?
- 9. If 9 men can perform a certain piece of labor in 17 days, how long would it take 3 men to do it?

  Ans. 51 days.
- 10. If 3 men can perform a piece of labor in 51 days, how many must be added to the number to perform the labor in 17 days?

  Ans. 6.
- 11. A rectangular piece of land containing an acre is 5½ rods in breadth. What is its length?

  Ans. 294 rods.
- 12. If \$100 gain \$6 in a year, how much will \$850 gain?
- 13. If \$100 gain \$6 in a year, how much would be sufficient to gain \$32 in a year?

  Ans. \$533.334.
- 14. If 20 gallons of water weigh 167lb., what will 180 gallons weigh?

  Ans. 1503lb.
- 15. If a staff 3 feet long cast a shadow of 2 feet, how high is that steeple whose shadow is 75 feet?

  Ans. 1123 feet.
- 16. If 5\frac{2}{5}cwt. be carried 36 miles for \$4.75, how far might it be carried for \$160?

  Ans. 1212\frac{1}{12} \text{ miles.}
- 17. If 100 workmen can perform a piece of work in 12 days, how many men are sufficient to perform the work in 8 days?

  Ans. 150.
- 18. If  $\frac{7}{12}$  of a yard cost  $\frac{7}{20}$  of a dollar, what will  $\frac{4}{5}$  of a yard cost?

  Ans. \$ 0.48.
- 19. What must be paid for 21A. 3R. 20p. of land, if 36A. 3R. cost \$ 1260?

  Ans. \$ 750.

- 20. What is the value of 2000lb. of standard gold, the eagle, or \$10 piece, weighing 10pwt. 18gr.?
- 21. If 4½ yards of cloth cost \$9.75, what will 13½ yards cost?

  Ans. \$29.25.
- 22. What is the length of a rectangle whose contents are 1 sq. ft. and whose breadth is  $2\frac{1}{3}$  inches? Ans.  $57\frac{2}{3}$  inches.
  - 23. If  $\frac{7}{16}$  of a ship cost 51£., what are  $\frac{32}{32}$  of her worth?

    Ans. 10£. 18s. 6\frac{2}{3}d.
- 24. If the moon moves 13° 10′ 35″ in one day, in what time does she perform one revolution?

  Ans. 27da. 7h. 43m. +
- 25. If 7lb. of sugar cost \(\frac{3}{4}\) of a dollar, what are 12lb. worth?

  Ans. \(\frac{9}{1.28\}\).
- 26. If \$1.75 will buy 7lb. of loaf-sugar, how much will \$213.50 buy?

  Ans. 8cwt. 2qr. 4lb.
- 27. If 7 ounces of gold are worth 30£, what is the value of 7lb. 11oz.

  Ans. 407£. 2s. 104d.
- 28. A friend borrowed of me \$500 for 6 months; how long ought he to lend me \$600, to requite the favor?
- 29. If the penny loaf weighs 7oz. when flour is \$8 perbarrel, how much should it weigh when flour is \$7.50 perbarrel?

  Ans.  $7\frac{7}{15}$  ounces.
- 30. If a regiment of soldiers, consisting of 1000 men, are to be clothed, each suit to contain  $3\frac{3}{4}$  yards of cloth that is  $1\frac{7}{4}$  yards wide, and to be lined with flannel  $1\frac{1}{4}$  yards wide, how many yards will it take to line the whole? Ans. 5625yd.
- 31. If by working 14 hours per day C. Simmons can plant half of a field in 9 days, in what time will he plant the remainder, working 10 hours per day at the same rate each hour?
- 32. If 75 gallons of water fall into a cistern containing 500 gallons, and 40 gallons run out, in an hour, in what time will it be filled?

  Ans. 14h. 17m. 8\$sec.
- 33. How many dozen pairs of gloves, at \$0.56 per pair, can be bought for \$120.96?

  Ans. 18 dozen.
- 34. A certain cistern has three pipes; the first will empty it in 20 minutes, the second in 40 minutes, and the third in 75 minutes; in what time would they all empty it?

Ans. 11m.  $19\frac{13}{53}$  sec.

35. A can mow a certain field in 5 days, and B can mow it
N 21

Ans. 122 days.

- in 6 days; in what time would both of them together mow it?

  Ans.  $2_7^{\beta_7}$  days.
- 36. A wall, which was to be built 32 feet high, was raised 8 feet by 6 men in 12 days; how many men must be employed to finish the wall in 6 days?
- 37. A can build a boat in 20 days, but with the assistance of C he can do it in 12 days; in what time would C do it alone?

  Ans. 30 days.
- 38. In a fort there are 700 men provided with 184000lb. of provisions, of which each man consumes 5lb. a week; how long can they subsist?

  Ans. 52 weeks 4 days.
- 39. If 25 men have  $\frac{3}{4}$  of a pound of beef each, three times in a week, how long will 3150lb. last them? Ans. 56 weeks.
- 40. How many tiles 8 inches square will lay a floor 20 feet long and 16 feet wide?

  Ans. 720.
- 41. How many stones 10 inches long, 9 inches broad, and 4 inches thick, would it require to build a wall 80 feet long, 20 feet high, and 2½ feet thick?

  Ans. 17280 stones.
- 42. If there be paid for 1 ton 7cwt. 3qr. 20lb. of coal \$9.50, what will 13 tons 5cwt. 2qr. cost?
- 43. If 61.3 pounds of tea cost \$44.9942, what is the price per pound?

  Ans. \$0.734.
  - 44. What is the value of .15 of a hogshead of lime, at \$2.39 per hogshead?

    Ans. \$0.3585.
    - 45. If .75 of a ton of hay cost \$15, what is it per ton?

      Ans. \$20.
  - 46. How many yards of carpeting that is half a yard wide will cover a room that is 30 feet long and 18 feet wide?

    Ans. 120 yards.
  - 47. If a man perform a journey in 15 days, when a day is 12 hours long, in how many days will he do it when a day is but 10 hours long?
  - 48. If 450 men are in a garrison, and their provisions will last them but 5 months, how many must leave the garrison that the same provisions may be sufficient to supply the remaining men 9 months?

    Ans. 200 men.
  - 49. The hour and minute hands of a watch are together at 12 o'clock; when will they next be together?

Ans. 1h. 5m. 27 3 sec.

50. A and B can perform a piece of work in 5.5 days, B and C in 63 days, and A and C in 6 days; in what time would each of them perform the work alone, and how long would it take them to do the work together?

Ans. A would do the work in 10 days; B, in 12 days; C, in 15 days; A, B, and C together, in 4 days.

- 339. To divide a number or quantity into parts, which are proportional to given numbers.
- Ex. 1. Divide \$250 into two parts which shall be one to the other as 2 to 3.

  Ans. \$100 and \$150.

2 + 3 = 5 at 5 : 3 :: \$250 : \$150, the greater part, 5 : 2 :: \$250 : \$100, the less part,

Since the parts are to be proportional to 2 and 3, whose sum is 5, it is evident that the sum of the

two numbers, 5, will have the same ratio to the greater of them, 3, as the amount to be divided, \$250, has to the greater of the required parts; and that the sum, 5, will have the same ratio to the less number, 2, as the \$250 has to the less of the required parts; we therefore make two statements, and then find the required term of each proportion as in Art. 337. Hence,

As the sum of the given numbers is to any one of them, so is the whole quantity to be divided to the part corresponding to the number used as the second term.

NOTE. — This application of proportion is sometimes called *Distributive* or *Partitive Proportion*.

# EXAMPLES.

2. A farmer divides between his three sons 246A. 1R. 32p. of land, sharing it between them as the numbers 3, 4, and 5. What were the shares?

Ans. 61A. 2R. 18p.; 82A. 0R. 24p.; 102A. 2R. 30p.

3. Divide 319 into four parts, that shall be to each other as the numbers 41, 62, 63, and 7.

Ans.  $55\frac{33}{48}$ ;  $85\frac{75}{487}$ ;  $86\frac{23}{48}$ ;  $91\frac{34}{48}$ .

4. Standard gold for coinage consists of 9 parts of pure gold, and 1 part alloy. Allowing the alloy to be silver and copper in equal parts, how much pure gold, silver, and copper are contained in a double eagle, its weight being 1oz. 1pwt. 12gr.?

Ans. 19pwt. 82gr. gold; 1pwt. 14gr. silver; 1pwt. 14gr. copper.

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- 5. The half-dollar of the United States coinage weighs 192 grains Troy, and consists of 9 parts pure silver and 1 part of copper. How much pure silver and how much copper in 20 half-dollars?

  Ans. 7oz. 4pwt. silver; 16pwt. copper.
- 6. Divide \$ 600 between three men, so that the second man shall receive one third more than the first, and the third man shall receive two thirds more than the second.
- 7. A, B, and C freight a steamer; A puts on board 98 tons, B 86 tons, and C 64 tons. Owing to danger of being wrecked, there were thrown overboard while at sea 93 tons. What should be the number of tons lost by each?

Ans. A, 363 tons; B, 321 tons; and C, 24 tons.

8. A and B start together by railroad from Chicago for Galena; A travels by freight train at the rate of 15 miles per hour, and B by passenger train at the rate of 25 miles per hour. C leaves Galena for Chicago at the same time by express train, whose velocity is at the rate of 32 miles per hour. Allowing the distance between the two places to be 160 miles, how far from Chicago will A and B each be, when C passes them?

Ans. A,  $51\frac{3}{47}$  miles; B  $70\frac{59}{19}$  miles.

### COMPOUND PROPORTION.

340. A Compound Proportion is an expression of equality between a compound and a simple ratio. Thus,

$$\left\{\begin{array}{l}4:3\\5:7\end{array}\right\}::60:63$$
, is a compound proportion.

Compound proportion is employed in the solution of such questions as would require two or more statements in Simple Proportion.

Ex. 1. If 8 men spend \$32 in 13 weeks, what will 24 men spend in 52 weeks?

Ans. \$384.

8 men : 24 men 13 weeks : 52 week	Mean. Extreme. \$ 32 : \$ —
8 4	
<b>24</b> × 52 × 3	$\frac{2}{-}$ = 384. Ans. \$384.
8 × 13	— — 504. Ans. \$ 504.

OPERATION.

Mean.

Extreme.

In stating the question, we make \$32, which is of the same kind as the required term, the third term. Then, taking of the remaining terms two of the same

kind, 8 men and 24 men, we inquire whether the answer depending on these alone must be greater or less than the third term; and since it must be greater, because 24 men will spend more than 8 men in the same time, we make 24 men the second term, and 8 men the first term. Again, we take the two remaining terms, and make 52 weeks the second term, and 13 weeks the first, since the same number of men would spend more in 52 weeks than in 13 weeks. We then find the continued product of the second and third terms, and divide by the product of the first terms.

By Analysis. — If 8 men spend \$32 in 13 weeks, 1 man will spend  $\$\frac{32}{4} = \$4$  in 13 weeks, and 24 men will spend  $\$4 \times 24 = \$96$  in 13 weeks. If 24 men spend \$96 in 13 weeks, in 1 week they will spend  $\$\frac{9}{16}$ , and in 52 weeks,  $\$\frac{9}{16} \times 52 = \$384$ , Ans.

By RATIO. — The ratio of  $8:24 = \frac{1}{8}$ ; the ratio of  $13:52 = \frac{1}{4}$ ;  $\$32 \div \frac{1}{8} \times \frac{1}{4} = \$384$ , Ans.

Note. — To have solved the question by simple proportion, two statements would have been required, which would have produced the following proportions: —

8 men : 24 men :: \$32 : \$96. 13 weeks : 52 weeks :: \$96 : \$384.

RULE. — Make that number which is of the same kind as the answer required the third term of a proportion. Of the remaining numbers, take any two, that are of the same kind, and consider whether an answer depending upon these alone would be greater or less than the third term, and place them as directed in Simple Proportion.

Then take any other two, and consider whether an answer depending only upon them would be greater or less than the third term, and ar-

range them accordingly; and so on until all are used.

Multiply the product of the second terms by the third, and divide the result by the product of the first terms. The quotient will be the fourth term, or answer.

### EXAMPLES.

- 2. If a man can travel 117 miles in 30 days, travelling 9 hours a day, how far can he go in 20 days, travelling 12 hours a day?

  Ans. 104 miles.
- 3. If 6 men in 16 days of 9 hours each build a wall 20 feet long, 6 feet high, and 4 feet thick, in how many days of 8 hours each will 24 men build a wall 200 feet long, 8 feet high, and 6 feet thick?
- 4. If \$100 gain \$6 in one year, how much would \$500 gain in four months?

  Ans. \$10.
- 5. If \$100 gain \$6 in one year, what must be the sum to gain \$10 in 4 months?

  Ans. \$500.

- 6. How long will it take \$500 to gain \$10, if \$100 gain \$6 in one year?

  Ans. 4 months.
  - 7. If \$500 gain \$10 in 4 months, what is the rate per cent?
- 8. If 5 compositors in 16 days, 11 hours long, can compose 25 sheets of 24 pages in each sheet, 44 lines in a page, and 40 letters in a line, in how many days, 10 hours long, can 9 compositors compose a volume (to be printed on the same kind of type), consisting of 36 sheets, 16 pages to a sheet, 50 lines to a page, and 45 letters in a line?

  Ans. 12 days.
- 9. If 12 men can build a wall 30 feet long, 6 feet high, and 3 feet thick, in 15 days, when the days are 12 hours long, in what time will 60 men build a wall 300 feet long, 8 feet high, and 6 feet thick, when they work only 8 hours a day?

Ans. 120 days.

10. If 16 horses consume 84 bushels of grain in 24 days, how many bushels will suffice 32 horses 48 days?

Ans. 336 bushels.

- 11. If the carriage of 5cwt. 3qr. 150 miles cost \$24.58, what must be paid for the carriage of 7cwt. 3qr. 64 miles, at the same rate?

  Ans. \$14.135-...
- 12. If 7½oz. of bread be bought for 4¾d. when corn is 4s. 2d. per bushel, what weight of it may be bought for 1s. 2d. when the price per bushel is 5s. 6d.?
- 13. If 496 men, in 5½ days of 11 hours each, dig a trench of 7 degrees of hardness, 465 feet long, 3½ wide, 2½ deep, in how many days of 9 hours long will 24 men dig a trench of 4 degrees of hardness, 337½ feet long, 5½ wide, and 3½ deep?

Ans. 132 days.

#### CONJOINED PROPORTION.

341. A Conjoined Proportion is a proportion which has each antecedent of a compound ratio equal in value to its consequent.

The first term of each pair of equivalent terms is an antecedent, and the term following a consequent.

Note. — This kind of proportion is sometimes called the Chain Rule.

Ex. 1. If 10 barrels of apples will pay for 5 cords of wood,

and 20 cords of wood for 4 tons of hay, how many barrels of apples will it take to purchase 50 tons of hay? Ans. 500.

OPERATION.
Antecedents. Consequents.
10  barrels = 5  cords.
20  cords = 4  tons.
50  tons = -barrels.
$\frac{10 \times 20 \times 50}{} = 500 \text{ barrels, Ans.}$
$5\times4$

We arrange the antecedents at the left of the sign of equality, and the consequents at the right of the same; and, as the answer is to be of the same kind as the first term, we place the odd term in the column of

antecedents. Then, to find the term corresponding in value with the odd term, we divide the continued product of the 10, 20, and 50 of the column of antecedents, by the product of the 5 and 4 of the column of consequents; and obtain 500 barrels, Ans.

By Analysis. — If 4 tons of hay equal in value 20 cords of wood, 1 ton equals in value 5 cords of wood. If 5 cords of wood equal 10 barrels of apples, then 1 ton of hay equals 10 barrels of apples, and 50 tons equal 50 times the value of 10 barrels of apples, or 500 barrels, answer as before.

RULE. — Place the antecedents in one column and the consequents in another, on the right, with the sign of equality between them. Divide the continued product of the terms in the column containing the odd term by the continued product of the other column, and the quotient will be the answer.

NOTE 1. — The odd term will always belong to that column which does not contain the other term of the same kind.

NOTE 2. — Conjoined proportion is nothing else than simple or compound proportion, exhibited in a different, and in many cases a more convenient form; and, to show the correctness of the rule, the following examples may also be performed by either simple or compound proportion.

#### EXAMPLES.

- 2. If 100 acres in Bradford be worth 120 in Haverhill, and 50 in Haverhill be worth 65 in Methuen, how many acres in Bradford are equal to 150 in Methuen? Ans. 96% acres.
- 3. If 10lb. of cheese are equal in value to 7lb. of butter, and 11lb. of butter to 2 bushels of corn, and 11 bushels of corn to 8 bushels of rye, and 4 bushels of rye to one cord of wood, how many pounds of cheese are equal in value to 10 cords of wood?

  Ans. 4324lb.
  - 4. If 12 men can do as much work as 25 women, and 5

women do as much as 6 boys, how many men would it take to do the work of 75 boys?

Ans. 30 men.

- 5. If 6 gallons liquid measure equal 5 English imperial gallons, and if 10 English imperial gallons equal 6 velts of France, and if 26 velts of France equal 16 Russian vedros of wine, how many vedros of wine equal 63 gallons liquid measure?
- 6. If 7 bushels of corn in Boston be worth 8 bushels in Buffalo, and 10 bushels in Buffalo be worth 14 bushels in Chicago, and 21 bushels in Chicago be worth 25 bushels at Davenport, how many bushels in Boston are worth 1200 bushels at Davenport?

  Ans. 630 bushels.
- 7. If 24 shillings in Massachusetts are equal to 32 shillings in New York, and if 48 shillings in New York are equal to 45 shillings in Pennsylvania, and if 15 shillings in Pennsylvania are equal to 10 shillings in Canada, how many shillings in Canada are equal to 100 shillings in Massachusetts?

Ans. 831 shillings.

# MISCELLANEOUS EXAMPLES IN PROPORTION.

- 1. A sets out on a journey, and travels 27 miles a day; 7 days after, B sets out and travels the same road 36 miles a day; in how many days will B overtake A?

  Ans. 21 days.
- 2. If I sell coffee at 2s. 3d. per pound and gain 35 per cent, what did I give per pound?
- 3. A detachment of 2000 soldiers were supplied with bread sufficient to last them 12 weeks, allowing each man 14 ounces a day; but on examination they find 105 barrels, containing 200lb. each, wholly spoiled; how much a day may each man eat, that the remainder may supply them 12 weeks?

Ans. 12oz.

4. In consequence of having a seventh part of their bread spoiled, 2000 soldiers were put on an allowance of 12 ounces of bread per day for 12 weeks; what was the whole weight of their bread (good and bad), and how much was spoiled?

Ans. The whole weight, 147000lb.; spoiled, 21000lb.

5. Two thousand soldiers, having lost 105 barrels of bread, weighing 200lb. each, were obliged to subsist on 12 ounces a

lay for 12 weeks; but had none been lost, they might have ad 14 ounces a day for the same time. What was the whole weight, including what was lost, and how much had they left to subsist on?

Ans. The whole weight, 147000lb.; left to subsist on, 126000lb.

6. Bought threescore pieces of Hollands for three times as many dollars, and sold them again for four times as many dollars; but if they had cost me as much as I sold them for, for what should I have sold them to gain at the same rate?

Ans. \$320.

- 7. Bought 20 pounds of tea at the rate of 1½ pounds for a dollar, and 62 pounds more at the rate of 15 pounds for \$12; and sold the whole at the rate of 3 pounds for \$4. How much did I gain, or lose?
- 8. Each of the four sides of a certain field appeared to be 2 furlongs 30 rods and 3 yards in length when measured by a line supposed to be 4 rods long; but the line was found to have been only 64 feet in length. Required the true distance round the field.

  Ans. 7074+4 feet.
  - 9. If 5 oxen or 7 cows eat 3 4 tons of hay in 87 days, in what time will 2 oxen and 3 cows eat the same quantity of hay?

    Ans. 105 days.
  - 10. If 360 men be placed in a garrison, and have provisions for 6 months, how many men must be sent away at the end of 4 months that the remaining provisions may last the garrison 8 months longer?

    Ans. 270 men.
  - 11. My tailor informs me it will take  $10\frac{1}{4}$  square yards of cloth to make me a full suit of clothes. The cloth I am about to purchase is  $1\frac{7}{6}$  yards wide, and on sponging it will shrink  $2\frac{1}{6}$  in width and length. How many yards of the above cloth must I purchase for my "new suit"?

Ans.  $6_{1}$ 83 yd.

12. If the price of a farm of 130A. 2R. 20p. be \$6537.50, what will be the price of another, containing 100A. 0R. 30p., if 4 acres of the latter be worth 5 of the former?

Ans. \$6267.71+.

13. How much each of copper, tin, zinc, and lead will be required to make a bell weighing 192 tons 17cwt. 16lb., the

weight of that at Moscow, bell-metal being composed of 800 parts copper, 101 tin, 56 zinc, and 43 lead?

Ans. Copper,  $308572\frac{4}{5}$ lb.; tin,  $38957\frac{79}{250}$ lb.; zinc,  $21600\frac{12}{125}$ lb.; lead.  $165854\frac{2}{5}$ lb.

- 14. The relative heating power per cord of white-oak to white-pine being as 81 to 42, and that of white-pine to pitchpine as 7 to 8; how many cords composed of white-pine and pitch-pine in equal parts will be required to produce as much heat as 20 cords of white-oak? Ans. 36 cords or 18 of each.
- 15. If a cask of wine, containing 85 gallons, cost \$116.95, what would be the value of a hogshead containing 63 gallons, and composed of four parts of the same wine and one part of water?

  Ans. \$69.3438.
- 16. If 8 horses in 30 days consume  $3\frac{1}{2}$  tons of hay, how long will  $4\frac{9}{10}$  tons last 10 horses, 15 cows, and 7 sheep, each cow consuming  $\frac{3}{4}$  as much as a horse, and each sheep  $\frac{1}{3}$  as much as a cow?
- 17. The relative nutritious matter per pound of potatoes to beets being as 25 to 14, that of beets to carrots as 7 to 5, and that of carrots to turnips as 5 to 2; how many pounds of turnips are equal in nutritious matter to 2000 pounds of potatoes? and allowing 60 pounds to a bushel, how much cheaper or dearer relatively per bushel are potatoes at 80 cents, and turnips at 20 cents, than either beets at 35 cents or carrots at 25 cents.

Ans. 12500 pounds; potatoes 17½cts. dearer; turnips 10cts. dearer.

18. A and B set out from the same place, and in the same direction. A travels uniformly 18 miles per day, and after 9 days turns and goes back as far as B has travelled during those 9 days; he then turns again, and, pursuing his journey, overtakes B 22½ days after the time they first set out. It is required to find the rate at which B uniformly travelled.

Ans. 10 miles per day.

19. Two men in Boston hire a carriage for \$25, to go to Concord, N. H., and back, the distance being 72 miles, with the privilege of taking in three more persons. Having gone 20 miles, they take in A; at Concord, they take in B; and when within 30 miles of Boston, they take in C. How much shall each man pay?

Ans. First man, \$7.609 $\frac{1}{188}$ ; second man, \$7.609 $\frac{1}{188}$ ; A \$5.873 $\frac{1}{100}$ ; B, \$2.864 $\frac{1}{12}$ ; C, \$1.041 $\frac{1}{12}$ .

### PERCENTAGE.

**342.** Percentage is an allowance for every hundred, consisting of a part of that on which it is reckoned.

Per cent., a contraction of per centum, signifies by the hundred. By 1 per cent. is to be understood 1 out of a hundred, or 1 hundredth; by 2 per cent. is to be understood 2 out of a hundred; or 2 hundredths, &c.

- 343. The rate per cent. is the number denoting the part allowed for every hundred, as 5 per cent., 6 per cent., &c.
- 344. The bas percentage is the number on which percentage is reckoned.
- 345. The amount is the basis of percentage increased by the percentage; and the remainder is the basis decreased by the percentage.
- **346.** The rates per cent. may be written decimally as in the following

т.		_
1 A	RI	. T.

1 per cent.		.01   9	per cent.		.09
2 per cent.	"		per cent.		.10
3 per cent.	"	.03 15	per cent.	"	.15
4 per cent.	" .	.04 20	per cent.	"	.20
5 per cent.	"	.05 25	per cent.	"	.25
6 per cent.	"	.06 63	per cent.	"	.63
7 per cent.	"	.07   100	per cent.	"	1.00
8 per cent.	"	.08   116	per cent.	"	1.16

#### And

 $\frac{1}{2}$  per cent. may be written .005, or .00 $\frac{1}{2}$ , and be read  $\frac{5}{10}$  of 1 per cent., or  $\frac{1}{2}$  of 1 per cent.

 $\frac{1}{4}$  per cent. may be written .0025, or .00 $\frac{1}{4}$ , and be read  $\frac{25}{100}$  of 1 per cent., or  $\frac{1}{4}$  of 1 per cent.

 $\frac{1}{8}$  per cent. may be written .00125, or .00 $\frac{1}{8}$ , and be read  $\frac{125}{1000}$  of 1 per cent., or  $\frac{1}{8}$  of 1 per cent.

 $7\frac{1}{2}$  per cent. may be written .075, or .07 $\frac{1}{2}$ , and be read  $7\frac{5}{10}$  per cent., or  $7\frac{1}{2}$  per cent.

Note. — If a fraction of 1 per cent. cannot be exactly expressed in decimal figures, it may be written as a part of a mixed decimal. Thus,  $4\frac{1}{3}$  per cent. may be written  $.04\frac{1}{3}$ ;  $6\frac{1}{7}$  per cent. may be written  $.06\frac{1}{7}$ ; and  $11\frac{2}{3}$  per cent. may be written  $.11\frac{2}{3}$ .

# EXERCISES.

1. Express decimally 19 per cent.

Ans. .19.

- 2. Express decimally 27 per cent.
- 3. Express decimally 13½ per cent.
- 4. Express decimally 13 per cent.
- 5. Express decimally 72 per cent.
- 6. Express decimally 77<sup>1</sup>/<sub>1</sub> per cent.
- 7. Express decimally 106 per cent.
- 8. Express decimally 107 per cent.
- 9. Express decimally 305 per cent.
- 10. Express decimally 99912 per cent.
- 347. To find the percentage any given rate per cent. is of any number or quantity.
- Ex. 1. Bought ½ of a ship for \$15650, and sold the same at a gain of 12 per cent. How much did I make by the transaction?

  Ans. \$1878.

OPERATION.

Basis of percentage, \$15650 Rate per cent. .12 Since 12 per cent. equals .12 of the original cost, we multiply \$15650 by the decimal expression .12.

Percentage, \$1878.00 Ans.

RULE. — Multiply the given number by the rate per cent. expressed decimally, and the product will be the percentage. Or,

As 100 per cent. is to the given rate per cent., so is the given basis of percentage to the percentage required.

# EXAMPLES.

2. What is 15 per cent. of 500 bushels? Ans. 75bu.

3. What is 20 per cent. of 75cwt.? Ans. 15cwt.

4. What is 30 per cent. of 150 tons? Ans. 45 tons.

5. What is 75 per cent. of \$500?

6. What is 95 per cent. of 700 chaldrons?

7. What is 2 per cent. of 40 miles? Ans. .8 mile.

8. What is 99 per cent. of \$1000? Ans. \$990.

9. What is 331 per cent. of 144 barrels? Ans. 48bbl.

10. What is 66% per cent. of 90 hogsheads?

11. What is \(\frac{1}{4}\) per cent. of \$100? Ans. \$0.25.

12. What is 7 per cent. of 1728lb.? Ans. 15.12lb.

13. A certain colonel, whose regiment consisted of 900 men,

lost 8 per cent. of them in battle, and 50 per cent. of the remainder by sickness. How many had he remaining?

Ans. 414 men.

- 14. A merchant, having \$1728 in the Union Bank, wishes to withdraw 15 per cent.; how much will remain?
- 15. A gentleman, who had an estate of \$25,000, in his will gave to his wife 40 per cent. of his property, and to his son Samuel 30 per cent. of the remainder. The residue he divided equally among his daughters, Marcia, Isabella, and Clara, after having deducted \$60 as a present to his clergyman. What did each receive?

Ans. Wife, \$10,000; son, \$4,500; daughters, \$3,480 each.

348. To find what rate per cent. one given number is of another.

Ex. 1. What per cent. of 50 is 12? Ans. 24 per cent.

$$\frac{\frac{12}{60} = \frac{6}{25} = .24, \text{ Ans.}}{2}$$
Or, 
$$\frac{12 \times \cancel{100}}{\cancel{500}} = 24 \text{ per cent.}$$

Since the percentage equals the product of the basis of percentage by the number denoting the rate per cent. (Art. 343), the quotient arising from dividing the percentage by the number

denoting the basis must equal the rate per cent. We therefore divide 12 by the 50, and obtain  $\frac{12}{50} = \frac{6}{25}$ , which, expressed decimally, equals .24, or 24 per cent. Since the question, evidently, is the same as to find  $\frac{12}{50}$  of 100 per cent., we multiply the 12 by 100, or annex two ciphers and divide by 50, and obtain the same result as before.

Rule. — Annex two ciphers to the number denoting the percentage, and divide by the number on which the percentage is reckoned; and the quotient will be the rate per cent. Or,

As the given basis of percentage is to the given percentage, so is 100

per cent. to the rate per cent. required.

# EXAMPLES.

- 2. What per cent. of 16 is 2? Ans.  $12\frac{1}{2}$  per cent.
- 3. What per cent. of 110 is 11? Ans. 10 per cent.
- 4. What per cent. of 21 is 1?
- 5. \( \frac{1}{3}\) of 18 per cent. is what per cent. of 24 per cent.?

  Ans. 25 per cent.
- 6. What per cent. of \$150 is 25 per cent. of \$36?

  N 22

  Ans. 6 per cent.

7. 36 bushels is what per cent. of 48 bushels?

Ans. 75 per cent.

- 8. What per cent. of 4 years is 1 year 6 months?
- 9. 31 gallons 2 quarts is what per cent. of 1 hogshead?

Ans. 50 per cent.

10. Of 160 yards of cloth there have been sold 128 yards; what per cent. of the whole remains unsold?

Ans. 20 per cent.

- 11. What per cent. of ½ of ½ of ¾ is ½? Ans. 20 per cent.
- 12. In a certain school the number of pupils studying geography is 40 per cent. more than the number studying grammar. What per cent. less is the number studying grammar than the number studying geography? Ans. 28\$ per cent.
- 13. If a miller takes out 4 quarts toll from every bushel he grinds, what per cent. does he take for toll?
- 14. If a certain coin is made of 22 parts copper and 3 parts nickel, what per cent. of it is copper, and what per cent. nickel?

  Ans. 88 per cent. copper; 12 per cent. nickel.
- 15. In a certain orchard there are 250 trees, of which 40 per cent. are apple-trees, 12 per cent. cherry-trees, 8 per cent. plum-trees, and the remainder, with the exception of 25 pear-trees, consist of peach-trees. What per cent. of the whole are the peach-trees?

  Ans. 30 per cent.
- 349. To find a number when a given number is known to be a certain per cent. of it.
- Ex. 1. I have bought two house-lots; for the one I paid \$300, which was 60 per cent. of what I paid for the other. What did I pay for the latter?

  Ans. \$500.

\$300 ÷ 60 = \$5; \$5 × 100 = \$500, Ans. is 60 per cent. of the unknown sum, 1 per cent. of it must equal 
$$\frac{1}{300}$$
 of \$300, or

\$5; and 100 per cent., or the whole of it, must equal 100 times \$5, or \$500, Ans. And since the question is evidently the same as to find the value of  $\frac{100}{60}$  of \$300, we multiply the \$300 by 100, and divide by 60, and obtain the same result as before.

RULE. — Annex two ciphers to the number denoting the percentage, and divide by that denoting the rate per cent. Or,

As the given rate per cent. is to 100 per cent., so is the given percentage to the basis of percentage required.

### EXAMPLES.

2.	25 is 10 per	cent. of what r	umber?	Ans. 250.
3.	161 is 8 per	cent. of what r	number?	Ans. 2031.

4. 72 is 12 per cent. of what number?

5.  $\frac{7}{8}$  is 40 per cent. of what number? Ans.  $2\frac{3}{16}$ .

- 6. \$1.62½ is 12½ per cent. of how many dollars? Ans. \$13.
- 7. \$  $1_{z_0}^9$  is 7 per cent. of what sum? Ans. \$ 140.
- 8. A flock of sheep has lost 15½ per cent. of its number, 17 sheep having been killed by the dogs, and 6 more having been drowned. What was the original number? Ans. 150.
- 9. If a man owning 45 per cent. of a mill should sell 331 per cent. of his share for \$450, what would be the value of the whole mill?
- 10. 12½ per cent. of the length of a certain railroad is equal to 3m. 1 fur. 1rd. What is its entire length?

Ans. 25m. 0fur. 8rd.

- 11. Gave to a benevolent society 19 bushels of corn, which was 17‡ per cent. of all I raised. How many bushels had I left?

  Ans. 91½ bushels.
- 12. Dalton says to Turner, \$36.89 is 13\(^2\) per cent. of the sum you borrowed of me; and Turner replies, It is just 16\(^2\) per cent. of the amount I have repaid you. How much of the money that was borrowed remains unpaid?

  Ans. \$57.66.
- 350. To find the number on which the percentage is reckoned, when the amount, or the remainder, and the rate per cent. are given.
- Ex. 1. Sold a horse for \$ 200, which was 25 per cent. more than he cost. What did he cost?

  Ans. \$ 160.

OPERATION.

Since the \$1+.25 = 1.25; \$200 \(\div \) 1.25 = \$160, Ans. \$200 is evidently the cost and 25 per cent. of the cost, it must equal the cost taken 1 25 times. Therefore, the given amount, \$200, divided by 1 increased by .25, or the given per cent. expressed decimally, equals \$160, the

basis of percentage required.

2. Sold 160 cords of wood, which was 20 per cent. less than the whole number owned. How many cords were owned?

Ans. 200 cords.

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1 - .20 = .80;  $160 \div .80 = 200$  cords, Ans. Since the 160 cords are 20 per cent.

less than the whole number owned, it must equal the whole number of cords taken .80 times. Therefore the given remainder, 160 cords, divided by 1 decreased by .20, or the given per cent. expressed decimally, equals 200 cords, the basis of percentage required.

RULE. — Divide the given amount by 1 increased by the given per cent., or the given remainder by 1 decreased by the given per cent., expressed decimally. Or,

As 1 increased by the given per cent., or 1 decreased by the given per cent., expressed decimally, is to 1, so is the given amount or the given re-

mainder to the basis of percentage required.

# EXAMPLES.

- 3. 126 is 5 per cent. more than what number? Ans. 120.
- 4. 328½ is 10 per cent. less than what number?

Ans. 365.

- 5. \$19.50 is 7 per cent. less than how many dollars?
- 6. \$34.40 is § per cent. more than how many dollars?

Ans. \$  $34.27\frac{119}{803}$ .

- 7. A man expends in a week \$24, which exceeds by 33½ per cent. his earnings in the same time. What were the earnings?

  Ans. \$18.
- 8. D. Chandler lives distant from the village 2m. 6fur. 24rd., which is 12½ per cent. nearer than is J. Mitchel's residence. At what distance does Mitchel live from the village?

Ans. 3m. 1fur. 334rd.

- 9. Bought a carriage for \$123.16, which was 16 per cent. less than I paid for a horse. How much was paid for the horse?
- 10. A carpet having been cut from a piece of carpeting, there were left 6yd.  $1_{1}$  orr., which was 75 per cent. less than the quantity cut off. How many yards were there in the piece at first?

  Ans. 31yd. 1qr. 2na.
- 11. A steamer with a cargo of flour, having been lightened, during a storm, of 10 per cent. of her freight, at the end of the

voyage had but 279 barrels to deliver. How many barrels were taken aboard, and how many were lost?

Ans. Taken 310 barrels; lost 31 barrels.

12. A and B each received the same sum. A spent 86½ per cent. of his money for land, and B lost of it by gambling as much as uld equal 27½ per cent. of what both received. They then together had left just \$36.85½. What was the sum received by each, and how much had each left?

Ans. Each received \$63; A had left \$8.50½; and B had left \$28.35.

### MISCELLANEOUS EXAMPLES.

1.	What is 7 per cent. of 1672?	Ans. $117_{\frac{1}{25}}$ .
2.	What is 5 per cent. of \$ 3266?	Ans. $174\frac{1}{5}$ .
3.	Find 312 per cent. of $\frac{1}{2}$ of $2\frac{1}{4}$ .	
4.	Find 1½ per cent. of 180.	Ans. $2\frac{7}{10}$ .
5.	9½ is what per cent. of 3?	Ans. $316\frac{2}{3}$ .
6.	$\frac{1}{8}$ is what per cent. of $\frac{5}{16}$ ?	Ans. 40.
7.	Find what per cent. 3.50 is of 50.	
8.	13 is 16 per cent. of what number?	Ans. 811.
9.	\$ 66 is 100 per cent. of what number?	Ans. \$ 66.
10.	½ is 6½ per cent. of what number?	Ans. 533.
11.	\$21.28 $\frac{1}{2}$ is $3\frac{1}{4}$ per cent. less than what	sum?
		Ans. \$ 22.00.
12.	19lb. 12oz. is $16\frac{3}{4}$ per cent. of how man	y pounds?

- Ans. 1176 pounds.

  13. A grocer bought 6 boxes of eggs, each containing 30
- dozen, and found that 15 per cent. of the whole were bad; how many eggs did he lose?

  Ans. 324 eggs.
- 14. There is paid for sawing a cord of wood \$0.69, which is 12 per cent. of the cost of the wood. What did the wood cost?
- `15. Three men agreed to excavate 40500 cubic feet of earth; by the first week's labor they excavate 200 cubic yards; by the second, 6000 cubic feet; and by the third, 25 per cent. of what remained at the end of the second week. They then called the work half done; but how many cubic feet did the job lack of being half completed? Ans. 1575 cubic feet.

16. 25 per cent. of ½ of a ship is how many per cent. of ½ of it?

Ans. 16% per cent.

17. If molasses cost 20 per cent. less than \$0.50 per gallon, and it be sold at 25 per cent. more per gallon than it cost, at what price is it sold?

Ans. \$0.50 per gal.

18. I have 20 yards of yard-wide cloth, which which which sponging 4 per cent. in the length, and 5 per cent. in the width; how much less than 20 square yards will there be of it after sponging?

Ans. 1½ yards.

19. A gentleman having a large farm gave 15 per cent. of it to his oldest daughter, 10 per cent. of what remained and  $\frac{1}{10}$  of an acre he gave to his oldest son, and 25 per cent. of the remainder he gave to his wife. The residue he divided equally among his other 5 children, who received each 39 acres. How many acres did his farm contain?

Ans. 340 acres.

20. If the population of the United States in 1858 be 30,500,000, what will it be in 1868, allowing the increase

should be at the rate of 341 per cent.?

21. In a certain battle in which the English, French, and Turks were allied against the Russians, there were 33½ per cent. more French than English, and the Turks were 8½ per cent. more than the French and 1600 more than the English. Required the whole number of the allies, and the per cent. the English, the French and the Turks each were of that number.

Ans. Whole number 13600; English, 2678, per cent.; French,

 $35_{17}^{5}$  per cent.; and Turks,  $38_{17}^{4}$  per cent.

22. Bought a cargo of flour, consisting of 560 barrels, at \$7.25 per barrel, less 10 per cent., and sold the same at 10 per cent. more than \$7.25 per barrel. At what per cent. above the cost was the flour sold? How much was made by the operation?

23. The population of a certain city, whose gain of inhabitants in 5 years has been 25 per cent., is 87500; what was it 5 years ago?

Ans. 70000.

24. Bought a horse, buggy, and harness for \$500. The horse cost 37½ per cent. less than the buggy, and the harness cost 70 per cent. less than the horse. What was the price of each?

Ans. Buggy \$  $275\frac{2}{5}$ ; horse, \$  $172\frac{1}{2}$ ; and harness, \$  $51\frac{2}{5}$ .

#### INTEREST.

351. Interest is the compensation which the borrower of money makes to the lender; and it is generally reckoned as a certain rate per cent for any given time, but usually for one year.

The *principal* is the sum lent, on which interest is computed. The *amount* is the interest and principal added together.

Simple interest is that reckoned on the principal only; and is that meant when the term interest is used alone.

Legal interest is the rate per cent. established by law.

Usury is a higher rate per cent. than is allowed by law.

The legal rate per cent. varies in the different States and in different countries.

In Maine, New Hampshire, Vermont, Massachusetts, Rhode Island, Connecticut, New Jersey, Pennsylvania, Delaware, Maryland, Virginia, North Carolina, Tennessee, Kentucky, Ohio, Indiana, Illinois, Iowa, Nebraska, Missouri, Kansas, Arkansas, Mississippi, Florida, District of Columbia, and on debts or judgments in favor of the United States, it is 6 per cent.

In New York, Michigan, Wisconsin, Minnesota, Georgia, and South Carolina, it is 7 per cent.

In Alabama and Texas, it is 8 per cent.

In California, it is 10 per cent.

In Louisiana, it is 5 per cent.

In Canada, Nova Scotia, and Ireland, it is 6 per cent.

In England and France, it is 5 per cent.

Note. — The legal rate, as above, in some of the States, is only that which the law allows, when no particular rate is mentioned. By special agreement between parties, in Ohio, Indiana, Michigan, Illinois, Iowa, Nebraska, Missouri, Kansas, Arkansas, Louisiana, and Mississippi, interest can be taken as high as 10 per cent.; in Florida, as high as 8 per cent.; in Texas and Wisconsin, as high as 12 per cent.; and in California and Minnesota, any per cent. In New Jersey, by a special law, 7 per cent may be taken in the city of Paterson, and in the counties of Essex and Hudson. In Vermont 7 per cent. may be taken on railroad bonds. Banks in Illinois cannot take above 7 per cent., and in Ohio, not above 6 per cent. In Mississippi, above 6 per cent. can be taken only for money lent.

352. When the rate of interest is 6 per cent. per annum, the interest for 1 year or 12 months will be 6 cents on every

100 cents on which it is reckoned, or  $76\pi$  of the principal Hence, for 2 months or  $\frac{1}{6}$  of a year, it will be 1 cent on every 100 cents, or  $76\pi$  of the principal; and for 1 month or  $\frac{1}{2}$  of 2 months it will be 5 mills, or  $\frac{1}{2}6\pi$  of the principal. Now, since the interest on 100 cents for 1 month, or 30 days, is 5 mills, for 6 days, or  $\frac{1}{6}$  of 30 days, it will be 1 mill, or  $\frac{1}{100\pi}$  of the principal; and since the interest on 100 cents for 2 months is 1 cent, or  $\frac{1}{100}$  of the principal, for 100 times 2 months, or 200 months, or 16 years 8 months, it will be 100 cents, or equal the whole principal; and in the same ratio for any other length of time.

The interest of \$1, at 6 per cent., and the ratio of the interest to the principal, for 200 months, and other convenient parts of time, is shown in the following

# TABLE.

# Interest of \$1

For	200 mo	16yr. 8mo.	is	\$ 1.00	eau	al the	e whole	principal.
"	100 mo. ==	8yr. 4mo.	"	0.50	•"	1	of the	principal
*	66§mo. ==	5yr. 62mo.	"	0.3331	46	1/8	66	• " ·
ĸ	50 mo. ==	4yr. 2mo.	"	0.25	"	ı	66	u
66	40 mo. =	8yr. 4mo.	"	0 20	"	ì	"	u
46	33½mo. ==	2yr. 91mo.	"	0.166	66	ı	66	"
"	25 mo. ==		"	0.125	"	ì	"	"
"	20 mo. ==	1yr. 8mo.	"	0.10	"	i 10	"	*
"	16 <del>2</del> mo. 🚃	1yr. 42mo.	"	0.0831	"	12	46	"
"	12 mo. ==		"	0.06	"	100	"	"
"	10 mo. 🕳	∮of a yr.	"	0.05	"	20	"	4
66	8 mo. =	a of a yr.	"	0.04	46	20	"	44
"	6 <del>3</del> mo. ==		"	0.0331	"	30 80	"	"
"	6 mo. =		"	0.03	"	180	66	u
"	5 mo. =		"	0.025	46	40	"	"
66	4 mo. =	8 7	"	0.02	"	50	"	."
"	2 mo. ==		"	0.01	"	100	"	u
66	1 mo. ==	$\frac{1}{12}$ of a yr.	"	0.005	"	200	66	44
"	6 d. =	of a mo.		0.001	"	1000	"	"
"	5 d. =	$\frac{1}{8}$ of a mo.		0.000		6000	46	. "
"	4 d. =	$\frac{2}{15}$ of a mo.		0.000		6000	"	"
"		$\frac{1}{10}$ of a mo.		$0.000\frac{1}{2}$		8000	"	"
"	2 d. =	$\frac{1}{15}$ of a mo.		$0.000\frac{7}{8}$		6000	"	u
"	1 d. ==	$\frac{1}{80}$ of a mo.	"	0.000 g	"	6000	"	«



- 353. The principal, the rate per cent., the time, and the interest have such a relation to each other that, any three of these terms being given, the fourth can be readily found. The computations in interest, therefore, admit of the following problems among others:—I. To find the interest; II. To find the principal; III. To find the rate per cent.; IV. To find the time.
- 354. To find the interest of any sum for any time at 6 per cent.
- Ex. 1. What is the interest of \$2640 for 2 years 7 months and 26 days, at 6 per cent.?

  Ans. \$420.64.

Interest of \$ 1 for 2 years = \$ 0.1 2 Interest of \$ 1 for 7 months = 0.0 3 5 Interest of \$ 1 for 26 days =  $0.004\frac{1}{3}$ Int. of \$ 1 for 2yr. 7mo. 26d. = \$  $0.159\frac{1}{3}$ 

Principal, \$ 2 6 4 0

159 ± 23760 13200 2640 880 Interest. \$420640 Ans.

The interest of \$ 2640 for 2yr. 26d. be 2640 times as much as the interest of \$1 for the given time. The interest of \$1 for 2 years will be twice as much as for 1 year, equal 12cts.; and since the interest for 2 months is 1 cent, for 7 months it will be 3½ cents, or

cents 5 mills. And as the interest for 6 days is 1 mill, for 26 days it will be 4\frac{1}{2} mills. These several sums added together give the interest of \$ 1 at 6 per cent. for the given time, equal \$ 0.159\frac{1}{2}, which taken 2640 times, by multiplying the given principal by it, gives \$ 420.64, the interest required.

Principal,  $\$ \frac{2640}{330}$   $\frac{1}{30}$  of the prin. =  $\frac{1}{30}$  of the prin. = 88 Interest for 6mo. 20d.  $\frac{1}{1000}$  of the prin. = 2.64 Interest for 6d.

Ans. \$420.64 Interest for 2yr. 7mo. 26d.

The time, 2yr. 7mo. 26d., is equal to 2yr. 1mo. + 6mo. 20d. + 6d. Now, since the interest on any sum, at 6 per cent., in 200 months equals the principal, for 2yr. 1mo., or \$\frac{1}{8}\$ of 200 months, it will equal \$\frac{1}{8}\$ of the principal. We therefore take \$\frac{1}{8}\$ of the principal, \$\frac{3}{8}\$ 2640, equal \$\frac{3}{8}\$ 330, as the interest for 2yr. 1mo. Of the balance of

time, 6mo. 20d., or 6\frac{2}{3}mo., being  $\frac{1}{36}$  of 200 months, we take  $\frac{1}{30}$  of the principal, equal \$88, as the interest for the 6mo. 20d.; and the 6d. being  $\frac{1}{1000}$  of 200 months, we take  $\frac{1}{1000}$  of the principal, equal \$2.64, as the interest for the 6d. We add together the interest for the parts of the whole time, and obtain, as by the first operation, \$420.64 as the whole interest.

Rule 1. — Find the interest of \$1 for the given time, by reckoning 6 cents for every YEAR, 1 cent for every TWO MONTHS, and 1 mill for every 6 DAYS; then multiply the given principal by the number denoting that interest, and the product will be the interest required. Or,

RULE 2. — Take such fractional part or parts of the principal as the number expressing the time is of 200 months.

NOTE 1. — To find the amount, add the principal to the interest.

Note 2. — In computing interest for a fractional part of a month, the month is considered as consisting of 30 days. This has the sanction of general usage and the decisions of the courts, though not entirely accurate.

NOTE 3. — Questions in interest, like other exercises in percentage, may be solved by proportion. The foregoing example admits of a statement and solution by the rule of compound proportion (Art. 340).

Note 4. — It is customary among merchants to reject the mills in the results of their computations of interest, increasing, however, the number of cents by 1 when the decimal of a cent exceeds 5.

### EXAMPLES.

- 2. What is the interest of \$675 for 1 year? Ans. \$40.50.
- 3. What is the interest of \$3967.87 for 2 years?

Ans. \$ 476.144.

4. What is the interest of \$896.28 for 3 years?

Ans \$161.33.

- 5. What is the amount of \$716.57 for 4 years?
- 6. What is the amount of \$76.47 for 7 years?

Ans. \$ 108.587.

7. What is the interest of \$123.45 for 6 years?

Ans. \$ 44.442.

8. What is the interest of \$750 for 12 years?

Ans. \$540.

- 9. What is the interest of \$130 for 2 months?
- 10. What is the interest of \$85 for 3 months?

Ans. \$ 1.275.

11. What is the interest of \$19.62 for 7 months?

Ans. \$ 0.6867.

12. What is the interest of \$637 for 10 months?

- 13. What is the interest of \$1671.32 for 14 months?

  Ans. \$116.99.
- 14. What is the interest of \$891.24 for 9 months?

Ans. \$40.10.

- 15. What is the interest of \$91 for 5 days? Ans. \$0.0758.
- 16. What is the interest of \$324.66 for 18 days?
- 17. What is the interest of \$3246 for 27 days?.

Ans. \$14.607.

- 18. What is the interest \$1364.24 for 1 day?
- 19. What is the interest of \$ 6444 for 29 days?
- 20. What is the amount of \$18.60 for 24 days?
- 21. What is the interest of \$386.19 for 100 months?

  Ans. \$193.09.
- 22. What is the interest of \$0.75 for 75 years?

Ans. \$3.373.

- 23. What is the interest of \$396.15 for 1 year 1 month and 9 days?

  Ans. \$26.343.
- 24. What is the interest of \$36.18 for 3 months and 7 days?

  Ans. \$0.584.
- 25. What is the interest of \$97.15 for 2 years 11 months and 27 days?
- 26. What is the interest of \$76.89\frac{1}{2} from January 11, 1852, to July 27, 1863?

  Ans. \$53.262.
- 27. What is the interest of \$98.25 from July 4, 1856, to October 19, 1859?

  Ans. \$19.404.
- 28. What is the interest of \$22.763 from February 19, 1836, to July 18, 1860?
- 29. What is the interest of \$175.07 from January 7, 1855, to October 12, 1859?

  Ans. \$50.04.
- 30. What is the interest of \$197.28½ from December 6, 1852, to January 11, 1854?

  Ans. \$12.987.
  - 31. What is the amount of \$4377.15 for 3 years?
- 32. What is the interest of \$444.60 for 5 years and 6 months?

  Ans. \$146.718.
- 355. To find the interest of any sum of money at any rate per cent. for any given time.
- Ex. 1. What is the interest of \$84.50 at 7 per cent. for 2 years 5 months and 12 days?

  Ans. \$14.49.

FIRST OPERAT	ON.						
Principal,	\$8	4	.5	0			
Rate per cent.	_	_ •	0.	7			
Interest for 1 year,	5.9	1	5	0 2			
	1.8	3	0	0			
Int. for 4mo, or $\frac{1}{4}$ of 1yr.	1,9	7	1	6	+		
	.4	9	2	9	+		
Int. for 10d., or 1 of 1mo.	.1	6	4	3	+		
Int. for 2d., or $\frac{1}{6}$ of 10d.	.0	3	2	8	+		
Int. for 2yr. 5mo. 12d. \$1	4.4	9	1	6	+	A	ns

Having found the interest for 1 year. and then for 2 years, the interest for 5 months is obtained by first taking 1 of year's interest. for 4 months, and then 1 of this last interest for 1 month. And since 10 days are 1 of 1 month, we take 1 of 1 month's interest for the interest of 10

days; and since 2 days are 1 of 10 days, we take 1 of the last interest for 2 days. The interest as found for the several parts of the whole time, added together, gives the interest required.

SECOND OPER	AT	101	۲.				
Principal,		\$	8	4.	.5	0	
Int. of \$1 at 6 per cent.				.1	4	7	
		$\overline{5}$	9	1	5	0	
	3	3	8	0	0		
•	8	4	5	0			
Int. at 6 per cent. 1	2	.4	2	1	5	0	
	2	0.	7	0	2	5	
Int. at 7 per cent. \$1	4	4	9	1	7	5	Ans

We first find the interest on the given sum at 6 per cent, and then add to this interest the fractional part of itself, denoted by the excess of the rate above 6 per cent. This excess is 1 per cent.; therefore we add 1 of the interest at 6 per cent. to that interest for the answer. If the rate per cent. had been less than 6 per cent, we should

RULE 1. — First find the interest for one year by multiplying the principal by the rate per cent. expressed decimally; and for two or more years multiply this product by the number of years.

Find the interest for months by taking the most convenient fractional

part or parts of ONE year's interest.

have subtracted the fractional part.

Find the interest for days by taking the most convenient fractional part or parts of QNE month's interest.

RULE 2. - Find the interest of the given sum at 6 per cent., and then add to this interest, or subtract from it, such a fractional part of itself as the given rate is greater or less than 6 per cent.

Note 1.  $-\frac{1}{6}$  of the interest at 6 per cent. may be taken for that at 1 per cent.; 1, for that at 11 per cent.; 1, for that at 2 per cent.; 1, for that at 3 per cent. From the interest at 6 per cent. may be taken & of itself for that at 4 per cent.; 1 of itself for that at 41 per cent.; and 1 of itself for that at 5 per cent. To the interest at 6 per cent. may be added I of itself for that at 7 per cent.; Fof itself for that at 7½ per cent.; Fof itself for that at 8 per cent; and for itself for that at 9 per cent. If the rate is 12 per cent., the interest at 6 per cent. may be taken twice; if 18 per cent., the interest at 6 per cent. may be taken three times, etc.

NOTE 2. — When in this book the rate of interest is not given, 6 per cent. is to be understood...

#### EXAMPLES.

- 2. What is the interest of \$16.75 for 7 months and 17 days, at 7 per cent.?

  Ans. \$0.739.
- 3. What is the interest of \$11.10½ from April 17, 1852, to December 7, 1852, at 7 per cent.?

  Ans. \$0.496.
- 4. What is the interest of \$12.69, from January 2, 1853, to August 30, 1854, at 7 per cent.?

  Ans. \$1.47\frac{1}{2}.
- 5. What is the interest of \$5000 for 2 years 5 months 26 days, at 7 per cent.?
- 6. What is the amount of \$416 for 3 years 16 days, at 7 per cent.?

  Ans. \$504.64.
- 7. What is the interest of \$336 for 15 days, at 5 per cent.?

  Ans. \$0.70.
- 8. What is the interest of \$17869.75 from February 7, 1852, to January 11, 1860, at 5 per cent.?
  - Ans. \$7083.3703.
- 9. What is the interest of \$300.50 for 1 year 2 months and 15 days?

  Ans. \$21.786.
  - 10. What the interest of \$37 for 29 days, at 5 per cent.?
- 11. What is the interest of \$35.61 from November 11, 1861, to December 15, 1863?

  Ans. \$4.474.
- 12. What is the interest of \$ 16.76 from December 17, 1841, to January 17, 1852?

  Ans. \$ 10.139+
- 13. What is the interest of \$1728.19 from May 7, 1854, to July 17, 1860, at \(\frac{1}{4}\) per cent.? Ans. \$26.762.
- 14. What is the interest of \$397.16 for 1 year 6 months and 1 day, at 5½ per cent.?

  Ans. \$32.826.
- 15. What is the amount of \$100.25 for 2 months and 29 days, at 4 per cent.?

  Ans. \$101.241.
- 16. What is the interest of \$51.17 for 9 months and 29 days, at 4 per cent.?

  Ans. \$1.699.
- 17. What is the interest of \$42.20 for 1 year and 16 days, at 41 per cent.?

N 23

- 18. What is the interest of \$16.25 for 2 years, at 3 per cent.?

  Ans. \$0.975.
- 19. What is the interest of \$96.84 from November 27, 1849, to July 3, 1852, at 7½ per cent.? Ans. \$18.883.
- 20. What is the interest of \$786.97 from October 19, 1857, to August 17, 1861, at 74 per cent.?
- 21. What is the interest of \$71.091 from July 29, 1853, to June 19, 1857, at 12 per cent.?

  Ans. \$33.175.
- 22. What is the amount of \$369.29 for 2 years 3 months and 1 day, at 9 per cent.?

  Ans. \$444.163.
- 23. What is the interest of \$76.35 for 1 year 8 months and 18 days?

  Ans. \$7.864.
- 24. What is the interest of \$47.15 for 1 month and 19 days, at 13½ per cent.?

  Ans. \$0.886.
- 25. What is the interest of \$36.72 from May 16, 1829, to February 18, 1857, at 7 per cent.? Ans. \$71.342.
- 26. What is the interest of \$35.50 for 3 years 5 months and 20 days, at 7 per cent.?

  Ans. \$8.628.
- 27. What is the amount of \$496.30 for 6 months and 20 days, at 7 per cent.?

  Ans. \$515.60.
- 28. What is the interest of \$691.04 for 1 month 3 days, at 5 per cent.?

  Ans. \$3.167.
- 29. What is the interest of \$9750 for 4 months, at 2 per cent. a month?

  Ans. \$780.
- 30. What is the interest of \$9162 for 3 months, at 1½ per cent. a month?

  Ans. \$412.29.
- 31. What is the interest of \$1500 for 7 months 20 days, at 10 per cent.?

  Ans. \$95.833.
- 32. What is the interest of \$640.50 for 10 months and 26 days, at 10 per cent.?

  Ans. \$58.00.
- 33. What is the interest of \$3178 for 15 months and 15 days?
- 34. If a banker borrow \$10,000 in Boston at 6 per cent., and let the same in Wisconsin at 7 per cent., how much does he make by the operation in that way in 2 years and 63 months?

  Ans. \$255.555.
- 356. To reckon interest for any number of days, when 12 months of only 30 days each, or 360 days, are considered a year.

# Ex. 1. What is the interest of \$460 for 93 days?

Ans. \$7.13.

Principal, \$460  $\frac{1}{6}$  of 93d. = 15 $\frac{1}{2}$ d.;  $\frac{0.015\frac{1}{2}}{2300}$  $\frac{230}{7.130}$  Ans. We multiply the principal by  $\frac{1}{6}$  of the number of days, considered as thousandths; since the interest of \$1 at 6 per cent. for 6 days is 1 mill or 1 thousandth of a dollar, (Art. 352,) and for any other number of days, at the same per cent., must be

one sixth as many mills or thousandths of a dollar as there are days

BECOND OF	ERATION.	
Principal,	\$460	٠
100 of the principal,	4.60	Interest for 60d.
$\frac{1}{2}$ of the interest for 60d.	2.30	Interest for 30d.
10 of the interest for 30d.	.2 3	Interest for 3d.
Ang	\$7.13	Interest for 93d

93 days = 60 days + 30 days + 3 days. Now, since the interest of any sum at 6 per cent. for 2 months, or 60 days, equals  $\frac{1}{100}$  of the principal (Art. 352),  $\frac{1}{100}$  of \$460 = \$4.60 will be the interest for 60 days;  $\frac{1}{2}$  of the interest for 60 days, or \$2.30, will be that for 30 days; and  $\frac{1}{10}$  of the interest for 30 days, or \$0.23, will be that for 3 days. The interest for the several parts of the time added together must give the interest for the whole time, or 93 days.

Rule. — Multiply the principal by the number denoting one sixth of the number of days expressed decimally as thousandths. Or,

Divide the principal by 100, and take such a part or parts of the quo-

tient as the given number of days is of 60 days.

The result will be the interest at 6 per cent., from which it may be found for any other rate, as in Art. 355.

Note 1. — One sixth of the number of days in any number of months of 30 days each is equal to 5 times the number of months. Thus, one sixth of the number of days in 7 months equals  $7 \times 5$ , or 35 days; and one sixth of the number of days in 9 months and 13 days equals  $(9 \times 5) + (13 \div 6)$ , or  $45 + 2\frac{1}{6} = 47\frac{1}{6}$  days.

Note 2. — It is a common practice among mercantile men, in calculating interest, to consider a year to consist of only 360 days, but the laws of some of the States require 365 days to a year. 360 days are in fact only  $\frac{265}{365} = \frac{72}{13}$  of a common year, and therefore are  $\frac{1}{13}$  less than what perfect accuracy would require. Hence, when the year is considered to be one of 365 days, the interest a found by the rule must be diminished by  $\frac{1}{13}$  of itself; Or, we may multiply the principal by the number of days, and, if the rate be 6 PER CENT., divide by 6083\frac{1}{2}; if the rate be 7 PER CENT., divide by 5214; and if the rate be 5 PER CENT., divide by 7300.

#### EXAMPLES.

- 2. What is the interest of \$96 for 33 days, at 7 per cent.?

  Ans. \$.616.
- 3. What is the interest of \$320.40 for 63 days, at 5 per cent.?

  Ans. \$2.803.
  - 4. What is the interest of \$131.20 for 123 days?

Ans. \$ 2.689.

- 5. What is the interest of \$26.60 for 78 days?
- 6. What is the interest of \$5780 for 153 days, at 10 per cent.?

  Ans. \$245.65.
- 7. What is the interest of \$105.10 for 48 days, at 12 per cent.?

  Ans. \$1.681.
- 8. What is the interest of \$13.62 for 93 days, at 7½ per cent.?
- 9. What is the interest of \$4580 for 253 days, at 5½ per cent.?

  Ans. \$177.029½8.
- 10. What is the interest of \$3140 for 273 days, at 7 per cent.?

  Ans. \$166.6813.
- 11. What is the interest of \$10550 for 243 days, at 8 per cent.?

  Ans. \$569.70.
  - 12. What is the amount of \$33.44 for 333 days?

Ans. \$35.295.

- 13. What is the amount of \$71.60 for 3 months and 18 days?
- 14. What is the interest of \$92.96 for 4 months and 3 days, at 7 per cent.?

  Ans. \$2.223.
- 15. What is the interest of \$144.50 for 144 days, allowing 365 days to the year, at 5 per cent.?

  Ans. \$2.85.
- 16. What is the interest of \$761.81 for 165 days, allowing 365 days to the year?

  Ans. \$20.662.
- 17. What is the interest of \$560 for 183 days, allowing 365 days to a year, at 7 per cent.?

  Ans. \$19.65.
  - 18. What is the interest of \$1960 for 93 days?
- 19. What is the amount of \$1000 for 1 month and 3 days?

  Ans. \$1005.50.
- 20. What is the amount of \$7300 for 18 days, at 6 per cent.?

  Ans. \$7321.90.

- 357. To find the interest on sterling money, at any rate per cent., for any time.
- Ex. 1. What is the interest of  $576\pounds$ . 5s. 6d. for 1 year and 10 months? Ans.  $63\pounds$ . 7s. 9d.

 $576\pounds$ . 5s.  $8\frac{1}{2}d$ . =  $\frac{576.255\pounds}{57.6255}$ . Principal. Int. for 1yr. 8mo. 5.76255 Int. for 2mo.

Ans. 63.38805£. =

63£. 7s. 9d.+ Int. for 1yr.10mo.

We reduce the 5s. 6d. to the decimal of a pound (Art. 279), and annex it to the pounds; we then find the interest as though the sum expressed dollars and cents; and, reducing the decimal in the answer to shillings and pence, we have 63£. 7s. 9d.+ as the interest required. Hence,

Reduce the shillings, pence, and farthings, if any, in the principal to the decimal of a pound. Then proceed as in United States money; and, if there be a decimal in the result, reduce it to a compound number.

#### EXAMPLES.

- 2. What is the interest of 179£. 12s. 11d. for 1 year and 7 months, at 5 per cent.?

  Ans. 14£. 4s. 51d.
- 3. What is the interest of 25£. for 1 year and 9 months, at 5 per cent.?
- 4. What is the interest of 5440£. 10s. for 3 years and 11 months, at 6 per cent.?

  Ans. 1278£. 10s. 4d.
- 5. What is the interest of 943£. 1s. 8d. from May 1, 1857, to October 21, 1857, allowing 365 days to a year, at 5½ per cent.?

  Ans. 23£. 9s. 4d.
- 358. To find the PRINCIPAL, the interest, the time, and the rate per cent. being given.
- Ex. 1. What principal will gain \$120 in 4 years, at 6 per cent.?

  Ans. \$500.

\$120.00 \(\displays \). 24 \(\displays \)\$500 Ans. The interest for the given time and rate is 24 cents on a principal of \$1; therefore, the required principal will be as many times \$1 as the number denoting the given interest contains times .24.

RULE. — Divide the given interest by the interest of \$1 for the given time, at the given rate.

#### EXAMPLES.

- 2. What principal will gain \$10.08 in 1 year, at 7 per cent.?

  Ans. \$144.
- 3. What principal will yield an income of \$13.20 in 1 year and 4 months, at 8½ per cent.?

  Ans. \$120.
- 4. What sum should be paid for a ground rent paying yearly, at 6 per cent., \$40.50?
- 5. If the interest on a sum borrowed at 2 per cent. a month is \$24 for 90 days, what is the sum?

  Ans. \$400.
- 6. What principal at 7½ per cent. is sufficient to produce \$206.38½ interest in 183 days?

  Ans. \$5600.
- **359.** To find the RATE PER CENT., the principal, the interest, and the time being given.
- Ex. 1. If the interest of \$500 for 4 years is \$120, what is the rate per cent.?
- OPERATION.

  We find the interest on a given principal for the given term to be \$20, at 1 per cent.; then the required rate will be as many per cent. as 20 is contained times in 120, or 6 per cent.

RULE. — Divide the given interest by the interest of the given principal at 1 per cent. for the given time.

# EXAMPLES.

- 2. The interest of \$144 for 1 year is \$10.08; what is the rate per cent.?

  Ans. 7 per cent.
- 3. At what rate per cent. must \$120 be on interest to amount to \$133.20 in 1 year 4 months?
- 4. At what rate per cent. must \$1, or any other sum, be on interest to double itself in 14\frac{2}{7} years?

  Ans. 7 per cent.
- 5. At what rate per cent. must any sum of money be on interest to quadruple itself in 33\frac{1}{3} years?

  Ans. 9 per cent.
- 6. I receive yearly \$232.50 interest on \$4650 loaned to the State of Massachusetts; what is the rate per cent.?
  - Ans. 5 per cent.
- 7. At what rate per cent. must \$7500 be loaned to gain \$60.00 in 48 days?

- 8. At what rate per cent. must \$280 be on interest to amount to \$411.95 in 6 years and 6 months?

  Ans.  $7\frac{1}{4}$  per cent.
- 9. At what rate per cent. must \$480 be on interest to amount to \$529.60 in 1 year 3 months and 15 days?

Ans. 8 per cent.

- 360. To find the TIME, the principal, the interest, and the rate per cent. being given.
- Ex. 1. How long must \$500 be on interest, at 6 per cent., to gain \$120?

  Ans. 4 years.

OPERATION.

We find that the given principal at the given rate will produce \$30 interest in 1 year; therefore to produce. \$120 interest there will be required as many years as 30 is contained times in 120, or 4 years.

Rule. — Divide the given interest by the interest of the given principal for 1 year.

#### EXAMPLES.

- 2. How long must \$120 be on interest, at 8½ per cent., to gain \$13.20?

  Ans. 1yr. 4mo.
- 3. In what time will \$144 produce \$10.08 interest, at 7 per cent.?

  Ans. 1 year.
- 4. In what time will \$240 at interest amount to \$280, at 6 per cent.?

  Ans. 2yr. 9mo. 10d.
- 5. In what time will \$1, or any other sum, double itself, at 5 per cent. interest?

  Ans. 20 years.
- 6. In what time will any sum double itself at 10 per cent. interest?

  Ans. 10 years.
- 7. In what time will \$1500 amount to \$2250, at 5 per cent. interest?
- 8. In what time will \$480 at 4½ per cent. amount to \$561.60?

  Ans. 3yr. 9mo. 10d.
- 9. In what time, at 12 per cent., will \$1728 amount to \$3853.44?

  Ans. 10yr. 3mo.
- 10. In what time, at 6 per cent., will \$240 amount to \$720?

  Ans. 33yr. 4mo.
- 11. Borrowed, May 16, 1857, the sum of \$400, payable as soon as the principal, increased by the interest at 6 per cent., shall equal \$500. At what date is it payable?

Ans. July 16, 1861.

# PROMISSORY NOTES.

**361.** A PROMISSORY NOTE, or note of hand, is an engagement, in writing, to pay a specified sum, either to a person named in the note, or to his order, or to the bearer.

A joint note is one signed by two or more persons, who together are holden for its payment.

A joint and several note is one signed by two or more persons, who separately and together are holden for its payment.

A negotiable note is one so made that it can be sold or transferred from one person to another.

362. The maker or drawer of a note is the person who signs it.

The payee, promisee, or holder of a note is the person to whom it is to be paid.

The indorser of a note is the person who writes his name upon its back to transfer it, or as a guaranty of its payment. If the indorser, however, wishes only to transfer the note, he may write before his name the words "without recourse," and then, though by his name he guarantees the genuineness of the note, he is not liable for its payment, should the maker not pay it when due.

- 363. The face of a note is the sum for which it is given.
- 364. A note should contain the words "value received," and the sum for which it is given should be expressed in written words, or, as is the general custom, the dollars may be written in words and the cents, if any, be expressed as hundredths of a dollar in the form of a common fraction.

Note. — The laws of Pennsylvania require that the words "without defalcation" should be inserted in a promissory note; and in Indiana, notes generally contain the words "without any relief whatever from valuation or appraisement laws."

- 365. A note is said to be one on time, when it is made payable on or after a certain date, and the day on which it becomes legally due is called the day of maturity.
- 366. According to the laws of many of the States, when a particular day is specified in the note for its payment, three

days additional are allowed, called days of grace, within which the maker may pay the note, unless it states "without grace." Should the third day of grace, however, fall on Sunday, or some public day, the day of maturity will be a day earlier.

- 367. When a note is written for months, calendar months are understood. Thus, if a note be dated April 21, for one month, it will be nominally due May 21; and if dated January 29, 30, or 31, it being for one month, it will be nominally due February 28, should it be a common year, or February 29, should it be a leap year.
- 368. When a note is written without interest, it can only draw interest, if on time, after the time specified for its payment, and not then lawfully, in some States, till after a demand has been made; or if not on time, after payment has been demanded.
- NOTE. A note attested or witnessed, in Massachusetts and some other States, is taken out of the statute of limitation.

# PARTIAL PAYMENTS ON NOTES AND BONDS.

**369.** Partial or part payments on notes, bonds, or other obligations, being receipted for by entry on the back of the obligation, are termed *indorsements*.

### UNITED STATES RULE.

370. In the United States courts, and the courts of Massachusetts, New York, and several other States, interest on notes and bonds, when partial payments have been made, is reckoned according to the following

RULE. — Compute the interest on the principal to the time when the first payment was made, which equals or exceeds, either alone or with preceding payments, the interest then due.

Add that interest to the principal, and from the amount subtract the

payment or payments thus far made.

The remainder will form a new principal; on which compute the interest, proceeding as before.

Note. — This rule is on the principle that neither interest nor payment should draw interest.

### EXAMPLES.

# (1.) \$165.18. Boston, June 17, 1847.

For value received, I promise to pay Nathaniel Ford, or order, on demand, one hundred and sixty-five dollars and eighteen cents, with interest.

. Attest, JOSEPH FIELD.

JAMES PETERSON.

On this note are the following indorsements. December 7, 1847, received eighteen dollars and thirteen cents of the within note. October 19, 1848, received twenty-eight dollars and sixteen cents. September 25, 1849, received thirty-six dollars and twelve cents. July 10, 1850, received three dollars and eighteen cents. June 6, 1851, received thirty-six dollars and twenty-eight cents. December 28, 1852, received thirty-one dollars and seventeen cents. May 5, 1858, received three dollars and eighteen cents. September 1, 1853, received twenty five dollars and eighteen cents. October 18, 1854, received ten dollars.

# How much remains due September 27, 1855?

Ans. \$ 15.417.

OPERATION.	
Principal, carrying interest from June 17, 1847,	<b>\$ 165.180</b>
Interest from June 17, 1847, to Dec. 7, 1847, 5mo. 20d.,	4.680
· Amount,	169.860
First payment, December 7, 1847,	18.130
Balance for new principal,	151.730
Interest from Dec. 7, 1847, to Oct. 19, 1848, 10mo. 12d.,	7.889
Amount,	159.619
Second payment, October 19, 1848,	28.160
Balance for new principal,	131.459
Interest from Oct. 19, 1848, to Sept. 25, 1849, 11mo. 6d.,	7.361
• Amount,	138.820
Third payment, September 25, 1849,	36.120
Balance for new principal,	102.700
Interest from Sept. 25, 1849, to June 6, 1851, 20mo. 11d.,	10.458
Amount,	113.158
Fourth pay't, July 10, 1850, a sum less than interest, 3.1 Fifth pay't, June 6, 1851, a sum greater than interest, 36.2	
	- 39.460
Balance for new principal,	73.698
Interest from June 6, 1851, to Dec. 28, 1852, 18mo. 22d.,	6.903
Amount,	80.601

Amount brought forward,	\$80.601
Sixth payment, December 28, 1852,	31.170
Balance for new principal,	49.431
Interest from Dec. 28, 1852, to May 5, 1853, 4mo. 7d.,	1.046
Amount,	50.477
Seventh payment, May 5, 1853,	3.180
Balance for new principal,	47.297
Interest from May 5, 1853, to Sept. 1, 1853, 3mo. 26d.,	.914
Amount,	48.211
Eighth payment, September 1, 1853,	<b>25.180</b>
Balance for new principal,	23.031
Interest from Sept. 1, 1853, to Oct. 18, 1854, 13mo. 17d.,	1.562
. Amount,	24.593
Ninth payment,	10.000
Balance for new principal,	14.593
Interest from Oct. 18, 1854, to Sept. 27, 1855, 11mo. 9d.,	.824
Balance due at the time of payment,	\$ 15.417

For value received, I promise to pay L. Swan, or order, on demand, seven hundred and sixty-nine dollars and eighty-seven cents, with interest.

SAMUEL Q. PETERS.

Attest, Moses Haynes.

(2.) \$769.87.

Payments: March 1, 1850, seventy-five dollars and fifty cents; June 11, 1851, one hundred and sixty-five dollars; September 16, 1851, one hundred and sixty-one dollars; Jan. 21, 1852, forty-seven dollars and twenty-five cents; March 5, 1853, twelve dollars and seventeen cents; December 6, 1853, ninety-eight dollars; July 7, 1854, one hundred and sixty-nine dollars.

What remains due September 25, 1855? Ans. \$226.297.

(3.) \$ 300. Chicago

Chicago, April 30, 1851.

St. Louis, June 17, 1849.

For value received, I promise Kimball & Hammond to pay them, or order, on demand, three hundred dollars, with interest.

SIMPSON W. LEAVET.

Payments: June 27, 1852, one hundred and fifty dollars; December 9, 1852, one hundred and fifty dollars.

What was due October 9, 1853?

Ans. \$ 26.735.

(4.) \$54.18. San Francisco, Feb. 11, 1852.

For value received, I promise to pay John Trow, or order, on demand, fifty-four dollars and eighteen cents, with interest.

LUKE M. SAMPSON.

Payments: July 11, 1853, twelve dollars and twenty-five cents; August 15, 1854, two dollars and ten cents; July 9, 1855, three dollars and twelve cents; August 21, 1855, thirty-seven dollars and eighteen cents.

What was due December 17, 1855?

Ans. \$ 10.222.

(5.) \$1728.

Philadelphia, Jan. 7, 1851.

For value received, we jointly and severally promise to pay Jones, Oliver, & Co., or order, one thousand seven hundred and twenty-eight dollars, on demand, with interest, without defalcation.

JOHN WICKERSHAM.

Attest, TIMOTHY TRUE.

JAMES THICKSTEIN.

(6.) \$1000.

New York, January 1, 1850.

For value received, I promise to pay James Johnson, or order, on demand, one thousand dollars, with interest at seven per cent.

• SAMUEL T. FORTUNE.

Indorsements: September 28, 1850, one hundred and forty-four dollars; March J, 1851, twenty dollars; July 17, 1851, three hundred and sixty dollars; August 9, 1851, one hundred and ninety dollars; September 25, 1852, one hundred and seventy dollars; December 11, 1853, two hundred dollars; July 4, 1855, seventy-five dollars.

What was due June 1, 1857?

Ans. \$7.61.

# CONNECTICUT RULE.

371. The Supreme Court of the State of Connecticut has adopted the following

Rule. — Compute the interest to the time of the first payment, if that be one year or more from the time the interest commenced; add it to the principal, and deduct the payment from the sum total. If there be after payments made, compute the interest on the balance due to the next payment, and then deduct the payment as above; and in like manner from one payment to another, till all the payments are absorbed; provided the time between one payment and another be one year or more.

If any payments be made before one year's interest has accrued, then compute the interest on the principal sum due on the obligation for ONE YEAR, add it to the principal, and compute the interest on the sum vaid from the time it was paid up to the end of the year; add it to the sum paid, and deduct that sum from the principal and interest added

as above.

If any payments be made of a less sum than the interest arisen at the time of such payment, no interest is to be computed, but only on the principal sum for any period.

NOTE. — If a year extends beyond the time of settlement, find the amount of the remaining principal to the time of settlement; find also the amount of the indorsement or indorsements, if any, from the time they were paid to the time of settlement, and subtract their sum from the amount of the principal.

### EXAMPLE.

(1.) \$ 900. New Haven, June 1, 1858.

For value received, I promise to pay J. Downs, or order, nine hundred dollars, on demand, with interest.

JAMES L. EMERSON.

Ans. \$ 417.822.

Indorsements: June 16, 1859, two hundred dollars; August 1, 1860, one hundred and sixty dollars; November 16, 1860, seventy-five dollars; February 1, 1862, two hundred and twenty dollars.

What was due August 1, 1862?

What was due in	ug usu	-, -				*******	W 110	.022.
		OPE	RATION				•	
Principal,						•	. \$	900.00
Interest from June 1,	1858	, to J	une 1	6, 185	9, 12	1 mo	onths,	56.25
								956.25
First payment, .		•	•		•		•	200.00
								756.25
Interest from June 16	, 185	9, to .	Augus	t 1, 18	360, 1	13 <del>1</del> n	onths,	51.046
								807.296
Second payment,		•	•	•	•	•	•	160.000
				,				647.296
Interest for 1 year,	•	•	•	•	•		٠.	38.837
								686.133
Am't of 3d pay't, from	Nov	16, 1	860, t	o Aug	. 1, 1	861,	8 <del>1</del> mo.,	78.187
								607.946
Interest from Aug. 1,	1861,	to A	ug. 1,	1862	, 12 r	nont	hs,	36.476
								644.422
Am't of 4th pay't, from	n Feb	. 1, 18	362, to	Aug.	1, 18	36 <b>2</b> ,	6mo.,	226.600
Bålance due A	ugust	1, 18	362,	•	•		. \$	417.822

# MERCHANTS' RULE.

372. It is customary with merchants and others, when partial payments are made of notes or other debts, when the note or debt is settled within a year after becoming due, to adopt the following

Rule. — Find the amount of the principal from the time it became due until the time of settlement. Then find the amount of each indorsement from the time it was paid until settlement, and subtract their sum from the amount of the principal.

#### EXAMPLES.

(1.) \$ 1728. Baltimore, January 1, 1853.

For value received, I promise to pay Riggs, Peabody, & Co., or order, on demand, one thousand seven hundred and twenty-eight dollars, with interest.

JOHN PAYWELL, JR.

Indorsements: March 1, 1853, three hundred dollars; May 16, 1853, one hundred and fifty dollars; September 1, 1853, two hundred and seventy dollars; December 11, 1853, one hundred and thirty-five dollars.

What was due at the time of payment, which was December 16, 1853?

Ans. \$ 948.03

Principal,	,				\$ 1728.00
Interest for 11 months and 15 days,					99.36
·					\$ 1827.36
First payment,		•	\$ 3	00.00	
Interest for 9 months and 15 da	ays,			14.25	
Second payment,	•		1	50.00	

879.33

Balance remaining due, December 16, 1853, \$948.03

(2.) \$ 700. Montpelier, February 4, 1854.

For value received, we jointly and severally promise to pay James Thomas, or order, on demand, seven hundred dollars, with interest.

SAMPSON PHILLIPS,

RICHARD FLETCHER.

Payments: March 18, 1854, one hundred and sixty dollars; June 24, 1854, two hundred dollars; September 11, 1854, one hundred and twenty dollars; October 5, 1854, sixty dollars.

What was due on this note Nov. 28, 1854? Ans. \$180.43.

(3.) \$ 500. Detroit, January 1, 1857. For value received, three months after date I promise to pay

For value received, three months after date I promise to pay to the order of James Francis five hundred dollars.

WILLIAM AMSDEN.

Indorsement: July 1, 1857, two hundred dollars.

What was due April 1, 1858, the rate of interest being 7 per cent.?

Ans. \$324.50.

#### COMPOUND INTEREST.

- 373. COMPOUND INTEREST is interest on the original principal with its interest added when remaining unpaid after becoming due.
- 374. When the interest is added to the principal at the end of every year, and a new principal is thus formed yearly, it is said to compound annually; when the interest is added to the principal so as to form a new principal half-yearly, it is said to compound semiannually.
- 375. Compound interest is based upon the principle, that, if the borrower does not pay the interest as it becomes due at stated times, it is no more than just for him to pay interest for the use of it, so long as he shall have it in his possession.

Note. — Compound interest is not favored by the laws, though it is not usurious. A contract or promise to pay money with compound interest cannot generally be enforced, being only valid for the principal and legal interest.

376. To find the compound interest of any sum of money at any rate per cent. for any time.

Ex. 1. What is the compound interest of \$ 300 for 3 years?

Principal for 1st year,	\$300	
Interest of \$ 1 for 1 ye	ar, .0 6	
Interest for 1st year,	18.00	8
•	3 0 0.0 0	i
Principal for 2d year,	3 1 8.0 0	
	.0 6	1
Interest for 2d year,	19.0800	1 1 1
	318.00	1
Principal for 3d year,	3 3 7.0 8	
	.06	ŀ
Interest for 3d year,	$\overline{20.2248}$	t
22202000 101 0 m j 0 m j	33708	3
Amount for 3 years,	357.3048	í
First principal,	300.00	t
Comp. int. for 3 years,	\$578048 Ans	_
comp. ma for o years,	world and and	•

We first multiply the given principal by the number denoting the interest of \$1 for one year, and add the interest thus found to the principal for the amount; on which as a new principal we find the interest for the second year, and proceed as before; and so also with the third year. From the amount of the last year we subtract the first principal, and obtain the compound interest for 3 years.

Principal for 1st year

SECO	ND O	PERA	TION

\$30000

I inicipal for Ist Jour,	<b>\$ 0</b> 0 0.0 0	
σ of the principal,	1 5.0 0	
† of the interest at 5 per ce	nt., 3.00	
Principal for 2d year,	318.00 A	mount for 1st year at
20 of the principal,	1 5.9 0	[6 per cent.
tof the interest at 5 per ce	nt., 3.18	
Principal for 3d year,	337.08 A	mount for 2d year at
of the principal,	1 6.8 4 5	[6 per cent.
† of the interest at 5 per ce	nt., 3.3 7 0	`
•	357.304	Amount for 3d year
First principal,	3 0 0.0 0	[at 6 per cent.

In the second operation the work is somewhat abridged, by finding the interest for each year at 6 per cent. by taking  $\frac{1}{20}$  of the principal for the interest at 5 per cent., and  $\frac{1}{6}$  of that for interest at 1 per cent.

Compound interest for 4 years, \$57.304 Ans.

RULE. — Find the interest of the given sum to the time the interest becomes due, and add it to the principal. Then, find the interest on this amount as a new principal, and add the interest to it, as before. Proceed in the same manner for each successive period when the interest becomes due until the time of settlement.

Subtract the principal from the last amount, and the remainder will

be the compound interest.

Note. — When partial payments have been made on notes at compound interest, it is customary to find the amount of the given principal, and from it to subtract the sum of the several amounts of the indorsements.

### EXAMPLES.

- 2. What is the amount of \$500 for 3 years at compound interest?

  Ans. \$595.508.
- 3. What is the compound interest of \$970 for 2 years 9 months and 24 days?

  Ans. \$173.295.
- 4. What is the compound interest of \$300 for 4 years 6 months, at 7 per cent.?

  Ans. \$107.001.
- 5. What is the compound interest of \$316 for 3 years 4 months and 18 days?

  Ans. \$69.017.
- 377. The computation of compound interest is rendered more expeditious by means of the following

TABLE,

SHOWING THE AMOUNT OF ONE DOLLAR AT COMPOUND INTEREST FOR ANY
NUMBER OF YEARS NOT EXCREDING FIFTY.

No.	8 per cent.	8j per cent.	4 per cent.	5 per cent.	6 per cent.	7 per cent.
1	1.030 000	1.035 000	1.040 000	1.050 000	1.060 000	1.070 000
_2	1.060 900	1.071 225	1.081 600	1.102 500	1.123 600	1.144 900
3	1.092 727	1.108 718	1.124 864	1.157 625	1.191 016	1.225 043
4	1.125 509	1.147 523	1.169 859	1.215 506	1.262 477	1.310 796
5	1.159 274	1.187 686	1.216 653	1.276 282	1.338 226	1.402 552
6	1.194 052	1.229 255	.1.265 319	1.340 096	1.418 519	1.500 730
7	1.229 874	1.272 279	1.315 932	1.407 100	1.503 630	1.605 781
8	1.266 770	1.316 809	1.368 569	1.477 455	1.593 848	1.718 186
9	1.304 773	1.362 897	1.423 312	1.551 328	1.689 479	1.838 459
10	1.343 916	1.410 599	1.480 244	1.628 895	1.790 848	1.967 151
11	1.384 234	1.459 970	1.539 454	1.710 339	1.898 299	2.104 852
12	1.425 761	1.511 069	1.601 032	1.795 856	2.012 196	2.252 192
13	1.468 534	1.563 956	1.665 074	1.885 649	2.132 928	2.409 845
14	1.512 <b>5</b> 90	1.618 694	1.731 676	1.979 932	2.260 904	2.578 534
15	1.557 967	1.675 349	1.800 944	2.078 928	2.396 558	2.759 032
16	1.604 706	1.733 986	1.872 981	2.182 875	2.540 352	2.952 164
17	1.652 848	1.794 675	1.947 901	2.292 018	2.692 773	3.158 815
18	1.702 433	1.857 489	2.025 817	2.406 619	2.854 339	3.379 932
19	1.753 506	1.922 501	2.106 849	2.526 950	3.025 600	3.616 526
20	1.806 111	1.989 789	2.191 123	2.653 298	3.207 135	3.869 684
21	1.860 295	2.059 431	2.278 768	2.785 963	3.399 564	4.140 562
22	1.916 103	2.131 512	2.369 919	2.925 261	3.603 537	4.430 402
23	1.973 587	2.206 114	2.464 716	3.071 524	3.819 750	4.740 530
24	2.032 794	2.283 328	2.563 304	3.225 100	4.048 935	5.072 367
25	2.098 778	2.363 245	2.665 836	3.386 355	4.291 871	5.427 433
26	2.156 591	2.445 959	2.772 470	3.555 673	4.549 883	5.807 353
27	2.221 289	2.531 567	2.883 369	3.733 456	4.822 846	6.213 868
28	2.287 928	2.620 177	2.998 703	3.920 129	5.111 687	6.648 838
29	<b>2.356 5</b> 66	2.711 878	8.118 651	4.116 136	5.418 388	7.114 257
30	2.427 262	2.806 794	3.243 398	4.321 942	5.743 491	7.612 255
81	<b>2.</b> 500 080	2.905 031	3.373 133	4.538 039	6.088 101	8.145 113
82	2.575 083	3.006 708	3.508 059	4.764 941	6.453 387	8.715 271
83	2.652 335	3.111 942	3.648 381	5.003 189	6.840 590	9.325 340
34	2.731 905	3.220 860	3.794 316	5.253 348	7.251 025	9.978 114
85	2.813 862	3.333 590	3.946 089	5.516 015	7.686 087	10.676 581
36	2.890 278	3.450 266	4.103 933	5.791 816	8.147 252	11.423 942
37	2.985 227	3.571 025	4.268 090	6.081 407	8.636 087	12.223 618
38	3.074 783	3.696 011	4.438 813	6.385 477	9.154 252	13.079 271
39	3.167 027	3.825 372	4.616 366	6.704 751	9.703 507	13.994 820
40	8.262 038	3.959 260	4.801 021	7.039 989	10.285 718	14.974 458
41	3.359 899	4.097 834	4.993 061	7.391 988	10.902 861	16.022 670
42	3.460 696	4.241 258	5.192 784	7.761 588	11.557 033	17.144 257
43	3.564 517	4.389 702	5.400 495	8.149 667	T2.250 455	18.344 355
44	3.671 452	4.543 842	5.616 515	8.557 150	12.985 482	19.628 460
45	8.781 596	4.702 358	5.841 176	8.985 008	13.764 611	21.002 452
46	8.895 044	4.866 941	6.074 823	9.434 258	14.590 487	22.472 623
47	4.011 895	5.037 284	6.317 816	9.905 971	15.465 917	24.045 707
48	4.132 252	5.213 589	6.570 528	10.401 270	16.393 872	25.728 907
49	4.256 219	5.396 065	6.833 849	10.921 333	17.377 504	27.529 930
50	4.383 906	5.584 927	7.106 683	11.467 400	18.420 154	29.457 025

Note. — If each of the numbers in the table be diminished by 1, the remainder will denote the interest of \$1, instead of its amount.

Ex. 1. What is the compound inter	est of \$360 for 5 years				
6 months and 24 days?	Ans. \$ 138.14.				
OPERATION.					
Amount of \$ 1 for 5 years,	<b>\$ 1.3 3 8 2 2 6</b>				
Principal,	360				
	80293560				
	40146780				
Amount of \$360 for 5 years,	481.761360				
Amount of \$1 for 6mo. 24d.,	1.034				
·	1927045440				
	1445284080				
	481761360				
Amount of \$360 for 5y. 6mo. 24d.,	498.141246240				
Principal,	3 6 0.				
Comp. int. of \$360 for 5y. 6mo. 24d.,	\$138.14 Ans.				

We find the amount of \$1 for 5 years in the table, and, multiplying it and the number denoting the given principal together, obtain the amount of the \$360 for 5 years. On this amount as a new principal we find the amount for the remaining 6 months and 24 days, by multiplying by the number denoting the amount of \$1 for the same time. From the last amount subtracting the original principal, we have left Hence, the compound interest required.

Multiply the amount of \$1 for the given time and rate, as found in the table, by the number denoting the given principal. The product will be the required amount, from which subtract the given principal, and the remainder will be the COMPOUND INTEREST.

Note. — When the given time includes not only the regular periods at which interest becomes due, but also a partial period, as a succession of periods of a year each, followed by one containing months or days, or both, after finding the amount for the regular periods, multiply that amount by the amount of \$1 for the remaining time or partial period, and the product will be the required amount for the given time. In like manner, when the number of successive periods exceeds the limits of the table, make the computations for a convenient length of time by means of the table, and on the amount thus found make another computation by means of the table, and so on.

In making computation for a succession of periods shorter or longer than one year each, use the numbers in the table the same as if the periods were those of one year each.

#### EXAMPLES.

2. What is the compound interest of \$1200 for 11 years at 7 per cent.? Ans. \$1325.822.

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- 3. What is the compound interest of \$300 for 10 years 7 months and 15 days?

  Ans. \$257.401.
- 4. What is the compound interest of \$5 for 50 years at 7 per cent.?

  Ans. \$142.285.
- 5. What is the amount of \$480 for 40 years, at compound interest?

  Ans. \$4937.144.
- 6. What is the compound interest of \$40 for 4 years, at 7 per cent.?

  Ans. \$12.431.
  - 7. What is the compound interest of \$100 for 100 years?
    Ans. \$33830.20.
- 8. What is the difference between the simple and the compound interest of \$1000 for 33 years and 4 months?
- 9. To what sum will \$50, deposited in a savings bank, amount, at compound interest for 21 years, at 3 per cent., payable semiannually?

  Ans. \$173.034.
- (10.) \$ 100. Boston, September 25, 1853.

  For value received, I promise to pay J. D. Forster, or order, on demand, one hundred dollars, with interest, after six months.

  ALLEN T. DAWES.

On this note are the following indorsements: — June 11, 1854, received fifty dollars; September 25, 1854, received fifty dollars.

What was due, reckoning at compound interest, August 25, 1855?

Ans. \$ 2.247.

(11.) \$ 1000. St. Paul, January 1, 1850.

For value received, I promise to pay Stephen Howe, or bearer, on demand, one thousand dollars, with interest at 7 per cent.

WILSON GOODHUE.

Indorsements: — June 10, 1850, seventy dollars; September 25, 1851, eighty dollars; July 4, 1852, one hundred dollars; November 11, 1853, thirty dollars; June 5, 1854, fifty dollars.

- At 7 per cent. compound interest, what remains due April 1, 1855?

  Ans. \$ 1022.34.
- 378. To find the PRINCIPAL, the compound interest, the time, and the rate being given.
- Ex. 1. What principal at 6 per cent. compound interest will produce \$2370 in 10 years?

  Ans. \$3000.

\$ 2370 + .790 = \$ 3000 Ans. We find the compound interest of \$ 1 for the given time, and at the given rate; and proceed as in like cases in simple interest (Art. 359).

RULE. — Divide the given compound interest by the compound interest of \$1 for the given time at the given rate.

#### EXAMPLES.

- 2. What principal, at 7 per cent. compound interest, will produce \$205.90 in 6 years and 6 months? Ans. \$372.16.
- 3. What sum of money, at compound interest, will produce \$1026.54 in 3 years 2 months and 12 days? Ans. \$5000.
- 4. What sum of money must be invested at compound interest at a semiannual rate of 3½ per cent. to produce \$857.25 in 15½ years?

  Ans. \$450.
- 379. To find the RATE PER CENT., the principal, the interest, or the amount, and the time being given.
- Ex. 1. At what rate per cent. must \$500 be at compound interest to become \$703.55 in 7 years? Ans. 5 per cent.

#### OPERATION.

\$703.55 ÷ 500 = \$1.4071, which for 7 years, in the table, denotes a rate of 5 per cent.

Since \$500 becomes \$703.55 in 7 years at the required rate, \$1 in the same time at the same rate will amount to \$100 as much, or \$1.4071. Corresponding to this amount of \$1 for the given time, we find in the table (Art. 377) 5 per cent., the rate required.

RULE. — Divide the amount by the principal, and the quotient will be the amount of \$1 for the given time and the required rate; and in the table, over this amount, may be found the rate per cent. required.

Note. — If the given time contains a part over an exact number of periods, took in the table, against the number denoting the whole periods or years in the given time, for that amount of \$1 which comes the nearest to the one found by dividing. Then see if the approximate amount, increased by its rate of interest for the fractional period, will equal the other amount; if so, the rate corresponding to the approximate amount will be the rate per cent. required; if not, the rate of the approximate amount will be as much greater or smaller than the required rate, as the interest added to the approximate amount is greater or smaller than that required to produce the amount found by dividing. The rule can only be well applied when the rate per cent. sought is within the limits of the table.

#### EXAMPLES.

- 2. At what rate per cent. will \$400 amount to \$640.405, at compound interest, in 12 years?

  Ans. 4 per cent.
- 3. At what rate per cent. must \$2500 be loaned, to produce \$2096.147 compound interest, in 9 years? Ans. 7 per cent.
- 4. At what rate per cent. will any sum of money double itself at compound interest in  $11_{\frac{80}{100}}^{\frac{80}{100}}$  years? Ans. 6 per cent.
- 5. At what rate per cent. will \$10,000 amount to \$31479.70 in 19 years and  $8\frac{3}{4}$  months? Ans. 6 per cent.
- **380.** To find the TIME, the principal, the compound interest, and the rate per cent. being given.
- Ex. 1. In what time will \$500, at 7 per cent. compound interest, amount to \$655.398?

  Ans. 4 years.

#### OPERATION.

 $$655.398 \div 500 = $1.310796$ , which, at the given rate in the table, denotes 4 years' time.

Since \$500 amounts to \$655.398 at 7 per cent. in the required time, \$1 at the same rate, in the same time, will amount to  $\frac{1}{500}$  as much, or \$1.310796. Corresponding to this amount of \$1 at the given rate, we find in the table (Art. 377) 4 years, the time required.

Rule. — Divide the amount by the principal, and the quotient will be the amount of \$1 at the given rate for the required time; and in the table, against this amount, may be found the time required.

Note. — If the required time cannot be found exactly in the table, the number against that amount of \$1 which under the given rate is next less than the amount found by dividing, will denote the whole periods or years. Then, find the fractional period or part of a year, by dividing 1 whole period or year by the ratio of the difference between the amount corresponding to the whole periods or years and that found by dividing to the difference between the former of the amounts and that next larger in the table; and the value of the fraction obtained as the result may be expressed in months or days, or both.

### EXAMPLES.

- 2. In what time will \$400 amount to \$640.405 at 4 per cent. compound interest?

  Ans. 12 years.
- 3. In what time will \$6000 amount to \$9021.78 at 7 per cent. compound interest?



- 4. In what time will any sum double itself at 5 per cent. compound interest?

  Ans. 14y. 2mo. 13d.
- 5. In what time will any sum double itself at 6 per cent. compound interest?

  Ans. 11y. 10mo. 20—d.
- 6. A gentleman has deposited \$450, for the benefit of his son, in a savings bank, at compound interest at a semiannual rate of 3½ per cent. He is to receive the amount as soon as it becomes \$1781.66½. Allowing that the deposit was made when the son was 1 year old, what will be his age when he can come in possession of the money?

  Ans. 21 years.

## DISCOUNT AND PRESENT WORTH.

- 381. DISCOUNT is an allowance or deduction made for paying money before it is due.
- 582. The PRESENT WORTH is the amount of ready money that will satisfy a debt before it is due. It is equivalent to a principal which, being put at interest, will amount to the debt at the time of its becoming payable. Thus, \$100 is the present worth of \$106 due one year hence at 6 per cent.; for \$100 at 6 per cent. will amount to \$106 in that time; and \$6 is the discount.
- 383. In discount, the rate per cent., the time, and the sum on which the discount is made, are given, to find the *present worth*, which corresponds precisely to the rate per cent., the time and the amount being given, in either simple or compound interest, to find the *principal*.
- \*384. The interest or percentage of any sum cannot properly be taken for the *true* discount; for we have seen (Art. 382) that the *interest* for one year is the fractional part of the sum at interest, denoted by the rate for the numerator, and 100 for the denominator; and the *discount* for one year is the fractional part of the sum on which discount is to be made, denoted by the rate for the numerator, and 100 plus the rate for the de-

nominator. Thus, if the rate of interest is 6 per cent., the *interest* for one year is  $_{100}^{6}$  of the sum at interest; but if the rate per cent. of discount is 6, the *discount* for one year is  $_{100}^{6}$  of the sum on which discount is made.

- 385. Business men, however, often deduct, or "take off," from the face of a bill or note due at some future time, a greater percentage than the interest would be for the given time at the given rate. Therefore, the true present worth and discount are not obtained by that method, but only a nominal present worth and a nominal discount. The true discount is equal to the interest on the true present worth of the debt, while the nominal discount is equal to the interest on the face of the debt.
- 386. To find the true present worth of any sum, and the discount, for any time, at any rate per cent.
- Ex. 1. What is the present worth of \$12.72, due one year hence, discounting at 6 per cent.? What is the discount?

  Ans. \$12 present worth; \$0.72 discount.

OPERATION.

Amount of \$1, 1.0 6) \$1 2.7 2 (\$1 2, Present worth.

106 212 '\$12.72, Given sum. 212 12.00, Present worth. \$0.72, Discount.

Since \$1 is the present worth of \$1.06 due one year hence, at 6 per cent., it is evident that the present worth of \$12.72 must be as many dollars as \$1.06 is contained times in \$12.72. We thus find the present worth to be \$12, which, subtracted from the given sum, gives \$0.72 as the discount.

RULE. — Divide the given sum by the amount of \$1 for the given time and rate, and the quotient will be the PRESENT WORTH.

From the given sum subtract the present worth, and the remainder will be the DISCOUNT.

### EXAMPLES.

- 2. What is the discount on \$802.50, at 7 per cent., due one year hence?

  Ans. \$52.50.
- 3. What is the present worth of \$117.60, due one year hence, at 12 per cent.?

- 4. What is the present worth of \$769.60, due 3 years and 5 months hence?

  Ans. \$638.672.
- 5. What is the present worth of \$678.75, due 3 years 7 months hence, at 7½ per cent.?

  Ans. \$534.975.
- 6. What is the discount on \$600, due 5 years hence, at 5 per cent.?
- 7. A merchant has given two notes; the first for \$79.87, to be paid January 21, 1856; the second for \$87.75, to be paid December 17, 1856. How much ready money will discharge both notes February 10, 1855?

  Ans. \$154.545.
- 8. C. Gardner owes Samuel Hall as follows: \$365.87, to be paid December 19, 1855; \$161.15, to be paid July 16, 1856; \$112.50, to be paid June 23, 1854; \$96.81, to be paid April 19, 1858. What should Hall receive as an equivalent, January 1, 1854?

  Ans. \$653.40.
- 9. What is the present worth of \$67.25 due 3 years hence?
- 10. What is the present worth of \$80.095, due 3 years hence at compound interest?

  Ans. \$67.25.
- 11. What is the discount on \$110.364 due 5 years hence, at 7 per cent. compound interest?

  Ans. \$31.677.
- 387. To find a nominal present worth of any sum due at some future time, and the discount on the same at a given rate, reckon the interest on the face of the debt for the given time and rate, and the same will be the nominal discount; and this discount subtracted from the face of the note will give the nominal present worth.

## EXAMPLES.

- 1. I have bought of Paine and Woodard a bill of goods amounting to \$960, on six months, but for ready money they take off from the face of the bill, for the time, 5 per cent. What was the amount paid?

  Ans. \$912.
- 2. How much more is the nominal than the true discount on \$5000 due one year hence, at 7 per cent.?
- 3. When money is worth 6 per cent. a year, how much may be gained by hiring money to pay \$4440 due 6 months hence, allowing the present worth of this debt to be reckoned by deducting the nominal discount?

  Ans. \$3.996.

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#### BANKING.

388. Banking is the general business transacted at banks. A bank is a joint-stock company, established for the purpose of receiving deposits, loaning money, dealing in exchange, or issuing bank-notes or bills, as a circulating medium, redeemable in specie at its place of business.

The capital of a bank is the money paid in by its stockholders, as the basis of business.

The affairs of a bank are usually managed by a board of directors chosen by the stockholders, and the principal officers are a president, a cashier, and one or more tellers.

NOTE. — The president and cashier sign the bills issued; the cashier superintends the bank accounts; and the tellers receive and pay out money. A bank check is an order drawn on the cashier for money.

**389.** Bank discount is the simple interest of a note, draft, or bill of exchange, deducted from it in advance, or before it becomes due.

The interest is computed, not only for the specified time, but for three days additional, called days of grace.

The legal rate of discount is usually the same as the legal rate of interest; and the difference between bank discount and true discount is the same as the difference between interest and true discount (Art. 384).

**390.** A note is said to be *discounted* at a bank, when it is received as security for the money that is paid for it, after deducting the interest for the time it was given.

The money paid for a note is called its avails, proceeds, or present worth.

391. A note, though nominally due at the time specified in it, is not legally due till the days of grace have been counted.

The time a note has to run is counted in days from after the day of its being discounted to the day of its becoming legally due.

392. To find the bank discount and the avails or proceeds of a note or bill for any time or rate per cent.

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Ex. 1. What is the discount on a note for \$1000 having 63 days to run, discounted at a bank at 6 per cent.? How much are the proceeds?

Ans. Discount, \$10.50; proceeds, \$989.50.

Sum discounted,	\$1000.00	We find the interest on the sum discounted
$\frac{1}{100}$ of sum = int. for 60d. $\frac{1}{20}$ of int. for 60d. = int. for		as in Art. 354, and this interest is the bank discount (Art. 389),
Bank discount,	<b>\$10.50</b>	which subtracted from the sum discounted
Proceeds, or present worth	, \$989.50	gives the proceeds or present worth. Hence,

Find the interest on the note, or sum discounted, for the given rate and time, including THREE days of grace, and this interest will be the discount.

Subtract the discount from the face of the note, or sum discounted, and the remainder will be the proceeds, or present worth.

### EXAMPLES.

- 2. What is the bank discount on a note for \$7800 on 90 days' time?

  Ans. \$120.90.
- 3. What is the bank discount on a note for \$1200, payable in 60 days, at 7 per cent.?

  Ans. \$14.70.
- 4. What is the bank discount on \$8000, payable in 60 days? What are the proceeds?

Ans. Discount, \$84; proceeds, \$7916.

- 5. How much money should be received on a note for \$760, payable in 5 months, discounted at a bank?
- 6. A merchant sold a cargo of hemp for \$7860, for which he received a note payable in 6 months. How much money will he receive at a bank for this note?

  Ans. \$7620.27.
- 7. If the following note was discounted April 3, 1857, how long had it to run, and what were the proceeds?
- \$  $160_{100}^{40}$ . Boston, December 3, 1856.

Six months after date, for value received, we promise to pay Robert S. Davis & Co., or order, one hundred and sixty  $\frac{40}{100}$  dollars, at the Merchants' Bank.

HALLETT, OSGOOD, & Co. Ans. 63 days; proceeds, \$158.72.

8. Required the time when the following note will become legally due, and the bank discount, provided it was discounted May 16, 1856, the rate being 7 per cent.

\$890,500.

Chicago, April 16, 1856.

One hundred and twenty days after date, I promise to pay to the order of Keen and Lee eight hundred and ninety  $\frac{1}{100}$  dollars, at the Marine Bank.

Thomas L. Cook,

Ans. August 17, 1856; discount, \$16.103.

9. Required the legal time of maturity of the following note, and the proceeds, it having been discounted June 11, 1856.

\$ 1340. Philadelphia, May 1, 1856.

For value received, ninety days after date, we promise to pay J. B. Lippincott & Co., or order, one thousand three hundred and forty dollars, at the Girard Bank, without defalcation.

JOHNSON & POLLOCK.

10. The following note was discounted, at 2 per cent. a month, July 5, 1857; how long had it to run, and what were the proceeds?

\$ 9000.

New York, June 19, 1857.

Two months after date, for value received, I promise to pay to the order of Joseph Appleton, nine thousand dollars, at the Manhattan Bank.

L. T. ROBERTS.

Ans. 48 days; proceeds, \$8712.00.

- 393. To find the amount for which a note must be given that the proceeds shall be a specified sum.
- Ex. 1. For what amount, payable in 60 days, must a note be given to a bank discounting at 6 per cent., to obtain \$989.50?

  Ans. \$1000.

OPERATION.

\$1.0000

Interest of \$ 1 for 63 days, .0105 Proceeds of \$ 1, \$ 0.9895

 $$989.50 \div .9895 = $1000.$ 

Since \$ 0.9895 of proceeds requires \$ 1 to be discounted, \$ 989.50 will require as many dollars as \$ 0.9895 is contained times in \$ 989.50, or \$ 1000. Hence.

Divide the given sum by the proceeds of \$1 for the given time and rate of bank discount, including THREE days of grace, and the quotient will be the required amount.

### EXAMPLES.

- 2. What sum, payable in 90 days, if discounted at 7 per cent. bank discount, will produce \$680? Ans. \$692.52.
- 3. What must be the face of a note, which, when discounted at a bank for 120 days, shall give as its proceeds \$540.50?

Ans. \$551.81.

- 4. For what amount must a note be given, in order that the proceeds of the note discounted, when having 6 months to run, shall be \$ 1938?
- 5. The avails of a note, having 4 months to run, discounted at a bank, were \$1631.60; what was the face of the note?

Ans. \$ 1665.74. 1500, for what sum

- 6. If a gentleman wishes to obtain \$1500, for what sum must he give his note payable in 30 days, allowing it is to be discounted at 1 per cent. a month?

  Ans. \$1516.68.
- 394. To find the rate of interest corresponding to a given rate of bank discount.
- Ex. 1. What rate of interest is paid when a note payable in 60 days is discounted at 6 per cent.?

Ans.  $6_{1979}^{126}$  per cent.

OFERATION.

.06 ÷ .9895 = .06<sub>1879</sub>

Every \$1 discounted for the given time and rate yields as its proceeds \$0.9895. Then, if \$1 in the given time will yield the same interest at 6 per cent., \$0.9895 in the same time will yield the same interest, at as many per cent. as the given rate, .06, contains times .9895.

RULE. — Divide the given rate per cent., expressed decimally, by the number denoting the proceeds of \$1 for the given time and rate. The quotient will be the rate of interest required.

### Examples.

- 2. What rate of interest is paid when a note payable in 30 days is discounted at 6 per cent.? Ans  $6\frac{6}{18}$  per cent.
- 3. What rate of interest is paid when a note payable in 90 days is discounted at 6 per cent.?

  Ans. 6 186 per cent.
- 4. A note payable in 4 months is discounted at 2 per cent. a month; what rate of interest is paid?
  - 5. When a note, payable in 6 months, is discounted at 7 per

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- cent., to what rate of interest does the bank discount correspond?
- 6. When a note payable in one year, without grace, is discounted, to what rate per cent. of interest does the bank discount correspond?

  Ans. 647 per cent.
- 395. To find the rate of bank discount corresponding to a given rate of interest.
- Ex. 1. At what rate of bank discount must a note, payable in 60 days, be discounted, to produce 6 per cent. interest?

Ans.  $5\frac{1895}{2021}$  per cent.

Since bank discount is reckoned on an amount which equals the proceeds for the given time and rate (Art. 389), the bank discount for every \$1 of proceeds is equivalent to the interest on \$1 + .0105 = \$1.0105. Then, if \$1 yield a certain interest in a given time at 6 per cent. per annum, \$1.0105 must yield the same interest in the same time, at as many per cent. per annum as the given rate, .06, contains times 1.0105.

RULE. — Divide the given rate per cent., expressed decimally, by the number denoting the amount of \$1 for the given time and rate. The quotient will be the rate of bank discount required.

### EXAMPLES.

2. At what rate must a note, payable in 30 days, be discounted, to produce 6 per cent. interest?

Ans.  $5\frac{1845}{1845}$  per cent.

3. At what rate must a note, running 60 days, be discounted, to yield 2 per cent. a month interest? Ans.  $23\frac{17}{527}$  per cent.

4. At what rate must a note, running 90 days, be discounted, to produce 6 per cent. interest?

- 5. At what rate must a note, running 120 days, be discounted, to produce 8 per cent. interest?
- 6. At what rate must a note, running 6 months, be discounted, to produce 7 per cent. interest?
- 7. At what rate must a note, payable 1 year hence, without grace, be discounted, to produce 6 per cent. interest?
- 8. What rate of bank discount on a note, payable 8 years and 4 months hence, without grace, corresponds to 5 per cent. interest?

  Ans. 3 per cent.

## MISCELLANEOUS EXAMPLES.

1. The gold coinage of the United States contains 9 parts of pure gold to 1 part of alloy, and the alloy is 1 part copper to 1 part silver. What per cent. of the whole is each metal?

Ans. Gold 90 per cent.; silver 5 per cent.; copper 5 per cent.

- 2. How much grain must be sent to the miller that a bushel of meal may be returned, the miller taking  $\frac{1}{16}$  part for toll?

  Ans.  $34\frac{2}{15}qts$ .
- 3. If a gentleman, possessing \$25000, has a net yearly income of 4 per cent. of that sum, how much may he spend each year in order that his expenses shall just equal his income?
- 4. The half-dollar of coinage previous to 1853 contains 2061 grains of standard silver, and that of present coinage contains 192 grains; what per cent. more of standard silver does the one contain than the other?

  Ans. 7#1.
- 5. My horse is worth 50 per cent. more than my buggy; how many per cent. is the buggy worth less than the horse?

  Ans. 334.
- 6. How many years longer will it take \$10 to become \$20 at 5 per cent. than at 6 per cent. simple interest?

Ans. 33 years.

- 7. What is the present worth of \$500 due 4 years hence, at 5 per cent. compound interest?

  Ans. \$411.351.
- 8. Bought cloth at \$5.00 per yard. What must be "the asking price," in order to fall on it 10 per cent., and still make 10 per cent. on the purchase?

  Ans. \$6.11 \frac{1}{6}.
- 9. A merchant sold a cargo of hemp for \$7860, for which he received a note payable in 6 months. How much money will he receive for the note at a bank?

  Ans. \$7620.27.
- 10. What is the difference between the true discount and that taken by banks on \$1500 due one year hence without grace?

  Ans. \$5.09\frac{23}{33}.
- 11. When a note payable in 60 days is discounted at the rate of 2 per cent. a month, what rate of interest is bank discount equal to?

  Ans. 25<sub>479</sub> per cent.
- 12. The proceeds of a 45 day note discounted at  $\frac{1}{2}$  per cent. a month yielded \$36.40. What was the face of the note?

  Ans. \$1617.77\forall.

- 13. At what rate per cent. of bank discount should a 30 day note be discounted, that interest may be received at the rate of 12 per cent.?

  Ans. 11233.
- 14. How much more can a bank make in 693 days with \$50000 by discounting notes on 60 days' time, than by discounting those on 30 days, the rate of discount being 6 per cent., and the profits in both cases to be retained in the bank till the end of the time?
- 15. A merchant bought 450 quintals of fish at \$3.50 cash, and sold them immediately for \$4.00 on 6 months' credit, for which he received a note. If he should get this note discounted at a bank, what will he gain on the fish?

  Ans. \$170.10.
- 16. A bank by discounting a note at 6 per cent. receives for its money a discount equivalent to 6½ per cent. interest. How long must the note have been discounted before it was due?

  Ans. 1yr. 3mo. 12d.

### STOCKS.

396. STOCKS is a general name given to government bonds, and to money capital invested in corporations.

The capital of banks, and of insurance, railroad, manufacturing, mining, and like companies, is usually divided into equal shares, the market value of which is often variable.

- 397. Stocks are said to be at par when they sell for their original value; above par, or at a premium, when for more than their original value; below par, or at a discount, when for less than their original value.
- 398. The premium and discount on stocks are generally computed at a certain per cent on the original or nominal value of the shares.

NOTE. — The original value of a share of bank, insurance, railroad, or like stock, is usually \$ 100, but sometimes \$ 50, and rarely any other sum than one of these.

399. A dividend is the interest or profit on stocks, dis-

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tributed to the shareholders, and is reckoned on the par or nominal value of the shares.

- **400.** To find the *market value* of stocks when they are at a premium, or at a discount.
- Ex. 1. What is the value of 20 shares of bank stock, at 9 per cent. premium, their nominal or par value being \$100 each?

# $\$100 \times 20 = \$2000; \$2000 \times 1.09 = \$2180.$

Since the par value of 1 share is \$100, that of 20 shares is \$2000; then, as \$1 at 9 per cent. premium equals \$1.09, \$2000 will equal 2000 times as much, or \$2180.

RULE. — Multiply the par value of the given stock by 1 increased by the rate per cent. premium, or by 1 decreased by the rate per cent. discount, expressed decimally, and the product will be the value required.

Note. — The difference between the par and market value gives the per cent. of premium or discount.

### EXAMPLES.

- 2. What is the value of \$24360 of stock, at 35 per cent. premium.

  Ans. \$32886.
- 3. Sold 15 shares of the Camden and Amboy Railroad, the par value being \$100 per share, at 13 per cent. advance. To what did they amount?

  Ans. \$1695.
- 4. What must be paid for 10 shares of the Old Colony and Fall River Railroad, at 85 per cent., the original value being \$100 each?
- 5. Sold 30 shares, \$100 each, in the Boston Bank, at 83 per cent. advance. To what did they amount, and how much was the premium?

Ans. Amount, \$3262.50; premium, \$262.50.

- 6. What must be given for 25 shares of insurance stock, par value being \$50, at 3 per cent. discount? Ans. \$1212.50.
- 7. What must be paid for 22 shares of the Iron City Manufacturing Stock, par value being \$250, at 95 per cent., and how much is the discount? Ans. \$5225; discount, \$275.
- 8. What will be the cost of \$50000 of United States government stock, at 17 per cent. advance?

  Ans. \$58500.

- 9. Bought \$19500 of State stocks at 93 per cent., and sold the same at 103 per cent.; how much was gained by the operation?

  Ans. \$1950.
- **401.** To find the *par value* of stocks, when they are at a premium, or at a discount.
- Ex. 1. Bought Ocean Insurance Company stock, at 7 per cent. premium, for \$535; what is its par value? Ans. \$500.

\$535 \div 1.07 \div \$500, Ans. Since \$1 at 7 per cent. premium equals \$1.07, the par value of the stock must be as many dollars as 535 contains times 1.07, or \$500.

Rule. — Divide the market value of the given stock by 1 increased by the rate per cent. premium, or by 1 decreased by the rate per cent. discount, expressed decimally, and the quotient will be the value required.

#### EXAMPLES.

- 2. Bought Massachusetts State stock, at 3½ per cent. premium, for \$6210; what is its par value? Ans. \$6000.
- 3. Sold 11 shares of Reading Bank, at 5 per cent. premium, for \$1155; what is the par value of its shares? Ans. \$100.
- 4. Bought 41 shares of canal stock at 40 per cent. below par for \$1230; what is the par value of its shares?
- 5. Bought 19 shares of bank stock, at a premium of 8 per cent., for \$2052; what was the amount of premium paid?
- 6. When government stocks are at 5 per cent. discount, how much par value will \$16245 purchase, and what is amount of discount?

  Ans. \$17100; discount, \$855.
- 7. When railroad stock at 15 per cent. advance is selling at \$57.50 per share, how many shares may be bought for \$862.50, and what will be the amount of premium?

Ans. 15 shares; \$112.50 premium.

- 8. How many State bonds of \$1000 each, at 12 per cent. discount, can be purchased for \$7920, and how much may be gained by the operation, if the selling price should afterwards advance to par?

  Ans. 9 bonds; \$1080 may be gained.
- 402. To find the rate of interest to which a dividend on any stock bought at a premium or discount corresponds.

Ex. 1. Received 12½ per cent. dividend on an investment in stocks at 25 per cent. above par; to what rate per cent. interest did it correspond?

Ans. 10 per cent.

OPERATION. Since the stock was bought at 25 per cent. above par, every \$1.25 of investment must represent only \$1 of par value. Then, since every \$1 of par value pays a dividend corresponding to  $12\frac{1}{2}$  per cent. interest, every \$1.25 of investment pays as many per cent. interest as  $12\frac{1}{2}$  per cent. = .125 is contained times in 1.25, or 10 per cent.

RULE. — Divide the rate per cent. of dividend, expressed decimally, by 1 increased by the rate per cent, premium, or by 1 decreased by the rate per cent. discount, expressed decimally, and the quotient will denote the rate of interest required.

Note. — If it be required to find at what price a stock, paying a certain rate per cent. dividend, should be bought in order that the investment shall pay a given rate of interest, divide the rate per cent. of dividend, expressed decimally, by the given rate of interest, expressed decimally, and the quotient will be the price required of each \$1 of the given stock.

### EXAMPLES.

- 2. Received 6 per cent. dividend on factory stock, purchased at 25 per cent. below par. What rate per cent. interest did the investment pay?

  Ans. 8 per cent.
- 3. When railroad stock paying 11 per cent dividend is worth \$110 per share, or \$10 per share above par, to what rate of interest would the income from an investment in its shares correspond?

  Ans. 10 per cent.
- 4. How much advance must be paid for stocks paying 12 per cent. dividends, in order that the investment shall pay exactly 8 per cent. interest?

  Ans. 50 per cent.
- 5. Which is the better investment, the buying of 9 per cent. stocks at 25 per cent. advance, or 6 per cent. stocks at 25 per cent. discount?
- 6. At what per cent. discount must government 5 per cent. stock be bought that the investment may yield 7 per cent.?

  Ans. 28‡ per cent.
- 7. How much more income yearly may be derived from \$20000 invested in 5 per cent. stock bought at 20 per cent. discount, than by letting the same sum at 6 per cent. interest?

  Ans. \$50.

### BROKERAGE AND COMMISSION.

- 403. BROKERAGE is the percentage paid to a dealer in money and stocks, called a *broker*, for making exchanges of money, negotiating different kinds of bills of credit, or transacting other like business.
- **404.** Commission is the percentage paid an agent, factor, or commission merchant for buying or selling goods, making collections, or transacting other business.
- **405.** When the person transacting the commission business lives in a foreign country, he is frequently called a *correspondent* or *consignee*.

The goods shipped or forwarded to a consignee to be sold on commission are termed a *consignment*, and the person sending or consigning the same is called the *consignor*.

406. The rate per cent of brokerage or commission is not regulated by law, but varies in different places, and with the nature of the business transacted.

Brokerage and commission are computed in the same manner.

- 407. To find the brokerage or commission on any given sum.
- Ex. 1. Paid a broker, for exchanging \$896 uncurrent bills for par funds, 2 per cent. brokerage. How much was the brokerage?

  Ans. \$17.92.

Since brokerage is a percentage on the given sum, the brokerage on \$896 at 2 per cent. will be  $$896 \times .02 = $17.92$ .

RULE. — Find the percentage on the given sum at the given rate per cent., and the result will be the brokerage or commission.

Note. — When the brokerage or commission, and the sum on which it is reckoned, are given, the rate per cent. may be found as in Art. 348.

#### EXAMPLES.

2. My agent in New Orleans has purchased cotton, on my account, to the amount of \$18768. What is his commission at 1\frac{3}{4} per cent.?

Ans. \$328.44.

- 3. I have engaged a broker to purchase for me 12 shares in the New York Central Railroad, at \$112.25 per share; what is his commission at 4 per cent.?

  Ans. \$3.363.
- 4. My agents, Hilton and Marcy of Cincinnati, advise me that they have purchased on my account a cargo of pork, consisting of 700 barrels, at \$12.25 per barrel; what is their commission at 13 per cent.?

  Ans. \$150.06.
- 5. What rate per cent. of brokerage does a broker charge who takes \$50 for investing \$10000?
- 6. What is the commission on the sale of 173cwt. of sugar, at \$8.95 per cwt., at 17 per cent.?

  Ans. \$29.03.5.
- 7. My factor at Mobile advises me that he has purchased on my account 37 bales of cotton, at \$107.75 per bale; what is his commission at  $\frac{3}{8}$  per cent.? Ans.  $$14.95\frac{1}{32}$ .
- 8. A consignee in London writes that he has purchased for his employer goods to the amount of 395£. 15s. 5d.; what is his commission at 21 per cent.?

  Ans. 8£. 18s. 150πd.
- 9. Paid G. Willis \$5.46 for exchanging \$364 of depreciated currency; what was the rate of brokerage?

Ans. 1½ per cent.

- 498. When the given amount includes both the brokerage or commission, and the sum to be invested.
- Ex. 1. A gentleman intrusts \$20050 to a broker in New York City, with instruction, after deducting his brokerage of ‡ per cent., to invest the balance in government bonds. What will be the sum invested, and how much will be the brokerage?

OPERATION.

 $$20050 \div $1.0025 = $20000$ , Investment. \$20050 - \$20000 = \$50, Brokerage.

Since the broker is entitled to 1 per cent. of the sum he invests, it is evident he requires \$1.0025 in order to invest \$1. Hence, the investment he can make will be as many dollars as \$20050 contains times 1.0025, or \$20000; which, being subtracted from the amount forwarded him, leaves as his brokerage \$50.

Rule. — Divide the given amount by 1 increased by the rate per cent. of brokerage or commission, expressed decimally, and the quotient will be the investment.

Subtract the investment from the given amount, and the remainder will be the brokerage or commission.

NOTE. — When the brokerage or commission and the rate per cent. of the same are given, the sum on which it is reckoned may be found as in Art. 349.

#### EXAMPLES.

2. An agent receives \$ 1976, which includes the sum he is to lay out in goods and also his commission at 4 per cent. How much of the amount is to be expended for the goods, and how much is his commission?

Ans. \$1900 for the goods; \$76 commission.

- 3. A broker receives \$8341.50, which includes a sum to be invested in railroad shares at \$83 each, and his brokerage at per cent. How many shares can be purchase, and how much is his brokerage?

  Ans. 100 shares; \$41.50 brokerage.
- 4. Sent to my agent, John Crowell, Rochester, \$8960, to purchase a quantity of flour; his commission is 2 per cent. on the purchase, which he is to deduct from the money; what is his commission?

  Ans. \$175.6824.
- 5. A town has levied a tax of \$5150, which sum includes the amount voted for the repairs of a bridge and the collector's commission of 3 per cent. How much was voted for the bridge, and how much does the collector receive for his commission?

  Ans. \$5000 for the bridge; \$150 commission.
- 6. What amount of money has been invested when the broker's charges, at 1½ per cent. for making the investment, amount to \$285?
- 7. A commission merchant purchases for me in New Orleans 34 boxes of sugar, pays for cartage and freight \$7.50, and his commission is  $1\frac{1}{2}$  per cent. on the amount of purchase, making the whole bill \$740.83\frac{3}{4}. How much was his commission, and, allowing 250 pounds to a box, how much a pound did he pay for the sugar? Ans. \$10.83\frac{3}{4} commission; \$0.08\frac{1}{2}\$ per lb.
- 8. Sent a cargo of flour to Liverpool, which my factor sold for 987£. 18s. 6d. He invested this sum in broadcloths, at 1£. 3s. 8d. per yard. His commission for selling the flour is 2½ per cent., and for purchasing the broadcloth 1½ per cent., and he is to receive his commissions, for selling and buying, out of the proceeds of the flour. Required the number of yards of broadcloth that I should receive.

  Ans. 801,3576,7yd.

### ACCOUNT OF SALES.

460. An Account of Sales is an account of goods sold, which a commission merchant or consignee, &c. furnishes to his employer. It contains the quantity and price of the goods disposed of, charges attending the sales, and the net proceeds, or the sum to which the owner, or consignee, is entitled after all charges are deducted.

The names of purchasers are often included in the account of sales, as in Example 1, and sometimes the mark of the box, bale, &c., as in Example 2.

410. The gross amount of sales is the sum of all the quantities at the prices given; and the net proceeds are found by deducting the commission at the given rate, and the other charges, from the gross amount.

Find the commission and net proceeds required in the following

### EXAMPLES.

 Account of sales of flour received by the steamer Calvert, from Baltimore, sold on account of James Taylor, Ellicott's Mills.

Date.	To whom sold.	Description.	Barrels.	Price.	
1857. Jan. 1.	William Hooper, Sidney Pope,	Superfine, Extra Eagle,	100 50	8.00 7.50	
" Jan. 3.	Albert Rollo, S. S. Coe,	Baltimore,	25 30	7.50 7.00	
Feb. 1.		Extra Eagle,	60 25	7.00 7.50	
0.1	d. 1. bumpson,	22120204 2208209	,		\$ 2180.00

#### CHARGES.

Commission on \$2180.00, at 21 per cent.,	\$49.05
Freight and drayage,	41.63
Advertising,	4.50

\$ 95.18

Net proceeds to credit of James Taylor, Errors excepted. **\$ 2275.**18

Boston, May 15, 1857.

NILES, MARVIN, & Co.

2. Sales of goods, made by Haskell, Fargo, & Co., on account of Jones, Boker, & Co., New York.

Date.	Marks.	• Description.	Gals.	Price.	
1857.					
Feb. 7.	Α.	4 casks Sperm Oil, Fall,	208	1.50	1
"	В. з.	13 " " " Winter,	650	1.40	1
44	Z.	23 " Whale Oil, Crude,	874	.70	ŀ
Mar. 3.	X	20 bbls. Olive Oil,	320	1.20	1
"	19.	10 " Turpentine, Spirits,	315	.60	1
Apr. 11.	Y. Z.	10 " Varnish,	315	.20	ł
ű 12.	L. L.	5 " Olive Oil,	1571	1.20	1
" 12.	4. 10,	2 " Varnish,	63	.20	
			•	•	<b>`\$</b>

#### CHARGES.

	Feb. 19.	Freight,	40	casks	@ \$	\$ 2.50,	\$
	Mar. 20.					2.00,	
•	Apr. 29.	"	17	"	"	1.50,	
	-						

Commission, 3 per cent., on \$ Cartage, cooperage, &c., Storage and insurance,

26.00 63.24

Net proceeds due J., B., & Co., Errors and omissions excepted.

Chicago, August 7, 1857.

HASKELL, FARGO, & Co.

## PROFIT AND LOSS.

411. Profit and Loss is the process by which merchants and others estimate their gains or losses in business transactions.

Gains and losses are usually reckoned on the prime or first cost of articles.

- 412. To find the selling price when the cost and the gain or loss per cent. are given.
- Ex. 1. If I buy cloth at \$4 per yard, for how much per yard must I sell it to gain 25 per cent.?

  Ans. \$5.



 $$4 \times 1.25 = $5, \text{ Ans.}$ 

Since the selling price is to be a gain on the cost of 25 per cent., evidently it must be 125 per cent. of the cost, or  $4 \times 1.25$ .

RULE. — Multiply the cost price by 1 increased by the gain per cent., or by 1 decreased by the loss per cent., expressed decimally, and the product will be the selling price required.

### EXAMPLES.

2. If I buy cloth at \$5 per yard, for what must I dispose of it per yard to lose 20 per cent?

Ans. \$4.

3. Bargained for cheese at \$8.50 per cwt. How must it be sold to gain 10 per cent.?

Ans. \$9.35 per cwt.

- 4. Molasses having been bought at 42 cents a gallon, and not proving so good as expected, was sold at a loss of 5 per cent. For what was it sold a gallon?
- 5. Bought a house and lot for \$2500; at what price must it be sold to gain 20 per cent.?

  Ans. \$3000.
- 413. To find the cost when the selling price and the gain or loss per cent. are given.
- Ex. 1. If I sell cloth at \$5 per yard, and thereby make 25 per cent., what was its first cost?

  Ans. \$4 per yard.

\$5.00 \div 1.25 = \$4, Ans. Since the gain is 25 per cent. of the cost, the selling price, \$5, is equal to the cost increased by 25 per cent. of the cost, or 1.25 of the cost. Therefore, the cost must be as many dollars as 5 contains times 1.25.

RULE. — Divide the selling price by 1 increased by the gain per cent., or by 1 decreased by the loss per cent., expressed decimally, and the quotient will be the cost.

## EXAMPLES.

If I dispose of cloth at \$4 per yard, and by so doing lose
 per cent., required the prime cost of the goods? Ans. \$5.

3. Sold 10 barrels of flour for \$96, and made 20 per cent. What was the prime cost a barrel?

Ans. \$8.

4. If  $27\frac{1}{2}$ cwt. of sugar be sold at \$12.50 per cwt., and there is gained 17 per cent., what was the prime cost per cwt.?

5. Sold wood at \$6.12½ per cord, and by so doing lost 12½ per cent. a-cord. What was the original cost per cord?

Ans. \$ 7.00.

- 414. To find the gain or loss per cent. when the cost and selling price are given.
- Ex. 1. If I buy cloth at \$4, and sell it at \$5 per yard, what per cent. do I gain?

  Ans. 25 per cent.

$$$5 - $4 = $1; 1.00 \div 4 = .25, \text{ or } 25 \text{ per cent.}, \text{ Ans.}$$

Since the difference between the selling price and the prime cost is \$ 1, the gain is \( \frac{1}{2} \) of the cost, or, expressed decimally, is .25 of the cost, or 25 per cent. of it.

RULE. — Divide the number denoting the gain or loss by that denoting the prime cost, and the quotient, expressed decimally, will be the gain or loss per cent. required.

#### EXAMPLES.

- 2. Bought cloth at \$7 per yard, and sold it at \$6.12\frac{1}{2}.

  What per cent. did I lose?

  Ans. 12\frac{1}{2} per cent.
- 3. Bought a chaise for \$200, and sold it for \$225. What per cent. did I gain?

  Ans. 12½ per cent.
- 4. I sell a house that cost me \$2500 at an advance of \$500. What per cent. do I gain?
- 5. Bought 24 yards of cloth for \$64.86437, and sold it at \$2.50 per yard. What per cent. is the loss?

Ans.  $7\frac{1}{2}$  per cent.

- 415. The selling price of goods and the rate per cent. of gain or loss being given, to find what the gain or loss per cent. would be, if sold at another price.
- Ex. 1. If I sell cloth at \$5 per yard, and thereby gain 25 per cent., what would have been my gain if I had sold it at \$7 per yard?

  Ans. 75 per cent.

$$\$5.00 \div 1.25 = \$4; \$7 - \$4 = \$3.$$
 We find the prime cost of the cloth per yard, when sold at \$5, by Art. 413, to

be \$4. We then find what would have been the gain per cent. on the cost, if it had been sold at \$7 per yard, by Art. 414, and obtain 75 per cent. as the answer.

RULE. — Find the prime cost (Art. 413), and then the gain or loss per cent. on this cost at the proposed selling price.

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### EXAMPLES.

- 2. If I sell cloth at \$7 per yard, and thereby gain 75 per cent., do I gain or lose if I sell the same at \$3 per yard?

  Ans. Lose 25 per cent.
- 3. Sold wheat at \$1.25 per bushel, and lost 15 per cent. What per cent. should I have gained had I sold it for \$1.647 17 per bushel?

  Ans. 12 per cent.
- 4. Sold wheat at \$1.647<sub>17</sub> per bushel, and gained 12 per cent. What per cent. should I have lost had I sold it for \$1.25 per bushel?
- 5. I sold a horse for \$75, and by so doing I lost 25 percent.; whereas, I ought to have gained 30 percent. How much was he sold under his real value?

  Ans. \$55.00.
- 6. When tea, sold at a loss of 25 per cent., brings \$1.25 per pound, what would be the gain or loss should it bring \$1.40 per pound?

  Ans. 16 per cent. loss.

## MISCELLANEOUS EXAMPLES.

- 1. Bought stocks whose par value is \$100 per share, and sold the same at a premium of 15 per cent., and thereby gained \$120. How many shares were there?

  Ans. 8 shares.
- 2. Mining stock of a par value of \$250 per share was bought at 9 per cent. premium, and afterwards sold at a loss of \$25 per share on the price paid. At what rate per cent. was it sold?

  Ans. 1 per cent. discount.
- 3. How much greater income may be realized yearly from \$19200 invested in 7 per cent. stocks purchased at 96 per cent., than from the same amount in 5 per cent. stocks purchased at 80 per cent.?
- 4. The par value of the shares in a certain manufacturing company is \$250 each, and the regular yearly dividends are \$15 a share. At what price should the shares be bought that the investment may pay 10 per cent. interest?

Ans. \$ 150 each.

- 5. A broker charges me 1½ per cent. brokerage for purchasing some uncurrent bank-bills at 20 per cent. discount. Of these bills four of \$50 each become worthless, but the remainder I dispose of at par, and thus make by the operation \$364. What was the amount of the bills?

  Ans. \$3000.
- 6. A commission merchant sold a consignment of cotton for \$5640, and after deducting his commission, and \$76.50 for freight, storage, &c., remitted to his consignor \$5422.50 as the net proceeds. What per cent. was his commission?

Ans. 2½ per cent.

- 7. A horse which I bought for 30 per cent. less than his real worth, having become injured, I sold him for 25 per cent. less than what he cost, and thereby lost \$55 of his original value.

  What did I get for the horse?

  Ans. \$60.784\$.
- 8. A watch, which cost me \$30, I have sold for \$35, on a credit of 8 months. What did I gain by the bargain?

Ans. \$  $3.653\frac{1}{13}$ . \$ 112: but 15 gallon

- 9. Bought a hogshead of molasses for \$112; but 15 gallons having leaked out, the remainder was sold at \$2.216\frac{2}{3}\$ a gallon. What per cent. is the loss?

  Ans. 5 per cent.
- 10. A hogshead of molasses was bought for a certain sum; but 15 gallons having leaked out, the remainder was sold at \$2.216\frac{2}{3}\$ a gallon, and thereby the loss was 5 per cent. on the cost. What was the cost?

  Ans. \$112.
- 11. Bought 50 barrels of flour at \$9.00 per barrel, but a part of it having been damaged, half of it was sold at a loss of 10 per cent., and the remainder at \$9.50 per barrel. How much was lost by the operation?

  Ans. \$10.
- 12. My agent purchases flour for me, which, with his commission at 2½ per cent., cost \$ 6135, and I dispose of it at 20 per cent. advance upon the price he paid. What did I make by the operation, allowing freight and storage to have been \$ 31.63?
- 13. I have remitted to my correspondent a certain sum of money, which he invested in iron; and having reserved to himself 2½ per cent. on the purchase, which amounted to \$90, he purchased the iron at \$95 per ton. Required the sum remitted, and the quantity of iron purchased.

Ars. Sum remitted, \$3690. Iron purchased, 37T. 17cwt. 3qr. 1439lb.

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## PARTNERSHIP, OR COMPANY BUSINESS.

416. PARTNERSHIP is the association of two or more persons in business, with an agreement to share the profits and losses in proportion to the amount of the capital stock, or the value of the labor and experience of each.

Partners are the persons associated in business.

Company, or Firm, is the general name of the business association.

Capital, or Joint Stock, is the money or property invested in the company or firm.

Dividend is the profit or loss on the shares of the capital, or joint stock.

Note. — Partnership is an application of the principle of distributive or partitive proportion (Art. 839).

- 417. To find each partner's share of the profit or loss, when there is no regard to time.
- Ex. 1. Three men, A, B, and C, enter into partnership for two years, with a capital of \$ 1080. A puts in \$ 240, B \$ 360, and C \$ 480. They gain \$ 54. What is each man's part of the gain?

A's stock, \$240 \$240 
$$\times$$
 .05 = \$12, A's gain.  
B's " 360 360  $\times$  .05 = 18, B's gain.  
C's " 480 480  $\times$  .05 = 24, C's gain.  
Entire stock, \$ $1080$  Proof, \$54, entire gain.  
\$54.00  $\div$  1080 = \$0.05, gain on \$1.

Since the entire stock is \$ 1080 and the entire gain \$ 54, the gain on every \$ 1 of stock will be as many dollars as 54 contains times 1080, or \$ 0.05 on every \$ 1 of stock. Then, each man's stock multiplied by .05 gives his part of the entire gain. The same result also may be obtained, as follows,

```
$1080:$240::$54:$12, A's gain.
$1080:$360::$54:$18, B's gain.
$1080:$480::$54:$24, C's gain.
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Proof, \$54, entire gain.

RULE. — The entire gain or loss, divided by the number denoting the

entire stock, will give the gain or loss on each dollar of stock; and each partner's stock, multiplied by the number denoting the gain on \$1, will give his share of the entire gain or loss. Or,

As the whole stock is to each partner's stock, so is the whole gain or .

loss to each partner's gain or loss.

### EXAMPLES.

2. Jones, Weston, and Sprague traded in company, with a capital of \$10000; Jones put in \$3000, Weston \$2000, and Sprague \$5000; they gained \$4000. What was each man's part of the gain?

Ans. Jones's part, \$1200; Weston's part, \$800; Sprague's part, \$2000.

3. Two merchants, C and D, engaged in trade; C put in \$6780, and D put in \$12000; they gain \$2000. What was each man's part?

Ans. C's part, 722.044; D's part, \$ 1277.956.

- 4. Harvey, Blake, and Horsford entered into partnership with a capital of \$11000, of which Harvey put in \$2500, Blake \$3000, and Horsford \$5500; they lost by trading 5 per cent. on their capital. What was each partner's share of the loss?
- 5. Elliott, Mayhew, and Griswold engaged together in a speculation. Of the money employed Elliott furnished \$500; Mayhew \$350, and Griswold a cart and two horses; they gained \$332.50, of which Griswold's part was \$120. What were Elliott's and Mayhew's parts of the gain, and what was the value of Griswold's part of the stock?

Ans. Elliott's gain, \$125; Mayhew's gain, \$87.50; Griswold's stock, \$480.

6. A, B, and C traded in company; A put in \$5000, B put in \$6500, and C put in \$7500; they gain 40 per cent. on their capital, but receive the whole amount of their gains in bills, for which they are obliged to allow a discount of 10 per cent. How much was each man's net gain?

Ans. A's gain, \$1800; B's gain, \$2340; C's gain, \$2700.

7. A, B, C, and D are in partnership, with a joint capital of \$40,000; on dividing their profits, it is found that A's share is \$2000, B's share \$4500, C's share \$2500, and D's share \$1500. What was each partner's stock?

8. A, B, and C were associated in trade; A's part of the general stock was \$2000, B's part \$3000, and C's part \$7500. On dividing the profits, it was found that A's and B's gain together amounted to \$1000, which was \$500 less than C's gain. What was the gain of each?

Ans. A's gain, \$400; B's gain, \$600; C's gain, \$1500.

9. A, B, and C own a ship together, which cost them \$30000; of which A paid an unknown sum, B paid 1½ as much, and C paid 1½ as much. The profits were 25 per cent of the cost of the ship. What was each man's gain?

Ans. A's gain, \$ 2000; B's gain, \$ 3000; C's gain, \$ 2500.

10. Walker, Edwards, and Armstrong are partners, whose respective shares of joint stock are as the fractions  $\frac{1}{3}$ ,  $\frac{1}{4}$ , and  $\frac{1}{5}$ . They gain \$50000. If, on dividing the profits, Armstrong relinquishes his part of the gain, how much will each of the others receive?

Ans. Walker, \$28571.425; Edwards, \$21428.571.

- 418. To find each partner's share of the profit or loss, when the stock is employed for different periods of time.
- Ex. 1. A and B are associated in trade. A has furnished of the joint stock \$420 for 5 months, and B has furnished \$350 for 8 months; their net profits are \$84. What is each man's share of the gain? Ans. A's share, \$36; B's, \$48.

 $\$420 \times 5 = \$2100$   $\$2100 \times .01\$ = \$36$ , A's gain.  $\$350 \times 8 = \$2800$   $\$2800 \times .01\$ = \$48$ , B's gain. Proof, \$84, entire "

It is evident that \$420 for 5 months is the same as \$420  $\times$  5 = \$2100 for 1 month, since \$2100 would gain as much in 1 month as \$420 in 5 months; and \$350 for 8 months is the same as \$350  $\times$  8 = \$2800, for 1 month. The question then is the same as if A had furnished \$2100 and B \$2800 for equal times. Then, if \$2100 +\$2800 = \$4900, gain \$84, \$1 will gain  $\frac{1}{4\sqrt{100}}$ 00 +\$84 = \$0.01\frac{1}{2}, and \$2100  $\times$  .01\frac{1}{2} = \$36, A's gain; \$2800  $\times$  .01\frac{1}{2} = \$48, B's gain. The same results may be obtained, as follows,

\$4900: \$2100:: \$84: \$36, A's gain. \$4900: \$2800:: \$84: \$48, B's gain.

Proof, \$84, entire gain.

RULE. — Multiply each partner's stock by the time it was in trade, and divide the entire gain or loss by the sum of the several products; by the quotient multiply the product of each partner's stock and time, and the result will be his share of the gain or loss. Or,

Multiply each partner's stock by the time it was in trade; then, as the sum of these products is to each product, so is the whole gain or loss to

each partner's gain or loss.

## EXAMPLES.

2. Goodwin commenced business January 1, with a capital of \$3200; May 1, he took Blunt into partnership, with a capital of \$4200; and at the end of the year they had gained \$240. What was each partner's share of the gain?

Ans. Goodwin's gain, \$128; Blunt's gain, \$112.

- 3. Three men hire a pasture in common, for which they are to pay \$26.40. A put in 24 oxen for 8 weeks, B put in 18 oxen for 12 weeks, and C put in 12 oxen for 10 weeks. What ought each to pay?
- 4. Barclay, Hickman, and Oliver are partners. Barclay furnishes of the capital \$300 for 5 months, Hickman \$400 for 8 months, and Oliver \$500 for 3 months; they gain \$200. After paying \$50 for advertising and \$50 for agency, what will be each partner's share of the net profits?

Ans. Barclay's share, \$24.19 $\frac{1}{31}$ ; Hickman's, \$51.61 $\frac{9}{31}$ ; Oliver's, \$24.19 $\frac{1}{41}$ .

5. A, B, and C engaged in partnership, with a joint capital of \$1000, A putting in stock for 7 months, B for 8 months, and C for 12 months. Of the profits A's part was \$21; B's, \$40; and C's, \$24. Required the capital each put in.

Ans. A, \$300; B, \$500; C, \$200.

- 6. White and Daniels traded in company, with a joint stock of \$6300. White's money having been employed 12 months, and Daniel's 8 months, on dividing profits, each had gained exactly the same sum. How much of the capital did each furnish?
- 7. Three men engage in partnership for 20 months. A at first put into the firm \$4000, at the end of four months he put in \$500 more, and at the end of 16 months he took out \$1000; B at first put in \$3000, at the end of 10 months he took out \$1500, and at the end of 14 months he put in



\$3000; C at first put in \$2000, at the end of 6 months he put in \$2000 more, at the end of 14 months he put in \$2000 more, and at the end of 16 months he took out \$1500; they had gained by trade \$4420. What is each man's share of the gain?

Ans. A's gain, \$ 1680; B's gain, \$ 1260; C's gain, \$ 1480.

8. Grover and Thorndike are associated in trade, Grover contributing of the capital \$12000, and Thorndike \$18000. At the end of 6 months they reduce the joint stock \$5000, by each withdrawing an equal sum. 3 months afterwards Thorndike withdraws \$6000, and Grover \$1000. The business proving a losing one, they dissolve copartnership at the end of the year. Required what part of the stock then remaining, which was only \$15000, belonged to each of the partners?

Ans. To Grover, \$7276.69 $\frac{93}{103}$ ; Thorndike, \$7723.30 $\frac{10}{103}$ .

9. John Jones, Samuel Eaton, and Joseph Brown formed a partnership, under the firm of Jones, Eaton, & Co., with a capital of \$10,000; of which Jones put in \$4000, Eaton put in \$3500, and Brown put in \$2500; but at the end of 6 months Jones withdrew \$2000 of his stock, and at the end of 8 months Eaton withdrew \$1500 from the firm; but at the end of 10 months Brown added \$2000 to his stock. At the end of 2 years they found their gains to be \$1041.80. What was the share of each man?

Ans. Jones's gain, \$300.51 $\frac{1}{13}$ ; Eaton's gain, \$300.51 $\frac{1}{13}$ ; Brown's gain, \$440.76 $\frac{2}{13}$ .

10. James Bradshaw commenced trade, January 1, 1856, with a capital of \$10000, and after some time formed a partnership with John Parkman, who contributed to the joint stock \$2800. In course of time they admitted into the firm Joseph Delano, with a stock of \$3600. On making a settlement, January 1, 1857, it was found that Bradshaw had gained \$2250; Parkman, \$420; and Delano, \$405. How long had Parkman's and Delano's money been employed in trade, and what rate of interest per annum had each of the partners gained on their stock?

Ans. Parkman's, 8 months; Delano's, 6 months. Gained 222 per cent. interest.

#### BANKRUPTCY.

419. BANKRUPTČY refers to business failures and inability to meet pecuniary liabilities.

A bankrupt, or insolvent, is one who fails or becomes unable to pay his debts.

An assignee is a person selected to take charge of the property and effects of a bankrupt, to convert the same into cash, and, after deducting the necessary expenses of settling, to divide the net proceeds, as the law requires, among the creditors.

The distribution is generally made pro rata, each creditor receiving according to his respective demand, or just claim.

- 420. To find each creditor's dividend or share of the net proceeds of an insolvent estate.
- Ex. 1. A bankrupt owes to A \$500, to B \$1200, and to C \$4300; and the net cash proceeds of his estate amount to only \$1500. How much does he pay on \$1, and what dividend does each creditor receive?

Ans. 25 cents of a dollar. A receives \$125; B, \$300; and C, \$1075.

#### OPERATION.

A's claim, \$ 500 \$  $500 \times .25 = $ 125$ , A's share. B's " \$ 1200 \$  $1200 \times .25 = $ 300$ , B's share. C's " \$ 4300 \$  $4300 \times .25 = $ 1075$ , C's share.

Entire claims, \$6000 Proof, \$1500, entire proc'ds.  $$1500.00 \div 6000 = $0.25$ , or 25 cents on \$1.

RULE. — Divide the net proceeds of the insolvent estate by the number denoting the total amount of its indebtedness, to find the sum it pays on each dollar of the debts.

Multiply each man's claim by the sum the estate pays on a dollar, to obtain each man's dividend.

#### EXAMPLES.

2. Clarke, Soule, & Co. have failed. Their liabilities are \$63500, their assets have a cash value of \$52384, and the expenses of settling are \$1584. How much can they pay on a dollar, and what dividend should John Dayton receive, whose claim is \$8361.55? Ans. 80 cents on a dollar; \$6689.24.

- 3. A merchant failing in trade owes A \$600, B \$760, C \$840, and D \$800. The net proceeds of his effects are \$2275. What dividend does each of his creditors receive?
- Ans. A, \$455; B, \$576.33\frac{1}{3}; C, \$637; D, \$606.66\frac{2}{3}.

  4. J. Bonney owes A \$400, B \$300, and C \$1000. His effects are worth \$600. What sum can he pay each of his creditors?
- 5. A manufacturing company becomes insolvent. Its indebtedness amounts to \$300000. Its assets consist of factory buildings and machinery worth \$180000, stock worth \$40000, and bills receivable good for \$12875. The charges of the court of insolvency and the assignee will amount to 31 per cent. on the amount distributed to the creditors. How much will the company pay on a dollar, and what will be Amos Henderson's dividend on a claim of \$1360.60?

Ans. 75 cents on a dollar; \$1020.45.

### TAXES.

421. A TAX is a sum of money assessed on individuals by government, corporations, societies, districts, &c.

Taxes for government purposes are imposed on property, and in most States on persons.

A poll or capitation tax is one without regard to property, on the person of each male citizen liable by law to assessment. A person thus liable is termed a poll.

- 422. Immovable property, such as lands, mills, houses, &c., is called real estate. All other property, such as money, notes, mortgages, cattle, shipping, furniture, &c., is called personal property.
- 423. In assessing taxes, it is necessary to have an inventory of taxable property; and, if a levy on the polls is to be included, there should be also a complete list of taxable polls.

424. The method of assessing taxes, though not precisely the same in all the States, is yet in most of them virtually the same.

In some of the States, however, the public schools are supported in whole or in part by rate bills; that is, the expenses of the schools, in whole or in part, are apportioned among the inhabitants patronizing the same, according to the number of days of each pupil's attendance.

Note. — In Massachusetts one sixth part of the whole sum to be raised is assessed upon the polls; provided the whole poll tax assessed in any one year upon any individual for town, county, and State purposes, except highway taxes, shall not exceed one dollar and fifty cents.

- 425. To assess a state, county, town, or other tax.
- Ex. 1. The inhabitants of a certain town are to be taxed \$4109. The real estate of the town is valued at \$493000, and the personal property at \$177000. There are 506 polls, each of which is to be taxed \$1.50. What is the tax on each dollar of property? What is J. B. Tewksbury's tax, whose real estate is valued at \$3700, and his personal at \$2300, he paying for 2 polls?

OPERATION.

- $$1.50 \times 506 = $759$ , amount assessed on the polls.
- \$493000 + \$177000 = \$670000, amount of taxable property.
- \$4109 \$759 = \$3350, amount to be assessed on property.
- $$3350 \div 670000 = $.005$ , tax to be assessed on each dollar.
- \$3700 + \$2300 = \$6000, Tewksbury's taxable property.
- $$6000 \times .005 = $30$ , Tewksbury's property tax.
- $$1.50 \times 2 = $3$ , Tewksbury's poll tax.
- \$30 + 3 = \$33, amount of Tewksbury's tax.

RULE. — Multiply the number of taxable polls by the tax on each poll; and the product, subtracted from the whole sum to be raised, will give the sum to be raised on the property.

The sum to be raised on property, divided by the whole taxable property, will give the sum to be paid on each dollar of property taxed.

Each man's taxable property, multiplied by the number denoting the sum to be paid on \$1, with his poll tax added to the product, will give the amount of his tax.

NOTE. — The operation of assessing taxes may be facilitated by finding the tax on \$2, \$3, &c. at the rate of taxation on \$1, before making the assessment on the inhabitants of the town, &c., and arranging the numbers as in the following

TABLE.

Prop.	Tax.	Prop.	Tax.	Prop.	Tax.	Prop.	Tax.
\$1	\$ 0.005	\$ 20	\$ 0.10	\$ 300	\$ 1.50	\$ 4,000	\$20.00
2	0.010	80	0.15	400	2.00	5,000	25.00
8	0.015	40	0.20	500	2.50	6,000	30.00
4	0.020	50	0.25	600	8.00	7,000	85.00
5	0.025	60	0.30	700	8.50	8,000	40.00
6	0.030	70	0.35	800	4.00	9,000	45.00
7	0.035	80	0.40	900	4.50	10.000	50.00
8	0.040	90	0.45	1.000	5.00	20,000	100.00
9	0.045	100	0.50	2,000	10.00	30,000	150.00
10	0.050	200	1.00	3,000	15.00	40,000	200.00

#### EXAMPLES.

- 2. What is Samuel Forster's tax, by the above table, whose property, real and personal, is valued at \$1310, and who pays for 7 polls a tax of \$1.50 each?

  Ans. \$17.05.
- 3. What is the tax of a non-resident, having property in the same town, worth \$415.35?
- 4. A tax of \$14018.90 is to be levied on a certain city. The property is valued at \$3506300; and there are 3500 polls, each of which is taxed \$1 each. What is the assessment on a dollar, and what is A's tax, who has property worth \$29010, and pays for 2 polls? Ans. \$.003 on \$1; A's tax, \$89.03.
- 426. To find what sum must be assessed to yield a given net amount.
- Ex. 1. What sum must be assessed to yield a net amount of \$6240, the collectors receiving 2½ per cent. commission for collecting?

  Ans. \$6400.

\$ 6240 \div .975 \div \$ 6400. Since the commission is 2\frac{1}{2} per cent., the net value of each dollar of assessment will be \$ 0.975, and the amount to be assessed to net \$ 6240 will therefore be as many dollars as 6240 contains times .975. Hence,

The given net amount divided by 1 decreased by the rate of commission, expressed decimally, will give the amount to be assessed.

#### EXAMPLES.

2. What sum must be assessed to net \$10450, allowing the collector receives 5 per cent. for collecting? Ans. \$11000.

3. Allowing the net sum to be raised by a society to be \$9700, and the allowance for collection to be 3 per cent., what is the gross amount to be assessed, and how much will be the cost of collection?

Ans. Assessment, \$10000; cost of collection, \$300.

- 4. The taxable property in a certain district containing 450 polls is \$756000. It is proposed to raise \$18000 for building a union school-house. If the poll tax be limited to \$1.50 a poll, and the cost of collecting be 3 per cent., what will be the required assessment on a dollar, and how much will be A's tax, who pays for 3 polls, and has property to the amount of \$15600.

  Ans. Tax on \$1, \$.0236; A's tax, \$372.66.
- 427. To apportion school expenses according to each pupil's attendance, or to make out rate bills.
- Ex. 1. The expenses of a certain district school are \$464, and the aggregate of attendance 9280 days. What is A's tax, who has sent pupils amounting to 157 days?

\$  $464.00 \div 9280 = \$0.05$ \$  $0.05 \times 157 = \$7.85$ . Since 9280 days' attendance cost \$464, 1 day's attendance will cost as many dollars as 464 contains times 9280, and 157 days will cost 157 times as much as 1 day. Hence,

The whole expenses of the school, less the public money, if any, divided by the number of days' attendance, will give the rate for each day's attendance.

The rate for each day, multiplied by the number of days of attendance of each patron's pupils, will give the amount of his rate bill.

### EXAMPLES.

2. If a district expends for a teacher's salary \$500, for his board \$150, for repairs of school-house \$30; the public money being \$350, and the whole number of days of attendance 5500, what is the rate per day, and what is A's bill, who sends 2 pupils 60 days each, and 1 pupil 30 days?

Ans. Rate per day, \$0.06; A's bill, \$9.00.

3. In a certain district the teacher's wages amounted to \$150, the fuel cost \$18.50, the public money received was \$63.50, and the number of days' attendance was 3000. What was A's rate bill, whose pupils attended 121 days; and B's, whose pupils attended 173 days?

### GENERAL AVERAGE.

428. GENERAL AVERAGE signifies a contribution ratably made to a general loss by the three great mercantile interests, vessel, freight, and cargo, — when, on account of a common peril, one or more of these has been voluntarily sacrificed, in whole or in part, to effect the preservation of the rest.

Jettison is the term applied to the part of the vessel or cargo cast overboard, or otherwise sacrificed, to save the remainder.

- 429. Particular average means an average to which contribute only "some parts of the property when severed from the whole; as in the case of a boat-load saved from peril by a sacrifice, to which only the boat and its contents contribute." Practically, it hardly needs to be discriminated from general average, since it is adjusted upon its contributory interest by the application of the same principles.
- 430. To constitute a valid claim to general average, there are three essentials, all of which must be present: 1. A common peril impending at the time; 2. A voluntary and premeditated sacrifice of some property for the purpose of saving other property; 3. The success of the endeavor. (Parsons on Mercantile Law.)
- 431. In adjusting a general average, the property sacrificed, as well as that saved, is regarded as contributory to the general loss. The entire value of the freight, however, is not contributory; ½ of its value in New York, and ⅓ of its value in other parts of the United States, being reserved for seamen's wages.

The goods, whether lost, injured, or saved, are valued at the price they would have brought in ready money on the vessel's arriving at her port of destination, unless the average is adjusted at the port of lading, when they are valued at the invoice price.

In compensating for the expenses incurred in repairing damages done to the vessel, only  $\frac{2}{3}$  of the cost is allowed, as in general the new material is  $\frac{1}{3}$  better than the old.

- 432. To adjust the general average of losses at sea.
- Ex. 1. The ship America sailed on May 16, 1857, for New

Orleans, with an assorted cargo. In consequence of a violent gale in the Gulf of Mexico, the captain was obliged to throw overboard a portion of the cargo, amounting in 'value to \$4465.50, and the necessary repairs of the vessel cost \$423. The contributory interests were as follows: vessel, \$30000; gross freight, \$6225; cargo shipped by R. S. Davis & Co., \$3650; by Henry Mason, \$6500; by G. T. Sampson, \$2000; by J. Francis & Son, \$550; by Morton Brothers, \$5450; and by Sanborn & Carter, \$8500. Of the cargo thrown overboard, there belonged to Henry Mason the value of \$3000, and to Morton Brothers the remainder, \$1465.50. The cost of detention in port in consequence of repairs was \$116.50. How ought the loss to be apportioned among the contributory interests?

#### OPERATION.

CONTRIBUTORY IN	TERESTS.	LOSS FOR GENERAL BENEFIT.				
Vessel,	\$ 30000	Thrown overboard,	<b>\$ 4465.50</b>			
Cargo,	26650	Cost of detention,	116.50			
Freight, less $\frac{1}{3}$ ,	4150	Repairs, less $\frac{1}{3}$ ,	282.00			
Entire contrib. int	, \$60800	Entire loss,	\$ 4864.00			

 $$4864.00 \div 60800 = .08$ , the loss per cent.

- \$ 30000  $\times$  .08 = \$ 2400, am't payable by vessel. 4150  $\times$  .08 = 332, am't payable by freight.
  - $3650 \times .08 = 292$ , am't payable by R. S. Davis & Co.
  - $6500 \times .08 = 520$ , am't payable by Henry Mason.
  - $2000 \times .08 = 160$ , am't payable by G. T. Sampson.
  - $550 \times .08 = 44$ , am't papable by J. Francis & Son.
  - $5450 \times .08 = 436$ , am't payable by Morton Brothers.
  - $8500 \times .08 = 680$ , am't payable by Sanborn & Carter.

Proof, \$4864, entire amount payable.

\$ 2400.00 — \$ 398.50 = \$ 2001.50, balance payable by vessel. 3000.00 — 520.00 = 2480.00, bal. rec'ble by H. Mason. 1465.50 — 436.00 = 1029.50, bal. rec'ble by Morton Bro's.

Since the vessel lost \$ 116.50 + \$ 282, = \$ 398.50, that amount is deducted in finding the net amount the vessel must contribute to the general loss. Henry Mason lost \$ 520; the amount payable by him is therefore made so much less on that account; and Morton Brothers also, having lost \$ 436, have the amount of their contribution lessened by that sum.

- RULE. Multiply each contributory interest by the loss per cent., and the product will be its contribution to the general loss.
- Ex. 2. The ship Hope, in her passage from Liverpool to New York, was crippled in a storm, in consequence of which the captain had \$6500 worth of the cargo thrown overboard, and the necessary repairs of the vessel cost \$1050. The charges for board of seamen, pilotage, and dockage amounted to \$142. The contributory interests were: vessel, \$31500; gross amount of freight, \$4160; cargo shipped by Manning & Brother, \$2145; by Anderson & Fisk, \$1460; by Smidt & Huber, \$960; by Greenwood, Laporte, & Co., \$670; and by Allermann, Ritter, & Herr, \$1000. In adjusting the general average in New York, the deduction made from the gross amount of freight on account of seamen's wages was one half. Required the several shares of the general loss.

# EQUATION OF PAYMENTS.

- 433. EQUATION OF PAYMENTS is the process of finding the average or mean time when the payments of several sums, due at different times, may all be made at one time, without loss either to the debtor or creditor.
- 434. A strictly accurate method of determining, by Arithmetic, the true average time for the payment of more than two sums, due at different times, it is believed, has not yet been discovered; and, even when there are only two sums, the accurate method is the translation of an Algebraic Formula. The true equated time, however, may be readily found by Algebra.

Mercantile usage, however, gives its sanction to another method, and one which is not entirely correct though, when the sums are small and the terms short, the variation from the exact truth is practically of no consequence. But this method, being far the most convenient of application, is adopted among business men.

**435.** To find the average or mean time of paying several debts due at different dates.

Ex. 1. A owes B \$19, \$5 of which is to be paid in 6 months, \$6 in 7 months, and \$8 in 10 months. What is the average time of paying the whole?

Ans. 8 months.

	OPERATION.
5	$\times$ 6 = 30
6	$\times$ 7 = 42
8	$\times$ 10 = 80
$\overline{19}$	19) $\overline{152}$ (8 months.
	152

The interest of \$5 for 6 months is the same as the interest of \$1 for 30 months; and of \$6 for 7 months, the same as of \$1 for 42 months; and of \$8 for 10 months, the same as of \$1 for 80 months. Hence, the interest of all the sums to the time of pay-

ment is the same as the interest of \$1 for 30mo. +42mo. +80mo. = 152 months. Now, if \$1 require 152 months to gain a certain sum, \$5+\$6+\$8=\$19 will require  $\frac{1}{19}$  of 152 months; and 152mo.  $\div$  19 = 8 months, the average or mean time for the payment of the whole.

2. Purchased goods of Kendall & White at different times, and on various terms of credit, as by the statement annexed. What is the mean time of payment?

Due May 1, \$375.50

" June 20,  $168.75 \times 50 = 8437.50$ 

" June 4,  $386.25 \times 34 = 13132.50$ " Aug. 11,  $144.60 \times 102 = 14749.20$ 

" July 7,  $\frac{386.90}{\$1462.00} \times \frac{102}{67} = \frac{14749.20}{25922.30}$ 

 $62241.50 \div 1462.00 = 42\frac{1675}{2924}$  days.

May 1 + 43 days = June 13, Ans.

We first find the time when each of the bills will become due. Then, since it will shorten the operation and not change the result, we take the first time when any bill becomes due, instead of its date, for the point from which to compute the average time. Now, since May 1 is the period from which the average time is computed, no time will be reckoned on the first bill, but the time for the payment of the second bill extends 50 days beyond May 1, and we multiply its amount by 50. Proceeding in the same manner with the remaining

bills, we find the average time of payment to be 43 days, nearly, from May 1, or on June 13.

Rule. — Multiply each payment by its own time of credit, and divide the sum of the products by the sum of the payments.

NOTE 1. — When the date of the average time of payment is required, as in Example 2, find the time when each of the sums becomes due. Multiply each sum by the number of days intervening between the date of its becoming due and the earliest date on which any sum becomes due. Then proceed as in the rule, and the quotient will be the average time required, in days forward from the date of the earliest sum becoming due.

Note 2. — In the result, it is customary, if there be a fraction of a day less

than 1, to reject it; but if more than 1, to reckon it as 1 day.

In practice the work may be somewhat abridged, without varying materially the result, by disregarding, in performing the multiplications, the cents in the several sums, when they are less than 50, and by calling them \$1, when more than 50.

When a payment is made at the time of purchase, it has no product, but 'it must be added with the other products in finding the average time.

Note 3. — The method of the rule, or that generally adopted by merchants, as has been intimated (Art. 384), is not perfectly correct. For if I owe a man \$200, \$100 of which I am to pay down, and the other \$100 in two years, the equated time for the payment of both sums would be one year. It is evident that, for deferring the payment of the first \$100 for one year, I ought to pay the amount of \$100 for that time, which is \$106; but for the \$100 which I pay a year before it is due, I ought to pay the present worth of \$100, which is \$94.83\frac{3}{2}\_1, and \$106 + \$94.83\frac{3}{2}\_3 = \$200.83\frac{3}{2}\_3, whereas, by the mercantile method of equating payments, I only pay \$200.

# EXAMPLES.

- 3. There is owing a merchant \$1000; \$200 of it is to be paid in 3 months, \$300 in 5 months, and the remainder in 10 months. What is the equated time for the payment of the whole sum?

  Ans. 7mo. 3d.
- 4. I have bought a farm for \$6500; \$2000 of which is to be paid down, \$500 in one year, and the remainder in 2 years. But if a note for the whole amount had been preferred, in what time would it have become due?
- 5. A owes B \$300, of which \$50 is to be paid in 2 months, \$100 in 5 months, and the remainder in 8 months. What is the equated time for the payment of the whole sum?

  Ans. 6 months.
- 6. I have sold H. W. Hathaway several bills of goods, at different times, and on various terms of credit, as by the following statement. What is the average time for the payment of the whole?

Jan.	1, a	bill	an	ounting to	\$ 600,	on	4 months.
Feb.	7,	"		"	370,	on	5 months.
March	15,	"	٠	"	560,	on	4 months.
April	20,	"		"	420,	on	6 months.
_							Ans. July 11.

7. Purchased goods of J. D. Martin, at different times, and on various terms of credit, as by the statement annexed. What is the equated time of paying for the same?

March 1, 1855, a bill amounting to \$675.25, on 3 months. July 4, 376.18, on 4 months. Sept. 25, 821.75, on 2 months. Oct. 1. 961.25, on 8 months. Jan. 1, 1856. " 144.50, on 3 months. . Feb. 10, " 811.30, on 6 months. 567.70, on 5 months. " Mar. 12, " 369.80, on 4 months. April 15, " Ans. March 16, 1856.

- 436. To find the time of paying the balance of a debt, when partial payments have been made before the debt is due.
- Ex. 1. I have bought of Leonard Johnson goods to the amount of \$1728, on 6 months' credit. At the end of one month I pay him \$300, and at the end of 5 months, \$800. How long, in equity, after the expiration of 6 months, should the balance remain unpaid?

  Ans. 3mo. 20d.

$$300 \times 5 = 1500$$

$$800 \times 1 = 800$$

$$1100$$

$$2300$$

$$$1728 - $1100 = $628;$$

$$2300 \div 628 = 3mo. 20d.$$

The interest on the \$300 for 5 months is equal to the interest of \$1 for 1500 months, and the interest of \$800 for 1 month is equal to that of \$1 for 800 months; and thus the interest on both partial payments, at the expiration of the 6 months, is equal to the interest on the interest on the interest of the could be interest. To equal this credit

6 months, is equal to the interest of \$1 for 1500 + 800 = 2300 months. To equal this credit of interest, the balance of the debt, which we find to be \$628, should remain unpaid, after the 6 months,  $\frac{1}{628}$  of 2300 months, or 3 months and 20 days.

RULE. — Multiply each payment by the time it was made before it becomes due, and divide the sum of the products by the balance remaining unpaid; and the quotient will be the required time.

# EXAMPLES.

- 2. A merchant has \$144 due him, to be paid in 7 months, but the debtor agrees to pay one half ready money, and two thirds of the remainder in 4 months. What time should be allowed for paying the balance?

  Ans. 2y. 10mo.
- 3. March 23, 1856, I sold John Morse goods to the amount of \$8000 on a credit of 8 months. April 5, he paid me \$1200; July 4, \$1500; September 25, \$1800; October 1, \$1000; November 20, \$500. When, in equity, should I receive the balance?
- 4. There is due to a merchant \$800, one sixth of which is to be paid in 2 months, one third in 3 months, and the remainder in 6 months; but the debtor agrees to pay one half down. How long may the debtor retain the other half so that neither party may sustain loss?

  Ans. 8\frac{2}{3} months.
- 5. I have sold Charles Fox goods to the amount of \$3051, on a credit of 6 months, from September 25, 1856. October 4, he paid \$476; November 12, \$375; December 5, \$800; January 1, 1857, \$200. When, in equity, ought I to receive the balance?

  Ans. October 8, 1857.

#### AVERAGING OF ACCOUNTS.

437. An Account Current is a statement of the mercantile transactions of one person with another, when immediate payments are not made.

An account is marked Dr. on the left, to indicate that the person with whom the account is kept is *debtor* for the items on that side; and is marked Cr. on the right, to indicate that he is *creditor* for the items on that side.

Accounts current are generally made up or settled at the end of every six months or year.

- 438. To find the equated time when the balance of an account current will be due.
- Ex. 1. In the following account when did the balance become due, the merchandise articles being on 6 months' credit?

  Ans. December 22, 1856.



•

Dr. Messrs. James Dutton & Co. in account with David Hale. Cr.

1856.	*	1.	1856.	i		1
	merchandise,	\$ 96.51	Jan.30.	By	cash,	\$ 240.00
" 18. "	"	57.67	Apr. 8.	12	46	48.88
Feb. 4. "	cash paid draft,	80.00	May22.	46	" .	50.00
" " "	merchandise,		July 7.	"	Note, # June 22, 6mo.	410.01
" 9. "	cash paid draft,	50.26		i	• • • •	·
Mar. 3. "	merchandise,	154.46	1	1		ľ
" 24. "	u	42.80	l l			i
Apr. 9. "		23.60		l		Į.
May15. "	u	28.46		l		1
" 21. "	46	177.19	ii e	İ		1
i		2710.00		1		
_		\$ 748.89	Ü.		/	\$748.89

Errors excepted. Settled as above, Boston, July 7, 1856.

DAVID HALE, By John Davis.

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1856.
                    Debits.
                                                    Credits.
                                         Due Jan. 30, 240
Due July 4, 97 \times 151 = 14647
                                          " Apr. 8, 49 \times 64 = 8186
          18,
               58 \times 165 = 9570
                                          " May 22, 50 \times 113 = 5650
               80
     Feb. 4,
               89 \times 182 = 7098
     Aug. 4,
                                                                    8786 days.
     Feb. 9,
               50× 5 =
                                             8786 ÷ 839 = 25311 days.
     Sept. 8, 154 \times 212 = 82648
         24, 42 \times 233 = 9786
                                       Credits due 26 days from January 80.
 "
                                          or on February 25, 1856.
 " Oct. 9, 24 \times 248 = 5952
     Nov. 15, 28 \times 285 = 7980
                                       Difference between February 25 and
          21, 177 \times 291 = 51507
                                          August 8 = 165 days.
             $749
                          139438 days.
                                           $ 749 - $ 839 = $ 410, balance.
    139438 \div 749 = 186434 \text{ days.}
                                                 889 \times 165 = 55935:
Debits due 186 days from February 4,
                                              55985 ÷ 410 = 136  days.
  or on August 8, 1856.
```

FIRST OPERATION.

186 days forward from August 8, 1856 = December 22, 1856.

On equating each side of the account (Art. 435), we find the debits became due 186 days from February 4, or on August 8; and the credits became due 26 days from January 30, or on February 25.

If the account had been settled on February 25, it is evident the debits would have been paid 165 days, or the time from February 25 to August 8, before having become due. This would have been a loss of interest to the debit side of the account, and a corresponding gain to the credit side. Now, as the settlement should be one of equity, we find how long it will take the balance, \$410, to gain the same interest that \$339 would gain in the 165 days. If it take \$339 to gain a certain interest in 165 days, it would take \$1 to gain the same interest 339 times 165 days = 55935 days; and it would take \$410 to gain the same amount of interest  $\frac{1}{10}$  of 55935 days = 136

28

<sup>\*</sup> Included only to illustrate the manner of settling an account.

days nearly. Hence, the balance became due 136 days forward from

August 8, 1856, or on December 22, 1856.

The time was counted forward from the average date of the larger amount, since it became due last; but had that amount become due first, the time would have been counted backward from its average time.

				BECOND	OPERATION.	
		Debit	<b>3.</b>		Cr	edits.
Due	Feb. 4,	80 ×	5 =	400	Due Jan. 80, 240	•
"	" 9,	50 ×	10 ==	500	" April 8, 49	$\times$ 64 = 3136
44	July 4,	97 ×	156 =	15132	" May 22, 50	$\times$ 113 = 5650
44	" 18,	58 X	170 =	9860	\$ 339	8786 days.
4	Aug. 4,	89 ×	187 =	7293	•	
"	Sept. 8, 1	154 X	217 =	88418	<b>\$</b> 749 <b> \$</b> 839 <b>=</b>	= \$ 410, balance of the
44	" 24,	42 ×	288 =	9996	items.	
4	Oct. 9,	24 ×	253 ==	6072	143183 8786 =	= 134397, balance of
66	Nov. 15,	28 ×	290 🛥	8120	products.	
*	" 21,	177 X	296 =	52392	134897 ÷ 4	$10 = 327_{410}^{327} \text{ days.}$
	\$	749		143183 day	78.	•-•

828 days forward from January 80, 1856 = December 23, 1856.

In the second operation, we take the earliest date on which any sum becomes due in the account, for the starting point from which to reckon the days, by which to find the several products belonging - either to the debit or credit side (Art. 435, Note 1). The sum of the debit products, 143183, denotes the number of days required for \$1 to gain as much interest as all the items of debit would gain in the times of their becoming due, and the sum of the credit products, 8786, denotes the number of days required for \$ 1 to gain as much interest as all the items of debit would gain in the times of their becoming due. The difference between the sums of debit and credit products is 134397, and the difference between the debit and credit items is Then, if it requires 134397 days for \$1 to gain a certain interest, it will require \$410 to gain the same amount  $\frac{1}{\sqrt{100}}$  of 134397 days = 328 days nearly. 328 days forward from January 30, 1856 = December 23, 1856, the time of the balance of the account becoming due; thus varying one day in the result, on account of the fractions.

Rule 1. — Find the average time of each side becoming due.

Multiply the amount of the smaller side by the number of days between the two average dates, and divide the product by the balance of the account.

The quotient will be the time of the balance becoming due, counted from the average date of the larger side, FORWARD when the amount of that side becomes due LAST, but BACKWARD when it becomes due FIRST. Or,

RULE 2. — Multiply each sum of debit and credit by the number of days intervening between the date of its becoming due, and the earliest date on which any sum in the account becomes due.

Then, the difference between the sums of debit and credit products, divided by the difference between the debit and credit item, will give the

time, to be counted from the earliest date of any sum in the account becoming due, FORWARD when the larger sum of products is on the LARGER side of the account, but BACKWARD when it is on the SMALLER side.

Note. — The cash value of a balance of an account drawing interest, or whose items are on different terms of credit, depends upon the time of settlement, and is therefore either larger or smaller than the difference between the debit and credit items.

The average time of a balance becoming due being known, its cash value may be found, when the balance is due BEFORE the time of settlement, by adding to the balance interest up to the time of settlement; and when due AFTER that time, by deducting from the balance interest for the time intervening between the time of settlement and the time of the balance becoming due. The deduction of interest, in the latter case, is the mercantile method, instead of that of finding the true present worth by deducting the true discount.

# EXAMPLES.

2. Required the time when the balance of the following accounts becomes subject to interest, allowing that each item was due from its date.

Dr.	D. waasi	oorin in ac	count with	S. Aaams.	Cr.
1855. July 4, Aug. 20, " 29, Sept. 25, Dec. 5,	To balance, " merchandise, " " " " " "	\$ 375.90 815.58 178.25 887.20 418.70	1855. Aug. 10, Sept. 1, 25, Nov. 20, Dec. 1,	By cash, " "merchandise, " cash, " "	\$ 316.00 675.00 512.25 161.75 100.00
	•		A	ns. August 6, 1	855.

3. When did the balance of the following account become due, the merchandise items being on 6 months?

Dr.		John Gree	ene in acco	Cr.		
1856.	Ī_		1	1856.		Ī
March 1,	To	merchandise,	\$ 720.75	April 1,	By cash,	\$ 700.00
" 20,	66	"	815.30	May 30,	" merchandise,	569.89
April 11,	"	"	587.80	July 20,	" cash,	500.00
" 80.	66	44	800.00	Sept. 25.	""	100.00
June 15,	"	44	625.25	80,	" merchandise,	750.20
July 18,	16	u	560.00	Oct. 30.	" "	829.96
Aug. 80,	- 66	44	684.90	Nov. 20.		500.00
Sent. 25.	14	66	365.30	ll '	i	l

4. Allowing that each item of the following account draws interest from its date, at what time would the balance become due, and how much ready money should in equity discharge the same, September 21, 1856, interest being at 6 per cent.?

Dr.

I. Bradley in account with T. B. Fuller.

Cr.

1856.	1	1856.	
March 1. To merchandise,	\$ 86.25	April 1, By cash,	\$ 48.25
April 7, " "	18.15	May 20, " "	90.10
June 15, " "	48.26	June 17.  " "	12.50
July 21, " "	91.20	July 4, " "	20.00
Aug. 1, " "	80.00	10, " "	25.00

Ans. November 21, 1856; cash value of balance, \$27.73.

5. Required the time when the balance of the following account became subject to interest, allowing the merchandise to have been on 8 months; and the cash value of the balance on November 28,1857, provided it drew 6 per cent. interest from the time of its becoming due.

Dr. N. Chandler, 2d, in account with T. E. Lanman.

Cr.

1856.				1857.		1
	Τo	merchandise,	\$ 800.00		y cash,	\$500.00
July 7,	"	"	759.96	Feb. 18,	merchandise,	481.75
Sept. 11,	"	46	417.20	Mar. 19, '	cash,	750.25
Nov. 25,	"	44	287.70	Andrin 1,	draft,	210.00
Dec. 20,	"	и .	571.10	May 25, '	cash,	100.00

Ans. July 28, 1857; cash value of balance, \$ 299.88.

6. The following account was settled May 24, 1857; how long previous to date was the balance by average due, and what was the cash value of the balance at the time of settlement?

Dr. David Taggart in account with George Perry. Cr.

1856.	1				1 1	185	6.	1			•	·
Jan. 1,	To	m'dise	on	Бто.,	\$ 560.00	Marc	h 7,	By	m'dise	on	6mo.,	\$ 350.00
Feb. 11,	"	44	on	6mo.,	846.00	April	17,	a	66	"	" ′	820.00
Mar. 20,	"	"	on	6mo.,	728.00				cash,			100.00
July 80,		cash,			400.00				"			800.00
Sept. 12,		m'dise	on	8mo.,					m'dise	$\mathbf{on}$	6mo.,	630.00
Dec. 18,	"	cash,			600.00				cash,			400.00
1857.		-		-		Nov.	1,	"	m'dise	on	4mo.,	750.00
May 10,	"	u			500.00	1					•	ļ

Ans. May 19, 1856; cash value of balance, \$364.93.

- 439. To find the true balance of an account current whose items draw interest.
- Ex. 1. Required the true balance of the following account, on November 1, 1857, the time of settlement, allowing that each item drew interest from its date, at the rate of 6 per cent.

  Ans. \$ 430.04.

Cr.

# Wesley Scott in account with Seth Wilson.

1857	7.	,		1	1857.	1		i
Jan.	17.	To	merchandise.	\$ 144 05	Feb. 1	. Bv	cash.	\$ 200.00
Mar.	20,	46	"	874.95	March 9		" .	800.00
May	7.	, "	"	500.00	April 1	. "	"	600.00
July	4,	"	cash,	100.00	· 5	. "	merchandise.	180.00
"	7,	"	merchandise,	600.00	June 12	. "	"	700.00
Sept.	25,	"	"	250.00	Aug. 19	. "	cash.	400.00
Nov.	1,	"	*bal. new acc't,	430.04	Nov. 1	, "	*balance of int.,	19.04
		1		\$ 2399.04	l	1		\$ 2399.04

Errors excepted. Philadelphia, November 1, 1857.

SETH WILSON.

```
Debits.
                                                            Credits.
Due Jan.
               17, 144 \times 288 = 41472
                                             Due Feb.
                                                          1,200 \times 273 = 54600
       March 20, 875 \times 226 = 84750
                                                   March 9, 800 × 287 = 71100
  46
       Mav
                7,500 \times 178 = 89000
                                                   April 1, 600 \times 214 = 128400
       July
                4, 100 \times 120 = 12000
                                                           5,180 \times 210 = 87800
                7,600 \times 117 = 70200
                                                   June 12, 700 \times 142 = 99400
   44
       Sept.
               25, 250 \times 37 = 9250
                                                   Aug. 19, 400 \times 74 = 29600
                8 1969
                             6)806.672
                                                          £ 2380
                                                                        6)420.900
                                                                          $ 70,150
                               $ 51.112
```

OPERATION.

\$2380 — \$1969 = \$411, balance of items. \$70.15 — \$51.11 = \$19.04, balance of interest. \$411 + \$19.04 = \$430.04, true balance.

The time in days intervening between the date of each item and the time of settlement is, evidently, the number of days each item is on interest. Then, if each item be multiplied by the number denoting its number of days, and divided by 6000, the result will be its interest (Art. 356, Note 1), and the sum of the interest of the several items of debit will be the aggregate interest of the debit side, and the same principle will hold in finding the aggregate interest of the credit side. But the same result may be obtained in a shorter way by dividing the sums of the debit and credit products by 6000, which is done most readily by pointing off three decimal places at the right of each sum (Art. 78), and dividing by 6, as in the operation. Then the balance of items and the balance of interest we add together, and thus obtain the true balance.

2. Required the true balance of the following account, on October 1, 1857, the time of settlement, allowing the rate of interest to be 6 per cent.

Ans. \$189.81.

<sup>\*</sup> Introduced only to illustrate the manner of balancing the account28 \*

Dr.	Grant & Barker in account and interest with J. Ritchie.									
1857. Mar. 1 " 15 Mar. 20 Oct. 1	" merch'dise on 6mo.,	40.00 180.85	April 1,	" cash, " draft,	100.09 120.09 80.00 50.00					

Debits.	OPERATION.  Debit products.	Credit products.
Due March 1, 100 × 214 =	21400	-
" " 15, 40 × 200 =	8000	
" Sept. 20, 181 × 11 =	1991	
\$ 821	81891	
Credits.		
Due April 1, 100 × 188 =		18300
". June 1, 120 × 122 =	•	14640
" Sept. 1, $50 \times 30 =$		1500
" 15, 160 × 16 =		2560
" Oct. 10, 80 × 9 ==	720	
\$510	82111	87000
821	<b>V</b>	32111
\$ 189 balance of i	items.	6)4.889

\$ 0.81 bal. of interest. \$189 + \$0.81 = \$189.81, true balance.

We find the several products by multiplying, as in working the preceding example. The products belonging to items becoming due previous to settlement we arrange as belonging to their respective sides. One item, however, becomes due 9 days after the time of settlement, thus requiring its side to be diminished by the interest of that item for the 9 days, or that the opposite side should be increased by that interest. We, therefore, write the product of this item with the debit products. We may now find the interest of each side separately, and then subtract that of the one side from that of the other for the balance of interest; but we obtain the desired result by a shorter method by finding the interest corresponding to the difference of the sums of the debit and credit products. Hence the general

Rule. — Multiply each item of debit and credit by the number of days intervening between its becoming due and the time of settlement.

Place the product of each item becoming due BEFORE the time of settlement on its own side of the account, and place the product of each item

<sup>\*</sup> Introduced only to illustrate the manner of settlement.

becoming due AFTER the time of settlement on the side opposite to its

The difference of the sums of the debit and credit products, with three places pointed off on the right for decimals, divided by 6, will give the balance of interest.

Place the balance of interest on its own side of the account, and the

difference then between the two sides will be the true balance.

NOTE 1.—The interest as found by the rule is that at 6 per cent. From which the interest at any other rate may be found by aliquot parts.

NOTE 2. — Besides the above method of settling accounts drawing interest, accountants often make use of tables constructed to aid in equating and balancing accounts.

# EXAMPLES.

- . 3. Alfred Hicks is in account and interest with Keen & Lee, as follows: Debtor, January 1, 1857, to merchandise, on 6 months, \$156.10; February 3, to cash paid draft, \$100; March 20, to merchandise, on 4 months, \$316.90; March 30, to merchandise, on 4 months, \$162.00; May 15, to cash paid draft, \$100; August 20, to merchandise, on 6 months, \$213.00. Creditor, February 1, by cash, \$120.00; March 20, by merchandise, on 4 months, \$420.16; May 1, by merchandise, on 6 months, \$300; July 1, by merchandise, on 4 months, \$50; September 10, by merchandise, on 4 months, \$99.84. Required the true balance, if settled on December 1, 1857, interest being at 6 per cent.

  Ans. \$61.36.
- 4. Required the true balance due on the following account, on March 25, 1857, each item drawing 7 per cent. interest from its date. Benjamin Lyman in account and interest with John Russell: Debtor, July 4, 1856, to merchandise, \$200; September 8, to merchandise, \$300; September 25, to merchandise, \$250; October 1, to merchandise, \$600; November 20, to merchandise, \$400; December 12, to merchandise, \$500; January 15, 1857, to merchandise, \$100; March 11, to merchandise, \$120. Creditor, July 20, 1856, by cash, \$300; August 15, by cash, \$350; September 1, by cash, \$400; November 1, by cash, \$320; December 6, by merchandise, \$600; December 20, by cash, \$100; February 1, 1857, by cash, \$200; February 28, by merchandise, \$150.

Ans. \$50.64.

# ACCOUNTS OF STORAGE.

440. ACCOUNTS of storage of property contain an entry of the number of articles received and delivered, with the date of each transaction.

Storage is usually reckoned by the month of 30 days, at a certain price per barrel, bale, box, &c.

The number of articles on which storage is chargeable for one month, or any other time agreed upon, is usually determined by an average.

- 441. To find the average of storage for a month, or any other time.
- Ex. 1. What will be the cost for the storage of flour at 6 cents per barrel, which was received and delivered as follows: Received May 1, 1857, 1000 barrels; May 26, 2000 barrels. Delivered May 16, 500 barrels; June 1, 1000 barrels; June 12, 1100; July 2, 400.

  Ans. \$104.

	•	-						
		OPERATI	ON.					The storage of
1857.		bbl.		d.		p	rod.	1000bbl. for 15d.
May 1,	Rec.	1000	X:	15	=	15	000	+ 500bbl. for 10d.
	Deliv.		•					+ 2500bbl. for 5d.
<b>"</b> 26,	Bal. Rec.	$\begin{array}{r} \hline 500 \\ 2000 \end{array}$	×	1 0	=	5	000	is the same as the
June 1,		$\begin{array}{c} \overline{2500} \\ 1000 \end{array}$	×	5	=	12	500	storage of 57000bbl. for 1d., or 1900bbl. for a month of 30
<b>"</b> 12,	Bal. Deliv.	$\begin{array}{c} \overline{1500} \\ 1100 \end{array}$	×	11	=	16	500	days. And the storage of 1900bbl. at 6 cents each equals
	Bal.	400	×	2 0	=	8	000	\$104, the answer
July 2,	Deliv.	400		3	0)	57	0 0 0	
Charge	eable fo	r 1 mor	ıth,			19	00	customary, when the number of articles
1900bbl							•	upon which storage is chargeable, as
found, con	itains a i	raction t	ess t	han	a h	alf,	to reje	ect the fraction; but if
it is more than a half, to regard it as an entire article.								

Rule. — Multiply the number of barrels, or other articles, by the number of days they are in store, and divide the sum of the products by 30, or the number of days in any term agreed upon. The quotient "give the number of barrels, or other articles, on which storage is geable for that term.

# EXAMPLES.

- 2. What is the cost of storage of tea at 3 cents a chest per month, received and delivered as follows: Received, May 16, 1857, 4560 chests. Delivered, May 30, 564 chests; June 1, 904 chests; July 9, 1000 chests; August 3, 1500 chests; and August 16, the balance.
- 3. Received, and delivered, on account of Richard Gordon, sundry bales of cotton, as follows: Received, January 1, 1857, 2310 bales; January 16, 120 bales; February 1, 300 bales, Delivered, February 12, 1000 bales; March 1, 600 bales; April 3, 400 bales; April 10, 312 bales. Required the number of bales remaining in store on May 1, and the cost of storage up to that date, at the rate of 5 cents a bale per month.

  Ans. In store, 418 bales; cost, \$306.90.

# MISCELLANEOUS EXAMPLES.

- 1. The stocks of three partners, A, B, and C, are \$3500, \$2200, and \$2500, and their gains \$1120, \$880, and \$1200, respectively, and B's stock continued in trade 2 months longer than A's. Required how long the money of each was in trade.

  Ans. A's money, 8mo.; B's, 10mo.; C's, 12mo.
- 2. A merchant failed for \$15000. On settling up, his net assets, equably distributed, gave only \$540 to a creditor whose demand was \$660 more than that sum. How much did the bankrupt pay on a dollar, and how much did he owe more than he could pay?

Ans. 45 cents on a dollar; owed \$8250 more than he could pay.

- 3. A and B pay a poll tax each of \$1.50, and a property tax at the rate of 7 mills on a dollar. A's entire tax is \$64.50, which is just \$14 more than B's entire tax. What is the taxable property of each?
  - Ans. A's property, \$9000; B's property, \$7000.
  - 4. The expenses of a district school are, for fuel, \$20, for

repairs of the school-house \$30, and for teacher's salary \$150, and the public money amounts to \$50. If the rate bills require to be made out at the rate of 3 cents a day for each pupil's attendance, what was the aggregate attendance?

Ans. 5000 days.

5. J. Kimball had goods to the value of \$7000 on board a vessel, which, from stress of weather, required cargo to the amount of \$4000 to be thrown overboard, and of which the value of \$3000 belonged to Kimball. If, in adjusting the general average, the several contributory interests pay 5 per cent., to what sum will Kimball's loss be reduced?

Ans. \$ 350.

- 6. A owes B \$ 150, \$ 50 to be paid in 4 months, and \$ 100 in 8 months. B owes A \$ 250 to be paid in 10 months. It is agreed between them that A shall make present payment of his whole debt, and that B shall pay his so much sooner as to balance the favor. Required the time at which B must pay the \$ 250.

  Ans. 6 months.
- 7. A debt is to be paid \(\frac{1}{3}\) down, \(\frac{1}{2}\) in 6 months, \(\frac{1}{6}\) in 8 months, and the balance in 12 months. If the payments were all converted into one, on what credit should it be? Ans. 5 mo. 25 d.
- 8. A merchant proposes to admit a young man into business with him, on condition that, if he put into the stock \$2000, his pay shall be \$800 a year, or if he put in \$4000, he shall have \$1100 a year; in this offer what was allowed for his services only?

  Ans. \$500.
- 9. A gentleman is owing three notes to George Shannon, one of \$100 due in 4 months, another of \$100 due in 8 months, and a third of \$200 due in 12 months. Should the three notes be converted into two for the same amount, the one to run just twice as long as the other, when ought they to be made payable?

  Ans. The one in 6mo., and the other in 12mo.
- 10. An account settled January 1, 1858, showed a balance of debts to the amount of \$360, and a balance of interest in favor of the credit side to the amount of \$3.78; how long in equity ought the balance of debts to remain unpaid after the day of settlement, exactly to offset this balance of interest, allowing the rate of interest to be 6 per cent.? Ans. 63 days.

### INSURANCE.

442. Insurance is a contract of indemnity, by which one party engages, for a stipulated sum, to insure another against a risk or loss to which he is exposed.

The insurer or underwriter is the party taking the risk.

The *premium* is the sum paid for insurance, and is generally reckoned at a certain per cent. of the value of the property insured.

The policy is the written obligation or contract.

Note. — Insurance is generally made by an incorporated joint-stock company, and sometimes by individuals. When each person insured becomes a member and proprietor in the profit or loss of the concern, it is called a *mutual* insurance company. Many companies, as a security against fraud, do not insure property for its full value.

### FIRE AND MARINE INSURANCE.

- 443. Fire Insurance is that which indemnifies damage and loss caused by fire or lightning.
- 444. Marine Insurance is that which indemnifies damage and loss caused by the perils peculiar to navigation.
- 445. To compute the premium of insurance on any given amount at a specified rate.
- Ex. 1. What would be the premium for insuring a house, valued at \$5728, at 1\frac{1}{4} per cent.? Ans. \$100.24.

 $$5728 \times .013 = $100.24.$ 

RULE. — Find the percentage of the given sum at the rate of insurance, and the result is the premium.

NOTE. — When the amount insured and the premium are known, the rate of insurance may be found by Art. 850.

### EXAMPLES.

- 2. What is the premium for insuring a house valued at \$896, at 12 per cent.?

  Ans. \$107.52.
- 3. How much would be required to be paid to effect the insurance of a brig valued at \$17,289, at 1½ per cent.?

  Ans. \$216.11½.

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- 4. My ship Keystone State is valued at \$35000, and her cargo at \$75000. I procure an insurance on \(\frac{3}{5}\) of the value of the ship, at \(3\frac{1}{2}\) per cent., and on \(\frac{2}{3}\) of her cargo, at \(2\frac{1}{2}\) per cent. What is the amount of premium? Ans. \$1932.50.
- 5. My library, worth \$3675, I got insured by paying 47 per cent.; and the policy cost me \$1. The library having been destroyed by fire, what was my actual loss? Ans. \$180.15\frac{1}{2}.
- 6. The premium for insuring \$9870 was \$690.90. What was the rate per cent.?
- 7. White & Bigelow effect an insurance on their store and goods, worth \$47600, for 5 years. The first year they are to pay  $4\frac{1}{4}$  per cent.; the second year,  $3\frac{7}{4}$  per cent.; the third year,  $4\frac{7}{3}$  per cent.; the fourth year, 5 per cent.; and the fifth year, 5 per cent. What is the whole they are to pay for the insurance?

  Ans. \$11,007.50.
- 446. To find what sum must be insured, at a given rate, to cover both property and premium.
- Ex. 1. For what sum must a policy be taken out, to cover both property and premium, the value of the property being \$2475, and the rate of insurance 10 per cent.?

# $$2475 \div .90 = $2750.$

Since the sum to be covered by the insurance includes both the property and the premium, and as the premium is 10 per cent., or .10 of that sum, the property, evidently, must be .90 of the sum for which the policy is to be taken out.

Rule. — Divide the value of the property to be covered by 1 decreased by the rate of insurance expressed decimally, and the quotient will give the whole sum to be covered.

Note. — When the rate of insurance and premium are known, the amount insured may be found by Art. 850.

#### EXAMPLES.

- 2. A manufacturing company own a factory valued at \$26250. For what sum must a policy be taken out to cover the property and premium of insurance, the rate being 12½ per cent.?

  Ans. \$30,000.
- 3. A store and its goods are worth \$6370. What sum must be insured, at 2 per cent., to cover both property and premium?

  Ans. \$6500.

4. The premium for insuring a school-house, at the rate of  $1\frac{1}{4}$  per cent., was \$50. For what sum was it insured?

Ans. \$4000.

- 5. If a policy covering property and premium be taken for \$600, at 10 per cent, what is the value of the property covered?

  Ans. \$540.
- 6. A merchant adventured \$1000 from Boston to New Orleans, at 3 per cent.; thence to Chili, at 5 per cent.; thence to Canton, at 6 per cent.; and thence to Boston, at 7 per cent. For what sum must be take out a policy, to cover his adventure the voyage round?

  Ans. \$1241.348.

### LIFE INSURANCE.

447. Insurance on life is a contract which stipulates for the payment of a certain sum of money on the death of one or more individuals, in consideration of an immediate payment, or an annual premium, being made by the insured.

A temporary insurance on life is a contract to pay a certain sum in case a given individual dies within a given number of years.

- 448. The amount of premium required of the insured as a security for the payment of a certain sum at his death by the insurer, is based upon the expectation of life of the insured, and on the rate of interest or net profit the insurer may be able to make by investing the premium.
- 449. By expectation of life is meant the average number of years of life that remains to any individual of a given age, as determined by the rates of mortality.
- 450. The Carlisle Table of the Expectation of Life, which is in general use in England, has also been taken as a guide by some American companies in fixing their rates of insurance.

Other companies have been guided in fixing their premiums by a table prepared by Dr. Wigglesworth, with a special reference to mortality in this country, and which the Supreme Court of Massachusetts has adopted as a rule in estimating the value of life estates.

451. The expectation of life, according to the Carlisle Table
N 29

and according to that prepared by Dr. Wigglesworth, is shown in the following

TABLE.

Age.	Carlisle Table. Expectation.	Wigglesworth Table. Expectation.	Age.	Carlisle Table. Expectation.	Wigglesworth Table. Expectation.	Age.	Carlisle Table Expectation.	Wigglesworth Table. Expectation.	Age.	Carlisle Table. Expectation.	Wigglesworth Table. Expectation.
0	88.72	28.15	24	88.59	32.70	-48	22.80	22.27	72	8.16	9.14
1	44 68	86.78	25	87.86	32.33	49	21.81	21.72	73	7.72	8.69
2	47.55	88.74	26	37.14	31.93	50	21.11	21.17	74	7.33	8.25
3	49.82	40.01	27	36.41	31.50	51	20.39	20.61	75	7.01	7.88
4	50.76	40.73	28	85.69	31.08	52	19.68	20.05	76	6 69	7.40
4 5 6	51.25	40.88	29	35.00	30.66	53	18.97	19.49	77	6.40	6.99
	51.17	40.69	80	84.34	80.25	54	18.28	18.92	78	6.12	6.59
8 9	50.80	40.47	81	33.68	29.83	55	17.58	18.35	79	5.80	6.21
8	50.24	40.14	82	33.03	29.43	56	16.89	17.78	60	5.51	5.85
	49.57	89.72	33	82.86	29.02	57	16.21	17.20	81	5.21	5.50
10	48.82	39.23	84	31.68	28.62	58	15.55	16.63	82	4.93	5.16
11	48.04	38.64	85	31.00	28.22	59	14.92	16.04	88	4.65	4.87
12	47.27	88.02	36	30.32	27.78	60	14.34	15.45	84	4.39	4.66
13	46.51	87.41	37	29.64	27.34	61	13.82	14.86	85	4.12	4.57
14	45.75	86.79	88	28.96	26.91	62	13.31	14.26	86	8.90	4.21
15	45.00	36.17	39	28.28	26.47	63	12.81	13.66	87	3.71	3.90
16	44.27	85.76	40	27.61	26.04	64	12.30	13.05	88	3.59	3.67
17	43.57	35.37	41	26.97	25.61	65	11.79	12.48	89	8.47	3.56
18	42 87	34.98	42	26.34	25.19	66	11.27	11.96	90	3.28	3.73
19	42.17	84.59	43	25.71	24.77	67	10.75	11.48	91	3.26	3.32
20	41.46	34.22	44	25.09	24.85	68	10.23	11.01	92	8.87	3.12
21	40.75	33.84	45	24.46	23.92	69	9.70	10.50	93	3.48	2.40
22	40.04	33.46	46	23.82	23.37	70	9.18	10.06	94	8.53	1.98
23	89.31	33.08	47	23.17	22.83	71	8.65	9.60	95	3.53	1.62

Against any age given in the table may be found the expectation of age corresponding to it.

452. The transactions of life insurance companies extending as they do over a term of years, the value of money and the average rates of interest, no less than the expectation of life, have an important influence in fixing their rates of insurance.

453. The premiums of life insurance are generally reckoned at a certain sum on \$100, payable annually in advance.

The rates of annual premium for insuring a healthy life for one year, for seven years, or for the whole period of life, in the sum of \$100, by the Massachusetts Hospital Life Insurance Company, of Boston, and by the Girard Life Insurance Annuity and Trust Company, of Philadelphia, are given in the following

TABLE.

Age next Birthday.	Mass. Ho	spital Life Company	Insurance •	The Gir	Age next Birthday		
	l year.	7 years.	For Life.	1 year.	7 years.	For Life.	DI WICE,
15	.83	.85	1.44	.77	.88	1.56	15
16	.84	.86	1.47	.84	.90	1.60	16
17	.85	.87	1.51	.86	.91	1.65	17
18	.86	.88	1.54	.89	.92	1.69	18
19	.87	.90	1.58	.90	.94	1.73	19
20	.88	.91	1.62	.91	.95	1.77	20
21	.89	.92	1.66	.92	.97	1.82	21
22	.90	.93	1.70	.94	.99	1.88	22
23	.91	.95	1.74	.97	1.03	1 93	23
24	.92	.96	1.79	.99	1.07	1.98	24
25	.93	.98	1.84	1.00	1.12	2.04	25
26	.95	.99	1.89	1.07	1.17	2.11	26
27	.96	1.01	1.94	1.12	1.23	2.17	27
28	.98	1.03	2.00	1.20	1.28	2.24	28
29	.99	1.05	2.06	1.28	1.35	2.31	29
80	1.01	1.07	2.12	1.31	1.36	2.36	30
91	1.03	1.09	2.18	1.32	1.42	2.43	31
82	1.05	1.11	2.25	1.33	1.46	2.50	32
88	1.07	1.14	2.32	1.34	1.48	2.57	83
34	1.09	1.16	2.40	1.35	1.50	2.64	. 34
35	1.11	1.19	2.48	1.36	1.53	2.75	35
86	1.14	1.21	2.56	1.39	1.57	2.81	36
87	1.16	1.24	2.65	1.43	1.63	2.90	87
88	1.19	1.28	2.75	1.48	1.70	3.05	38
89	1.22	1.31	2.85	1.57	1.76	3.11	39
40	1.24	1.36	2.95	1.69	1.83	3.20	40
41	1 27	1.41	3.07	1.78	1.88	3.31	.41
42	1.31	1.47	8.19	1.85	1.89	3.40	42
43	1.35	1.54	3.32	1.89	1.92	3.51	43
44	1.40	1.62	8.45	1.90	1.94	8.63	44
45	1.47	1.71	8.60	1.91	1.96	8.73	45'
46	1.54	1.80	8.75	1.92	1.98	3.87	46
47	1.62	1.90	8.92	1.93	1.99	4.01	47
48	1.71	2.02	4.09	1.94	2.02	4.17	48
49	1.81	2.14	4.27	1.95	2.04	4.49	49
50	1.91	2.28	4.46	1.96	2.09	4.60	50
51	2.03	2.42	4.67	1.97	2.20	4.75	51
52	2.15	2.59	4.89	2.02	2.37	4 90	52
53	2.29	2.76	5.12	2.10	2.59	5.24	53
54	2.44	2.95	5.36	2.18	2.89	5.49	54
55	2.60	8.15	5.62	2.32	8.21	5.78	55
56	2.78	8.88	5.89	2.47	3.56	6.05	56
57	2.96	3.62	6.19	2.70	4.20	6.27	57
58	8.17	3.87	6.50	3.14	4.31	6.50	58
59	8.39	4.17	6.83	8.67	4.63	6.75	59
60	3.64	4.50	7.18	4.35	4.91	7.00	60

According to the table, a healthy man who is 42 years old next birthday, by paying the Massachusetts Hospital Insurance Company \$ 1.31, would secure to his family, heirs, or to whom-

soever he desires, \$100, should he die in one year, and in the same proportion for a larger sum. And if he would obtain a life insurance of the Girard Life Insurance Annuity and Trust Company, he must pay annually \$3.40. The New York Life Insurance Company has a schedule of rates like that of the Girard Company, given in the table.

- 454. To compute the premium of life insurance for any given amount.
- Ex. 1. What premium will the Massachusetts Hospital Life Insurance Company require for the insurance of a life one year for \$1728, the person being thirty years of age next birth-day?

  Ans. \$17.45.

By the table we find the premium on \$100 to be \$1.01.

the premium on \$1 for one year, and \$1728  $\times$  .0101 = the premium on \$1728 for the same time.

By the table we find the premium on \$100 to be \$1.01.

Therefore  $\frac{3}{1} \cdot \frac{10}{0} \cdot \frac{1}{0} = \frac{3}{10} \cdot \frac{10}{0} \cdot \frac{10}{0} = \frac{3}{10} \cdot \frac{10}{0} = \frac{3}{10} \cdot \frac{10}{0} \cdot \frac{10}{0} = \frac{3}{$ 

Compute the premium on the sum to be insured at a rate proportionate to the given premium on \$100.

# EXAMPLES.

- 2. What amount of premium must S. C. Kendall pay annually to the Massachusetts Hospital Life Insurance Company, to effect an insurance on his life for 7 years for \$8000, his age being 33 years?

  Ans. \$91.20.
- 3. Robert Vaux, 60 years of age, wishes to engage in a very profitable speculation; and being obliged to borrow the necessary funds, he effects an insurance on his life for 7 years, for \$78000, at the office of the Girard Life Insurance Company. Required the amount of the annual premium. Ans. \$3829.80.
- 4. What will be the yearly premium for insuring a person's life, who is 15 years old, for \$2000 for 7 years, at the New York Life Insurance Company?
- 5. A gentleman 45 years of age, being bound on a long and dangerous voyage, and wishing to secure a competence for his family, obtains an insurance for life of the Girard Life Insurance Company, for \$12000. By an act of Providence he dies in the third year. What is the net gain to his family?

Ans. \$ 10657.20.

- 6. Richard Sears, 50 years old, effects an insurance for life for \$5000, for which he pays an annual premium of \$4.50 on each \$100 insured. If he should die at the age of 80 years, how much less will be the amount of insurance than the payments, allowing the latter to be without interest? Ans. \$1900.
- 7. A gentleman, 56 years old, gets his life insured for \$4000, at the office of the Kentucky Mutual Life Insurance Company, by paying an annual premium of \$5.20 on each \$100 insured; and dies at the age of 60 years. Reckoning interest on his payments at simple interest, what is gained by the insurance?

  Ans. \$3043.20.
- 8. Alexander Murray, 28 years of age, effects an insurance on his life for \$10000, at the office of the Massachusetts Hospital Life Insurance Company. If the company loan the premium at 6 per cent. compound interest, and he should die at the age of 40 years, who will gain by the insurance?

Ans. The insured gains \$ 6626.01.

# CUSTOM-HOUSE BUSINESS.

455. Duties or customs are sums of money required by government to be paid on imported goods.

Ports of entry are ports into which merchandise may be imported, or from which it may be exported.

At each port of entry is an establishment, called a customhouse, at which certain officers, appointed by government, attend to the collection of the duties.

456. Duties are either specific or ad valorem.

A specific duty is a certain sum paid on a ton, hundredweight, yard, gallon, &c., without regard to the cost of the article.

An ad valorem duty is a certain percentage paid on the actual cost of the goods in the country from which they are imported.

Nor. — As evidence of the cost value of merchandise subject to duties, the importer, owner, or consignee is required to produce an invoice or manifest, if one has been received, made out in the currency of the place or country whence imported, and containing a true statement of the actual cost of such goods in such foreign currency. When the currency of a place has a depreciated value compared with that of the United States, it is necessary that a consular certificate showing the rate of depreciation should be attached to the invoice. When, however, an invoice has not been received, the fact must be testified to under oath, and then the imported articles will-be entered at an appraised value.

- 457. In this country, under the present tariff, duties are levied on the ad valorem principle.
- 458. Allowances are deductions required to be made before estimating the duties, on account of the weight of the cask, box, bag, &c. in which an article is imported, or on account of breakage, leakage, waste, or other damage.

Tare is the allowance made for the weight of the cask, box, &c. containing the commodity.

Draft is the allowance made for waste in the weighing of goods.

Leakage is the allowance made for waste on liquids imported in casks.

Gross weight is the weight of the commodity together with the cask, box, bag, &c. containing it.

Net weight is what remains after all allowances have been made.

459. No allowances for tare, draft, breakage, &c. are applicable to imports subject to ad valorem duties, except actual tare, or weight of a cask or box, and actual drainage, leakage, or damage. The collector may cause these to be ascertained, when he has any doubt as to what they are.

Note. — When the tariff laws of the United States required the collection of specific duties, the allowance for draft was on 112lb., 1lb.; above 112lb. and not exceeding 224lb., 2lb.; above 224lb. and not exceeding 336lb. and not exceeding 1120lb., 4lb.; above 1120lb. and not exceeding 2016lb., 7lb.; above 2016lb., 9lb. The allowance for tare was deducted after the draft had been deducted. The allowance for breakage was 10 per cent. on all beer and porter in bottles; and 5 per cent. on all other liquors imported in bottles; and a dozen bottles of common size were estimated to contain 23 gallons. The allowance for leakage was 2 per cent. on liquors imported in casks. In making the allowances for tare or leakage, a fraction, when equal to, or greater than, one half, was reckoned 1; when less, it was omitted.

Specific duties were calculated by deducting all allowances to be made from the given quantity of merchandise, and multiplying the remainder by the duty on a unit of the given quantity.

To calculate advalorem duties.

Ex. 1. What is the duty on 2565lb. of sugar, invoiced at \$ 256.50, at 24 per cent. ad valorem? Ans. \$ 61.56.

OPERATION.

# $$256.50 \times .24 = $61.56$ , duty.

RULE. — Find the percentage on the invoiced value of the goods, at the given rate of tariff, and the result will be the ad valorem duty.

Note 1. — Other questions in duties beside those requiring the finding of the amount of duty, are likewise solved by some one of the rules in percentage.

Note 2. - In custom-house calculations 2240lb. are considered a ton, and

112lb. a hundred-weight.

### EXAMPLES.

- 2. What is the duty at 8 per cent. on an importation of books invoiced at \$ 4350? Ans. \$ 348.00.
- 3. Required the duty at 19 per cent. on 7890 pounds of cordage, invoiced at 15 cents per pound. Ans. \$ 224.865.
- 4. What is the duty at 24 per cent. on an invoice of woollen goods, which cost in London 986£, the pound sterling being Ans. \$ 1145.3376. valued at \$ 4.84?
- 5. \$ 112.50 duty is paid on an importation of window-glass whose invoice value was \$750. What was the rate per cent. of duty?
- 6. Robinson & Brother of New York have imported wines from Havre, invoiced as follows: 60 baskets Champagne at 70 francs per basket; 36 baskets port at 35 francs per basket; 50 casks of sherry, each 31 gallons, at 4 francs per gallon. The allowance for breakage on the wine in baskets is 5 per cent.; and the actual waste of that in casks is 1 gallon to a cask. Required the duties at 30 per cent., allowing the value of a franc Ans. \$624.2346. to be 18 cents.
- 7. Paid \$53.76 duties, at the rate of 8 per cent., on 60 casks of raisins, after the deduction of 12lb. to a cask for tare. Allowing the gross weight of each cask of raisins to have been 112lb., what was their invoice value per pound?

Ans. 114 cents.

- \* 8. A portion of the cargo of the ship Cuba from Havana was invoiced as follows: 40 hogsheads of molasses, 63 gallons each, at 3 reals plate per gallon; 24 boxes of brown sugar, 400lb. each, at 1 real vellon per pound; 260 boxes of oranges, at \$2 per box; and 410 boxes of cigars, at \$7 per box. The tare on the sugar was 10 per cent., and the leakage of the molasses 2 per cent. Allowing the value of a real plate to be 10 cents, and a real vellon 5 cents, and the rate of duties on the molasses and the sugar to have been 24 per cent., on the oranges 8 per cent., and on the cigars 30 per cent., what was the whole amount of duties?
- 9. 270 tons of railroad iron, invoiced at \$50 per ton, cost, when the duties were paid, exclusive of other charges, \$16740. What was the rate per cent. Ans. 24 per cent.
- 10. A merchant of Baltimore makes an importation of goods invoiced at \$20560. On goods invoiced at \$3000 the duties were at the rate of 4 per cent.; on goods invoiced at \$4200 the duties were at the rate of 8 per cent.; on goods invoiced at \$2100 the duties were at the rate of 15 per cent.; goods invoiced at \$6000 were free of duty; and on the remainder the duties were at the rate of 30 per cent. What was the whole amount of the duties?
- 11. Willard, Fairbanks, & Co., of Boston, import from Liverpool 10 pieces of Brussels carpeting, 40 yards each, purchased at 5s. per yard, duty 24 per cent.; 200 yards of hair-cloth, at 4s. per yard, duty 19 per cent.; 100 woollen blankets, at 2s. 6d., duty 15 per cent.; and shoe-lasting to the cost of 60£., duty 4 per cent. Required the whole amount of duty, allowing the value of the pound sterling to be \$ 4.84.

  Ans. \$ 173.635.
- 12. A merchant imported from Bremen 32 pieces of linen of 32 yards each, on which he paid for the duties, at 24 per cent., \$122.88, and other charges to the amount of \$40.96. What was the invoice value per yard, and the cost per yard after duties and charges were paid?

Ans. Invoice value per yard, \$0.50; cost per yard, \$0.66.

# COINS AND CURRENCIES.

461. Coins are pieces of metal legally stamped, and issued for circulation as money.

The currency of a state or country is its money or circulating medium of trade.

462. The former currency of this country had the denominations of sterling money, viz. pounds, shillings, and pence. On the adoption by Congress, in 1786, of the present decimal currency, with the dollar as its unit, there were in circulation colonial notes, or bills of credit, which had depreciated in value. This depreciation, being greater in some States than in others, gave rise to the difference in the States as to the number of shillings equivalent to a dollar, as shown in the following

#### TABLE.

New Eng. States,  $= 6s. = \frac{8}{10} £.$ , called New Eng. currency; Virginia, Kentucky, of which 1£.  $= $3\frac{1}{8}$ ; 1s.  $= 16\frac{2}{8}$ cts. Tennessee, New York, = 8s. = \$£., called New York currency; Ohio, \$ 1 in · Michigan, of which if. =  $$2\frac{1}{2}$ ; 1s. =  $12\frac{1}{2}$ cts. North Carolina. Pennsylvania, New Jersey, = 7s. 6d. = \ £., called Pennsylvania ctr-\$1 in < Delaware, rency; of which  $1£. = $2\frac{2}{3}$ ; 1s. =  $13\frac{1}{3}$ cts. Maryland, =4s. 8d.=7£., called Georgia currency; Georgia, \$ 1 in } South Carolina, (of which 1£. = \$43; 1s. = 21\$cts.

· Note 1.— The State currencies are now merely nominal, accounts being nowhere kept in them. Articles, however, are sometimes priced in them, but much less often now than formerly.

Note 2.—\$1 equals about  $\frac{115}{121}$ s. =  $\frac{25}{121}$ £. of English or sterling money; and consequently 1£. = \$4.84; 1s. = \$0.24\frac{1}{2}\$. Also \$1 = 5s. = \frac{1}{4}£. of Nova Scotia, New Brunswick, Newfoundland, and Canada, called Canada currency; of which 1£. = \$4; 1s. = 20 cents.

**463.** The *legal tender* in payment of debts in the United States is gold and silver.

NOTE. — The gold coins of the United States of coinage prior to 1834 are a legal tender for the payment of all debts, at the rate of 94% cents per penny-

weight, or about \$ 10.66 for each eagle; and all the gold coins of a subsequent coinage are a legal tender of payment in any sums whatever, according to their nominal values.

Of the silver coins coined prior to April 1, 1853, the three-cent pieces are a legal tender of payment in any sums of thirty cents and under, and the dollars, half-dollars, quarter-dollars, dimes, and half-dimes are a legal tender of payment in any sums whatever, at their nominal value, and all the silver coins of a subsequent coinage are a legal tender in sums not exceeding five dollars.

- 464. Besides the coins of the country, foreign coins, whose value has been fixed by law, and bank-notes, redeemable in specie, pass as money. However, by the act of Congress passed February, 1857, all former acts authorizing the currency of foreign gold or silver coin, and declaring the same legal tender in payment of debts, were repealed.
- 465. The intrinsic value of foreign coins is the value depending upon the weight and purity of the metal of which they are made; their legal value is that fixed by law; their commercial value is the price they will bring in the market; and their exchange value is the nominal price assigned to them in reckoning exchange between one country and another.
- 466. The United States mint, at present, purchases standard silver at \$1.22½ per ounce, and fine silver at \$1.36 per ounce.

Note. — At these rates of purchase, five-franc pieces yield about 99 cents each; Mexican and South American dollars, 106½ cents each; old Spanish dollars, 105 cents each; half-dollars of the United States coined before 1837, 52½ cents each; half-dollars of the United States coined since 1837, and previous to the change of standard in 1853, 52½ cents each; German florins, 41½ cents each; Prussian and Hanoverian thalers, 72 cents each; best manufactured American plate, from 120 to 122 cents per ounce; and genuine British plate, 125 cents per ounce.

467. By the act of 1843 the gold coins of Great Britain, if not less than  $\frac{915\frac{1}{2}}{1000}$  in fineness, were rated at  $94\frac{6}{10}$  cents per pennyweight, and the gold coin of France of not less than  $\frac{899}{100}$  in fineness, at  $92\frac{9}{10}$  cents per pennyweight. By a previous act, the gold coins of Portugal and Brazil of not less than 22 carats fine were rated at  $94\frac{8}{10}$  cents per pennyweight, and the gold coins of Spain, Mexico, and Colombia, of a fineness of 20 carats  $3\frac{1}{10}$  grains, at the rate of  $89\frac{9}{10}$  cents per pennyweight.

Note. - The relative value of gold and silver in the United States, at pres-

ent, is nearly as 14 $\frac{7}{8}$  to 1; in England, as 14 $\frac{29}{100}$  to 1; in France and in Russia, as 15 to 1; in Spain, as 16 to 1; in China, as 14 $\frac{1}{8}$  to 1; and in Portugal, as 13 $\frac{56}{100}$  to 1.

468. The value of certain foreign coins and currencies, as fixed by law in the collection of duties at the United States custom-houses, is shown in the following

#### TABLE.

Pound Ster. of G. Britain, \$4.84	Florin of South of Germ., \$ 0.40
Pound Ster. of Br. Prov.,)	Ounce of Sicily, 2.40
Nova Scotia, N. Bruns., > 4.00	Pagoda of India, 1.84
and Newfoundland,	Star Pagoda of Madras, 1.84
Dollar of Mexico, Peru, 1.00	Tael of China, 1.48
Chili, and Cen. Amer., \ 1.00	Millrea of Portugal, 1.12
Specie Dollar of Sweden 1.06	Millrea of Azores, .831
and Norway,	Ducat of Naples, .80
Specie Dollar of Denmark, 1.05	Sicca Rupee of Bengal, or 3
Rix Dollar of Bremen, .783	of Bombay,
Thaler of Bremen of 72 \ .71	Rupee of British India, .441
grotes,	Mark Banco of Hamburg, .35
Rix Dollar, or Thaler of)	Franc of France and Belg., .18-6
Prussia and Northern .69	Livre Tournois of France, .181
States of Germany,	Leghorn Livre, .16
Ruble, silver, of Russia, .75	Lira of Lombardo-Vene-
Florin of the Austrian Em-)	tian Kingdom,
pire and city of Augs- \.481	Lira of Tuscany, .16
burg,	Lira of Sardinia, .18 6
Florin or Guilder of Noth-	Real Plate of Spain, .10
erlands, .40	Real Vellon of Spain, .05
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#### REDUCTION OF CURRENCIES.

- 469. Reduction of Currencies is the process of finding the value of the denominations of one currency in the denominations of another.
- 470. To find the value of the denominations of one currency in the denominations of another, when the values of a unit of each are known.
- Ex. 1. What is the value of 18£. 4s. 6d. of the New England currency in United States money?

  Ans. \$ 60.75.
- Ve reduce the shillings and pence to the decimal of a pound, and annex it to the pounds. We then divide the sum by  $\frac{3}{10}$ , because 6s., or a dollar in this currency, is  $\frac{3}{10}$  of a pound.

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Ex. 2. What is the value of \$60.75 in New England currency?

Ans. 18£. 4s. 6d.

 $60.75 \times \frac{3}{10} = 18.225 \pounds$ .  $18.225 \pounds$ .  $= 18 \pounds$ . 4s. 6d.

Since 6s., or a dollar of this currency, is  $\frac{2}{10}$  of a pound, we multiply the given sum by the fraction  $\frac{2}{10}$ , and find the value of the decimal in shillings and pence.

RULE. — Multiply or divide, as the case may require, by the value of the unit of the given currency expressed in United States money.

NOTE. — When one currency is to be changed to another, and neither of them is United States money, the value of one of them may be found in United States money, and the value of that result be found in the other currency.

# EXAMPLES.

- 3. Change 46£. 16s. 6d. of the currency of New York to United States money.

  Ans. \$117.06\frac{1}{2}.
  - 4. Change \$1032 to the currency of Pennsylvania.

Ans. 387£.

5. Find the value of \$515.70 in Canada currency.

Ans. 128£. 18s. 6d.

- 6. Change 160.50 francs to United States money.
- 7. Change \$728.41 to English money.

Ans. 150£. 9s.  $11_{72}^{61}$  d.

- 8. Find the value of 12£. 12s. of the currency of Georgia in United States money.

  Ans. \$54.
- 9. Find the value of 128£. 18s. 6d. of Canada in United States money.

  Ans. \$515.70.
- 10. Find the value of 740.45 rubles, silver, of Russia, in United States money.

  Ans. \$555.832.
- 11. Find the value of 46£. 16s. 6d. of New York currency in the currency of New England.

  Ans. 35£. 2s. 4½d.
  - 12. Change 151 millreas of Portugal to real plate of Spain.
- 13. Find the value of 1000 specie dollars of Norway in francs.
- 14. A merchant of Quebec bought in London 30 pieces of broadcloth of 30 yards each, at 15 shillings per yard; what is the amount of his bill in Canada currency? Ans. 816£. 15s.
- 15. A merchant of Prussia bought in Naples silks to the amount of 410 ducats; what was the amount of his purchase in thalers?

  Ans. 475% thalers.

### EXCHANGE.

- **471.** EXCHANGE, in commerce, is the paying or receiving noney in one place for an equivalent sum in another, by means of Arafts, or bills of exchange.
- 472. A BILL OF EXCHANGE is a written order, to some person at a distance, to pay a certain sum, at an appointed time, to another person, or to his order.

The maker or drawer of a bill is the person who signs it.

The buyer, taker, or remitter of a bill is the person in whose favor it is drawn.

The drawee of a bill is the person on whom it is drawn, who is also called the acceptor, after he has accepted it.

The indorser of a bill is the person who indorses it.

The holder or possessor of a bill is the person in whose legal possession it may be at any time.

473. Most mercantile payments are made in bills of exchange, since it is generally more convenient to discharge debts by means of them than by cash remittances. For example, suppose A, of Boston, is creditor to B, of Baltimore, \$100; and C, of Boston, is debtor to D, of Baltimore, \$100; both these debts may be discharged by means of one bill. Thus, A draws for this sum on B, and sells his bill to C, who remits it to D, and the latter receives the amount, when due, from B. Here, by a transfer of claims, the Boston debtor pays the Boston creditor, and the Baltimore debtor the Baltimore creditor; and no money is sent from one place to the other. The same would take place if D, of Baltimore, drew on C, of Boston, and sold his bill to B, of Baltimore, who should send it to A, of Boston; the effect, in either case, being merely a transfer of debt and credit.

Bills of exchange pass from hand to hand, until due, like any other circulating medium.

474. The terms of a bill vary according to the agreement between parties, or the custom of countries. Some bills are drawn at sight; others, at a certain number of days, or months, after sight or after date; and some, at usance, which is the customary or usual term between different places.

# DAYS OF GRACE.

Days of grace are a certain number of days granted the acceptor for paying the bill, after the term of a bill has expired. The usual time allowed in this country is three days.

Note 1.— In some States three days of grace are allowed on all bills of exchange payable at sight, or at a future day certain; but in other States sight drafts are excepted, as in New York, Pennsylvania, New Jersey, Maryland, Virginia, Missouri, Illinois, Michigan, Connecticut, Rhode Island, Delaware, &c.

No days of grace are allowed on bills written payable on demand, or which

have no time of payment expressed in them.

Note 2. — In reckoning when a bill, payable after date, becomes due, the day on which it is dated is not included; and if it be a bill payable after sight, the day of presentment is not included. When the term is expressed in months, calendar months are understood; and when a month is shorter than the preceding, it is a rule not to go in the computation into a third month.

Thus, if a bill be dated the 28th, 29th, 30th, or 81st of January, and payable one month after date, the term expires on the last day of February, to which the days of grace must, of course, be added; and therefore the bill becomes

due on the 8d of March.

# INDORSING BILLS.

- 475. An indorsement of a bill is the act by which the holder of it transfers his right to another. It is usually made on the back of the bill, and must be in writing.
- 476. Bills payable to order are transferred only by indorsement and delivery, but bills payable to bearer are transferred by either mode. On transfer by delivery, the person making it ceases to be a party to the bill; but on a transfer by indorsement, he is to all intents and purposes chargeable as a new drawer.
- 477. An indorsement may take place any time after the bill is issued, even after the day of payment has elapsed.
- Note. An indersement may be restrictive, giving authority to the indersee to receive the money for the inderser, but not to transfer the bill to another. The indersement for a part of the money only is not valid, except with regard to him who makes it. The drawer and acceptor are not bound by it. After the payment of a part, however, a bill may be indersed over for the residue. The indersement is said to be in blank, when the inderser simply writes his name upon the back of the bill to make it transferable by delivery; and is said to be special, when the inderser directs the money to be paid to some particular person, or to his order. If the inderser would avoid all liability, he must qualify his indersement by the words, "without recourse," or by others of the same import.

### ACCEPTING BILLS.

- 478. An acceptance is an engagement to pay a bill according to the tenor of the acceptance, which may be either absolute or qualified.
- 479. An absolute acceptance is an engagement to pay a bill according to its request, which is commonly done by the drawee writing his name at the bottom, or across the body of the bill, with the word accepted.
- **480.** A qualified acceptance is when a bill is accepted conditionally. But the holder is not obliged to receive a conditional or partial acceptance. He may act as if an acceptance had been entirely refused.
- Note. When a bill is drawn for the account of a third person, and is accepted as such, and he fails without making provision for its payment, the acceptor must discharge the bill, and can have no recourse against the drawer. When a holder, at his own risk, takes a conditional or partial acceptance, he must give immediate notice to all other parties to the bill, or he can have no resort to them in default of payment.

All bills payable at sight, or at a day certain, or on demand, should be presented within a reasonable time, or the holder may, from his default, be the loser.

# PROTESTING BILLS.

- 481. The holder of a bill, when acceptance or payment has been refused, should give regular and immediate notice to all the parties to whom he intends to resort for payment; since, if a loss should be incurred, on account of unnecessary delay, by the failure of any of the parties, he would be obliged to bear the loss. Such a notice of non-acceptance or non-payment, when made by a public officer called a notary or notary public, or by any other legal mode, is called a protest.
- 482. When the parties to a bill, which the drawee has failed to accept or pay, live in different countries, a protest is indispensably necessary, as this instrument is admitted in foreign countries as legal proof of the fact of the refusal, and that the holder intends to recover any damages which he may sustain in consequence.
- 483. In case of non-payment, when the parties to a bill live in the same country, it is not necessary, although frequently

practised, that there should be a regular protest by a public notary. But a notice simply of non-payment is sufficient to entitle the holder to claim interest.

484. The damages incurred by non-acceptance and non-payment of a foreign bill, besides interest commencing from the day of demand, consist usually of the exchange or re-exchange, commission, and postage, together with the expenses of protest and interest. The damages of protested bills in general, however, are regulated in a great measure by the local laws and usages of the different States and countries.

# LIABILITIES OF THE PARTIES.

- 485. The drawer, acceptor, and each and every indorser of a bill, are liable to the payment of it; and though the holder can have but one satisfaction, yet, till such satisfaction is actually had, he may sue any of them, or all of them, either at the same time or in succession, and obtain judgment against them all, till satisfaction be made.
- 486. Nothing will discharge an indorser from his engagement, but the absolute payment of the money; not even a judgment recovered against the drawer, or any previous indorser, or any execution against any of them, unless the money be paid in consequence.

Note. — When acceptance is refused, and the bill is returned by protest, an action may be commenced immediately against the drawer, though the regular time of payment be not arrived. His debt, in such a case, is considered as contracted the moment the bill is drawn.

In order, however, to make the indorsers liable, it is proper that the holder should present the bill for payment on the day it becomes due.

# PAR OF EXCHANGE.

- 487. The intrinsic par of exchange is the value the coins of one country have, when compared with those of another, with respect both to weight and fineness.
- 488. The commercial par of exchange is the value the coins of one country sell for in the markets of another; and is therefore not a fixed, but a variable value.

# Course of Exchange.

- 489. The course of exchange is the variable price of the money of one country, which is paid for a fixed sum of money of another country.
- 490. The fluctuations of exchange are occasioned by various circumstances, both political and commercial, but in general bills rise or fall in their prices, like any other salable articles, according to the relation existing for the time being between the demand and the supply.
  - 491. The limits within which the fluctuations of exchange range, correspond with the cost of making remittances in cash. Therefore, in the time of peace, exchange seldom remains long unfavorable to any country. When unfavorable, it has a tendency to correct itself, by giving an unusual stimulus to exportation, and by throwing obstacles in the way of importation; and when favorable, it produces the same effect, by restricting exportation and facilitating importation.

### INLAND BILLS.

492. An INLAND BILL of exchange, or draft, is one of which the drawer and drawee are residents of different parts of the same country.

Inland bills are seldom bought or sold at the precise sum specified upon their face, but, according to the course of exchange, are subject to a discount, or command a premium.

- 493. To compute inland exchange.
- Ex. 1. What is the value of the following bill of exchange or draft, at \( \frac{1}{8} \) per cent. premium ? Ans. \( \frac{2}{5} \) 2563.20.

\$ 2560. New York, April 14, 1857.

At sight, pay to Dura Wadsworth, or order, two thousand five hundred and sixty dollars, value received, and charge the same to the account of CAMERON, BASHFORD, & Co.

To Messrs. LAWRENCE & ASPINWALL, Merchants, Boston.

OPERATION.

 $\$2560 \times 1.00 = \$2563.20.$ 

RULE. — Multiply the face of the bill or draft by 1 increased by the rate per cent. of premium, or by 1 decreased by the rate per cent. of discount, expressed decimally. The product will be the value of the given bill or draft.

NOTE. — When there is interest to be computed, it must be reckoned on the face of the bill or draft. When other than the value or cost of the bill or draft is to be found, proceed as in percentage.

### EXAMPLES.

2. What is the value of the following bill, or draft, at \(\frac{1}{4}\) of 1 per cent. premium?

Ans. \\$1955.37.

\$ 1950-8%.

Chicago, August 3, 1857.

Thirty days after date, please pay to the order of Robert S. Davis & Co., one thousand nine hundred fifty  $\frac{50}{100}$  dollars, value received, and charge the same to our account.

KEENE & LEE.

To GEORGE REED, Broker, Boston.

3. What is the cost of a draft on Philadelphia, at ½ per cent. premium, the face being \$ 2000?

Ans. \$ 2010.

4. What must be the face of a draft, at 2 per cent. discount, to cost \$ 1744.40?

5. What is the cost of a 60 days' bill on Pittsburg to the amount of \$600, at 1 per cent. discount, and interest off at 6 per cent.?

Ans. \$587.70.

6. What is the cost of a 30 days' bill on Boston, at § per cent. premium, and interest off at 6 per cent., the face of the bill being \$ 9256.40?

Ans. \$ 9240.20.

7. What must be the face of a 30 days' bill which will yield \$9240.20 when sold at \( \frac{3}{2} \) per cent. premium, and interest off at 6 per cent.?

8. A 15 days' draft yielded \$1190.184 when sold at 1½ per cent. discount, and interest off at 6 per cent. What was the face of the draft?

Ans. \$1212.

### FOREIGN BILLS.

- 494. A FOREIGN BILL of exchange is one of which the drawer and drawee are residents of different countries.
  - 495. Foreign bills are usually drawn in sets; that is, at

the same time there are drawn two or more bills of the same tenor and date, each containing a condition that it shall continue payable only while the others remain unpaid. Each bill of a set is remitted in a different manner, and when one of the set has been accepted and paid, the others become worthless.

In reckoning the value of foreign bills of exchange, an acquaintance with the moneys of account of foreign countries is required.

496. The moneys of account of the principal places of commercial importance, with the par value of the unit, expressed in United States money, are shown in the following

TABLE.

Cities and Countries.	Denominations of Money.	Value.
London, Liverpool, &c.,	12 pence = 1 shilling; 20s. = 1 pound =	\$4.84
Paris, Havre, &c.,	100 centimes = 1 franc =	0.186
Amsterdam, Hague, &c.,	100 cents = 1 guilder or florin =	0.40
Bremen,	5 swares = 1 grote; 72gr. = 1 rix dollar =	0.78
Hamburg, Lubec, &c.,	12 pfennings = 1 schilling; 16s. = 1 mark	•
Berlin, Dantzic,	12 pfennings = 1 groschen; 80gr.=1 thaler=	0.35
Belgium,	100 centimes = 1 franc =	0.69
St. Petersburg,	100 kopecks = 1 ruble =	0.186
Stockholm.	12 rundstycks = 16 skillings; 48s. = 1 rix	0.75
o to canomi,	dollar specie = 10 skinnings; 40s. = 1 mx	
Copenhagen,	16 skillings = 1 mark; 6m. = 1 rix dollar =	1.06
Vienna, Trieste, &c.,	60 kreutzers = 1 florin =	1.05
Naples,		0.483
Venice, Milan, &c.,	10 grani = 1 carlino; 10car. = 1 ducat = 100 centesimi = 1 lira =	0.80
Florence, Leghorn, &c.,	100 centesimi = 1 lira =	0.16
Genoa, Turin, &c.,	100 centesimi = 1 lira =	0.16
Sicily,		0.186
Portugal,	20 grani = 1 taro; 30 tari = 1 ounce = 1000 reas = 1 millrea =	2.40
<b>.</b>		1.12
Spain,	34 maravedis = 1 real vellon =	0.05
Constantinople,	68 maravedis = 1 real plate =   100 aspers = 1 plaster =	0.10
British India.		0.05
	12 pice = 1 anna; 16 annas = 1 rupce =	0.44
Canton,	100  candarines = 1  mace; $10m. = 1  tael = 1$	1.48

NOTE 1. — Grani is the plural of grano, carlini of carlino, centesimi of centesimo, lire of lira, tari of taro.

Note 2. — The moneys of account in Brazil are of the same denominations as in Portugal; in Mexico, Centrul America, New Granada, Chili, Venezuela, Bolivia, Peru, and Buenos Ayres, accounts are kept in pesos, or dollars, of 8 reals each; in Cuba accounts are kept in dollars equal 8 reals plate, or 20 reals vellon, equal \$1; in British American Possessions accounts are kept in the denominations of sterling money, but of a depreciated value compared with the currency of the same denominations in England, except in Canada, where, by an act of the Colonial Parliament, passed in 1857, the decimal currency of the United States has been adopted.

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497. The exchange value, in United States money, of the pound sterling of Great Britain is that of its former legal value, or \$4\frac{1}{2} = \$4.44\frac{1}{2}\$, which is considerably below either its intrinsic or commercial value. The commercial value is generally about 9 per cent. more than this exchange, or nominal par value.

Thus, nominal par value being = \$4.44\$
Add premium at 9 per cent. = .40

The commercial par value will be  $= $4.84\frac{4}{3}$ .

Therefore, when the nominal exchange between the United States and Great Britain exceeds 9 per cent. premium, it is above true par; when less, it is below true par.

498. The quotations of the rates of exchange of the United States on England have always reference to the old par value of the pound sterling in United States money. The course of exchange of the United States on France is so many frances and centimes payable in France for a dollar paid here; on Holland it is so many cents a guilder (of Netherlands); on Hamburg, so many cents a mark banco; on Bremen, so many cents a rix dollar; and so on.

Note 1. — The course of exchange of London on France is so many francs and centimes payable in France for  $1\pounds$ .; on Amsterdam, it is so many florins for  $1\pounds$ .; on Hamburg, so many schillings for  $1\pounds$ .; on Vienna and Trieste, so many florins and creutzers; on Spain, so many pence paid in England for 1 dollar of plate (=  $8\frac{3}{6}$  reals plate) payable in Spain; on Lisbon, so many pence for 1 millrea; on Naples, so many pence for 1 ducat; and so on.

NOTE 2. — The value of  $1\pounds$  sterling from 7 to  $10\frac{3}{4}$  premium on the old par value of  $\$4.44\frac{4}{9}$ , is shown in the following

## TABLE.

7 pe	er cent.	premium,	\$4.756	9	per cent	premium,	\$ 4.844
71	46	- "	4.767	91	- "	- "	4.856
7₹	"	"	4.778	91	"	"	4.867
7∰	"	"	4.789	9\$	"	".	4.878
7₹ 8	"	"	4.80	10*	"	"	4.889
81	46	46	4.811	101	"	"	4.90
8 <del>1</del>	"	"	4.822	101	66	66	4.911
8 <del>\$</del>	"	"	4.833	104	"	ш	4.922

NOTE 3. — In this country the quotations of foreign exchanges are usually for bills payable 60 days after sight, and of inland or domestic exchanges, for bills payable at sight.

499. To compute foreign exchange.

Ex. 1. What should be paid for the following bill at 9½ per cent. premium?

Ans. \$486.666.

Exchange for £100.

Philadelphia, May 21, 1857.

Sixty days after sight of this, my first Bill of Exchange, (second and third of the same date and tenor unpaid,) pay to Langdon Shannon, or order, one hundred pounds sterling, value received, with or without further advice.

WILLIAM P. BROWN.

To Messes. Peabody & Co., Bankers, London.

OPERATION.

 $100 \times \frac{49}{9} = $444.44; $444.444 \times 1.095 = $486.666.$ 

 What is the cost at Amsterdam of a bill on New York for \$ 340.67, exchange being at \$ 0.38 to the guilder?
 Ans. 896.50 guilders.

OPERATION.

 $340.67 \div 38 = 896.5 = 896$  guilders 50 cents.

- 3. What must be paid in Boston for a bill on Paris for 3676 francs, exchange being 5 francs 20 centimes to the dollar?

  Ans. \$706.924.
- 4. What is the value of a bill on Hamburg for 3000 marks 10 schillings, exchange being at \$0.35 to a mark banco?

  Ans. \$1050.217.
- 5. How large a bill can be purchased on Liverpool for \$81727.75, exchange being at 9½ per cent. premium?
- 6. Paid \$14400.12 for a bill on Havre for 79000 francs; how much was exchange below par?

  Ans. 2 per cent.
- 7. What is the cost of a draft on St. Petersburg for 5763 rubles 75 kopecks, exchange being at 74 cents a ruble?

  Ans. \$ 4265.175.
- 8. What must the face of a bill on Lisbon be which costs \$550.66, exchange being at \$1.10 a millrea?

Ans. 500 millreas 600 reas.

- 9. What is the cost at Berlin of a draft on Philadelphia for \$ 10000, exchange being at \$ 0.68 a thaler?
  - Ans. 14705 thalers 26 groschen 5++ pfennings.
- 10. How much must be paid in Chicago for a bill on Stock-holm amounting to 400 specie rix dollars 12 skillings?

Ans. \$ 424.265.

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- 11. What is the value in St. Louis of a draft on Dantzic for 300 thalers 20 groschen 10 pfennings, exchange being at \$0.69 a thaler?

  Ans. \$207.4741.
- 12. What must be the face of a draft on Calcutta which costs in Boston \$5694, when exchange is at \$0.40 a company rupee?

  Ans. 14235 rupees.
- 13. How much must be paid in Naples for a draft on Baltimore amounting to \$615.60, the value of a ducat in exchange being \$0.80?

  Ans. 769 ducats 5 carlini.
- 14. There was paid in New Orleans \$7300 for 1500£. draft on Liverpool; at what per cent. of premium was it purchased?

  Ans. 9½ per cent.
- 15. What must be the face of a draft on Paris that can be bought in London for 868£. 17s. 6d., exchange at 23 francs 60 centimes a pound sterling? Ans. 20505 francs 45 centimes.
- 16. How much must be paid in Genoa for a bill on New York whose face is \$2640, when exchange is at \$0.18 a lire?

  Ans. 14666 lire 66% centesimi.
- 17. When a bill on Paris for 88128 francs costs \$17280, at what per cent is the rate of exchange above par?
- 18. Robert Anderson, of Cincinnati, has consigned a cargo of pork, valued at 17000£, to Richard Arkwright & Son, Liverpool. Robert S. Davis & Co., being about to import an invoice of books, have purchased of Anderson a bill of exchange, at 8½ per cent. premium, for the value of the said cargo. What should they pay for the bill?

  Ans. \$81984.444.

# ARBITRATION OF EXCHANGES.

500. Arbitration of Exchanges is the process of finding the proportional exchange of two places by means of one or more given intermediate exchanges.

Exchange effected thus, through one or more intermediate exchanges, is called *circular* exchange.

The exchange when made through a single intervening ex-

change is called *simple arbitration*, and when through two or more intervening exchanges is called *compound arbitration*.

- 501. Since the actual course or rate of exchange between any two places is almost always, from various circumstances, different from the arbitrated course, the object of arbitration is to enable an individual in one place to ascertain whether he can most advantageously draw and remit directly between his own place and another, or circuitously through other places.
- 502. Exchange of merchandise, and the different weights and measures of different countries, may be arbitrated in the same manner as bills of exchange and currencies.
- 503. The value, in the standards of the United States, of the principal weights and measures of the most important commercial places, is shown in the following

1 ABLE.					
			Value.		
England.		Fuder = 6 ahms,	229.5gal.		
	Value.	Last of grain,	89.64bu.		
Avoirdupois pound,	11b.	Stock = 13 last,	134.4bu.		
Old wine gallon,	1gal.	Brabant ell.	27.58in.		
Imperial gallon,	1.20gal.				
Old ale gallon,	1.22gal.	DENMARK AND NOR	WAY.		
Old Winchester bushel,	1bu.	Pound,	1.10lb.		
Imp. corn bu. (= 8 imp. ga	l.) <b>, 1.</b> 03bu.	Centner = 100 pounds,	110.28lb.		
Quarter of grain (=8 imp.b		Viertel of wine,	2.04gal.		
Imperial yard,	36in.	Anker of wine.	10gal.		
FRANCE.		Ahm = 4 ankers,	40gal.		
Kilogramme,	2.20lb.	Fuder of wine,	237.16gal.		
Quintal = 100kil.,	220.54lb.	Toende or barrel of grain,	8.95bu.		
Millier = 1000kil.	2205.48lb.	Last = 12 toende	47.50bu.		
Litre,	2.11pt.	Danish ell,	24.66in.		
Velt,	2gal.	•			
Decalitre = 10 litre,	2.64gal.	Sweden.	0011		
Hectolitre = 100 litre,	26.42gal.	Pound,	.93lb.		
Hectolitre = 100 litre,	2.84bu.	Pound of iron,	.75lb.		
Metre,	89.86in.	Anker of wine,	10.35gal.		
·		Eimer of wine,	20.75gal.		
HOLLAND AND BEL		Ahm = 2  eimers,	41.50gal.		
Pond,	1.08lb.	Pipe = 3 ahms,	124.25gal.		
Fr. kilogramme,	2.20lb.	Tun or barrel of grain,	4.16bu.		
Last, marine, $= 2000$ p.	4410lb.	Ell,	23.36in.		
Vat = 100kan = 1 hectol. I	r., 26.42gal.	Bremen.			
Ahm of wine,	41gal.	Pound,	1.09lb.		
Mudde=100 kop=1 hectoli	tre, 2.84bu.	Centner,	116lb.		
Last of grain,	85.25bu.	Viertel of wine,	1.93gal.		
Amsterdam ell,	27in.	Anker = 5 viertels,	9.65gal.		
Pound, HAMBURG.	1.06lb.	Oxhoft = 6 ankers,	58gal. <b>2</b> bu.		
·100 nonnda		Scheffel of grain,	80.70bu.		
100 pounds,	106.8lb.	Last = 40 scheffels,	22.75in.		
Ahm of wine,	38.25gal.	Tall,	44. ( UILL.		

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TRIESTE.	Value.	Cantaro or arroba of wine,	Value.
Pound,	1.23lb.	Moyo of wine = 16 arrobas.	4.25gal. 68gal.
100 pounds,	123.60lb.	Botta = $38ar$ . of wine = $38$	
Eimer of wine,	15gal.	of oil,	127.5gal.
Staro of grain,	234bu.		1.57bu.
E'l for silks,	25.20in.	Fanega of grain, Cahiz = 12 fanegas,	18.91bu.
Ell for woollens,	<b>2</b> 6.60in.	Vara or yard,	88.87in.
Constantinopl	E.	Cuba.	00.01 m.
Quintal,	124.45lb.		101.75lb.
Alma for liquids,	1.37gal.	Quintal,	4.1gal.
Kisloz of grain,	.94bu.	Arroba of wine,	3bu.
Pik, commercial,	27in.	Fanega of grain,	83.84in.
CALCUTTAL		Vara,	00.04III.
Maund,	74.66lb.	NAPLES.	<del></del>
Bazaar maund,	82.13lb.	Rottolo,	1.96lb.
Guz,	86in.	Cantaro grosso=100 rottolo	
		Cantaro piccolo,	106lb.
Russia.		Salma of oil,	42.75gal.
Pound,	.90lb.	Carro of wine	264gal.
Pood = 40 pounds,	86lb.	Carro of grain,	52.20bu.
100 pounds,	90.26lb.	Canna,	83in.
Wedro of wine,	8.25gal.	Sicily.	
Sorokovy = 40 wedros,	130gal.	Cantaro grosso,	192.50lb.
Chetwert of grain,	5.95bu.	Cantaro sottile,	175lb.
Arsheen,	28in.	100 Sicilian pounds,	70lb.
Sashen,	7ft.	Tonna,	9.38gal.
Prussia.		Salma grossa,	9.48bu.
Pound,	1.03lb.	Salma generale,	7.62bu.
100 pounds, Dantzic,	108.8lb.	Conna or yard,	38.40in.
Quintal = 110 pounds,	118.42lb.	GENOA.	
Eimer of wine,	18.14gal.	f	MA OFIL
Ahm,	39.66gal.	Cantaro grosso,	76.87lb.
Scheffel of grain,	1.52bu.	Cantaro sottile,	69.89lb.
Last of grain,	91bu.	Mezzarola,	39.25gal.
Berlin Ell,	25.5in.	Mina of grain,	8.50bu.
Prussian Éll,	26.28in.	Canna piccola,	87.50in.
PORTUGAL.		Canna grossa,	116.70in.
Pound or arratel,	1.01lb.	Venice.	
Arroba = 22 arratels,	22.26lb.	100 pounds, pesso grosso,	105.18lb.
Quintal = 4 arrobas,	89.05lb.	100 pounds, pesso sottile,	66.42lb.
100 pounds or arratels,	101.19lb.	Miro of oil,	4.02gal.
Almude of wine,	4.37gal.	Anfora of wine,	1.87gal.
Tonelado $= 52$ almude,	227.25gal.	Staja of grain,	2.27bu.
Moyo,	23.03bu.	Moggio = 4 staji,	9.08bu.
Vara,	48.20in.	Braccio for silks,	24.84in.
	2012022	Braccio for woollens,	26.64in.
Spain.	1 0111	CHINA.	
Pound,	1.01lb.		1.83lb.
Arroba = 25 pounds,	25.88lb.	Catty,	133.83lb.
Quintal = 4 arrobas,	101.52lb.	Pecul,	133.8310. 14.62in.
Cantaro or arroba of oil,	8.75gal.	i Ouviu,	TA.OAIII.

Note. — The weights and measures of Mexico, Central America, and of the republics of South America are the same generally as those of Spain; of Brazil, the same as those of Portugal; of the British North American Provinces, the same, in general, as in England; and of Hayti, the weights are the same as in the United States, except about 8 per cent. heavier, and the measures the same as in France.

- 504. To compute arbitration of exchanges.
- Ex. 1. When the exchange between New York and London is at a premium of 9 per cent., and that between London and Paris 25 francs to a pound sterling, how much must be paid in New York for a bill on Paris for 1000 francs?

Since \$4\frac{4}{2} = 1\hat{\Lambda}. of the nominal par of exchange (Article 497), \$40=9\hat{\Lambda}.; and 109\hat{\Lambda}. of the same value = 100\hat{\Lambda}. at 9 per cent. premium.

We write the terms of equivalent value as antecedent and consequent, and proceed as in conjoined proportion (Art. 341).

NOTE. — When it is required to find which of several routes of exchange is the most advantageous, the rate of exchange by each route may be determined first, and the results then compared.

### EXAMPLES.

2. When exchange at Lisbon on Paris is at the rate of 5 francs 95 centimes per millrea, and at Paris on the United States at 5 francs 20 centimes per dollar, how much must be paid in Lisbon to cancel a demand in New York for \$3500?

Ans. 3058 millreas  $823\frac{9}{17}$  reas.

- 3. A merchant in Boston wishes to pay 2000£ in Liverpool. Exchange on Liverpool he finds is at 10 per cent. premium, on Paris 5 francs 20 centimes to a dollar, and on Hamburg 35 cents to a mark banco; and the exchange between France and England at the same time is 24 francs to a pound sterling, and that of Hamburg on England 13¾ marks banco to a pound sterling. Which is the most advantageous course of remittance, that direct to Liverpool, or that through Paris or Hamburg?
- 4. A merchant of St. Louis wishes to pay a debt of \$5000 in New York. The direct exchange is 1½ per cent. in favor of New York, but on New Orleans it is ½ per cent. premium; and between New Orleans and New York at ½ per cent. discount. How much would he save by the circular exchange compared with the direct?

  Ans. \$87.56½.

- 5. A merchant in Boston owes a debt of 9760 thalers in Bremen, to pay which he purchases a bill on London, at a premium of 9 per cent., and remits the same to his agent in England, on whom his creditor is requested to draw. If the exchange between London and Bremen be at the rate of 34d. sterling per thaler, and the charges for brokerage ½ per cent., how much must have been the cost of the bill in New York?

  Ans. \$6731.74+.
- 6. When exchange between New Orleans and Hamburg is at 34 cents per mark banco, and between Hamburg and St. Petersburg is 2 marks 8 schillings per ruble, how much must be paid in St. Petersburg for a bill on New Orleans for \$650?

  Ans. 764 rubles 7019 kopecks.
- 7. When exchange in Philadelphia on Boston is at 4 per cent. premium, and on Chicago at 2 per cent. discount, if the exchange between Chicago and Boston is at par, how much better is the circuitous route of exchange between Philadelphia and Boston than the direct?
- 8. A merchant, about to import broadcloth, finds he can obtain the quality desired in Amsterdam at 8 guilders per Amsterdam ell; in Berlin, at 3 thalers 15 groschen per Berlin ell; and in England, at 15 shillings per yard. Exchange being on Amsterdam at 40 cents per guilder, on Berlin at 66 cents per thaler, and on England at  $9\frac{1}{2}$  per cent. premium, and the freight being the same in each case, from which place can he make the importation to the best advantage?

  Ans. Berlin.
- 9. When exchange between Washington and London is at 8 per cent. premium, and between London and Paris 25.25 francs per pound sterling, what sum in Washington is equal to 7000 frances in Paris?
- 10. A merchant in London remits to Amsterdam 1000£. at the rate of 18d. per guilder, directing his correspondent at Amsterdam to remit the same to Paris at 2 francs 10 centimes per guilder, less ½ per cent. for his commission; but the exchange between Amsterdam and Paris happened to be, at the time the order was received, at 2 francs 20 centimes per guilder. The merchant at London, not apprised of this, drew upon Paris at 25 francs per pound sterling. Did he gain or lose, and how much per cent.? Ans. Gain, 16‡‡ per cent.

### ALLIGATION.

**505.** Alligation is a process employed in the solution of questions relating to the compounding or mixing of articles of different qualities or values.

It is of two kinds: Alligation Medial, and Alligation Alternate.

### ALLIGATION MEDIAL.

- 506. Alligation Medial is the process of finding the mean or average rate of a mixture composed of articles of different qualities or values, the quantity and rate of each being given.
- 507. To find the average value of several articles mixed, the quantity and rate of each being given.
- Ex. 1. A grocer mixed 2cwt. of sugar worth \$9 per cwt. with 1cwt. worth \$7 per cwt. and 2cwt. worth \$10 per cwt.; what is 1cwt. of the mixture worth?

  Ans. \$9.

RULE. — Find the value of each of the articles, and divide the sum of their values by the number denoting the sum of the articles. The quotient will be the average value of the mixture.

### EXAMPLES.

- 2. If 19 bushels of wheat at \$1.00 per bushel should be mixed with 40 bushels of rye at \$0.66 per bushel, and 11 bushels of barley at \$0.50 per bushel, what would a bushel of the mixture be worth?

  Ans. \$0.727.
- 3. If 3 pounds of gold of 22 carats fine be mixed with 3 pounds of 20 carats fine, what is the fineness of the mixture?

  Ans. 21 carats.
- 4. If I mix 20 pounds of tea at 70 cents per pound with 15 pounds at 60 cents per pound, and 80 pounds at 40 cents per pound, what is the value of 1 pound of this mixture?

Ans. \$ 0.4713.

### ALLIGATION ALTERNATE.

- **568.** Alligation Alternate is a process of finding is what ratio, one to another, articles of different rates of quality or value must be taken, to compose a mixture of a given mean or average rate of quality or value.
- 509. To find the proportional quantities of the articles of different rates of value that must be taken to compose a mixture of a given mean rate of value.
- Ex. 1. A merchant has spices, some at 18 cents a pound, some at 24 cents, some at 48 cents, and some at 60 cents. How much of each sort must be taken that the mixture may be worth 40 cents a pound?

Ans. 1lb. at 18c.; 1lb. at 24c.; 1lb. at 8c.; 1½lb. at 60c.

Given mean, 
$$\begin{cases} 1 \text{lb. at } 18\text{c., gain } 22\text{c.} \\ 1 \text{lb. at } 24\text{c., gain } 16\text{c.} \end{cases} = 88\text{c.,} \\ 1 \text{lb. at } 24\text{c., gain } 16\text{c.} \end{cases} = 88\text{c.,} \\ 1 \text{lb. at } 24\text{c., gain } 16\text{c.} \end{cases} = 88\text{c.,} \\ 1 \text{lb. at } 48\text{c., loss } 8\text{c.} \\ 1 \text{lb. at } 60\text{c., loss } 20\text{c.} \end{cases} = 88\text{c.,} \\ 1 \text{lb. at } 48\text{c.} = 48\text{c.} \\ 1 \text{lb. at } 60\text{c., loss } 20\text{c.} \end{cases} = 88\text{c.,} \\ 1 \text{lb. at } 48\text{c.} = 48\text{c.} \\ 1 \text{lb. at } 60\text{c.} = 90\text{c.} \\ 1 \text{loss.} \end{cases} = 180\text{c.} \end{cases} = 180\text{c.}$$

Compared with the given mean value, by taking 1lb. at 18cts. there is a gain of 22cts., by taking 1lb at 24cts. a gain of 16cts., by taking 1lb. at 48cts. a loss of 8cts., and by taking 1lb. at 60cts. a loss of 20cts. Now it is evident that the mixture, to be of the mean or average value given, should have the several items of gain and loss in the aggregate exactly offset one another. This balance we effect by taking 1lb. more of the spice at 60cts.; and thus have a mixture of the required average value, by having taken, in all, 1lb. at 18cts., 1lb. at 24cts., 1lb. at 48cts., and 11lb. at 60cts. We prove the correctness of this result by dividing the value of the whole mixture by

$$40 \begin{cases} 18 & \frac{1}{2} & 20 & 10 \\ 24 & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{4}{8} & & \\ 60 & \frac{1}{20} & 22 & 11 \end{cases}$$

the number of pounds taken.

Having arranged in a column the rates of the articles, with the given mean on the left, we connect together terms denoting the rate of the articles; so that a rate less than the given mean is united

with one that is greater. We then proceed to find what quantity of each of the two kinds whose rates have been connected can be taken, in making a mixture, so that what shall be gained on the one kind shall be balanced by the loss on the other.

By taking 1lb. at 18cts. the gain will be 22cts.; hence it will require

 $\frac{1}{12}$ lb. to gain 1ct.; and by taking 1lb. at 60cts the loss will be 20cts.; hence it will require  $\frac{1}{20}$ lb. to lose 1ct. Therefore, the gain on  $\frac{1}{22}$ lb. at 18cts. balances the loss on  $\frac{1}{20}$ lb. at 60cts. The proportions at these rates are, then,  $\frac{1}{22}$  and  $\frac{1}{20}$  or (by reducing to a common denominator)  $\frac{20}{40}$  and  $\frac{22}{40}$ , or (by omitting the denominators, which do not affect the ratio) 20 and 22, which is obviously the same result as would be obtained by placing against each rate the difference between the rate with which it is connected and the mean rate. In like manner we determine the quantity that may be taken of the other two articles, whose rates are connected together.

We thus find that there may be taken  $\frac{1}{22}$ lb. at 18cts.,  $\frac{1}{16}$ lb. at 24cts.,  $\frac{1}{3}$ lb. at 48cts., and  $\frac{1}{20}$ lb. at 60cts.; or, 20lb. at 18cts., 8lb. at 24cts., 16lb. at 48cts., and 22lb. at 60cts. By dividing the last set by 2, we obtain another set of results, and by multiplying or dividing any of these results others may be found, all of which can be proved to satisfy the conditions of the question. Hence, examples of this

kind admit of an indefinite number of answers.

RULE 1. — Take a unit of each article of the proposed mixture, and note the gain or loss; and then take such additional quantity or quan-

tities of the articles as shall equalize the gain and loss. Or,

RULE 2. — Write the rates of the articles in a column, with the mean rate on the left, and connect the rate of each article which is less than the given mean with one that is greater; the difference between the mean rate and that of each of the articles, written opposite to the rate with which it is connected, will denote the quantity to be taken of the article corresponding to that rate.

Note. — When a rate has more than one rate connected with it, the sum of the differences written against it will denote the quantity to be taken. There will be as many different answers as there are different ways of connecting the rates; and, by multiplying and dividing, these answers may be varied indefinitely.

#### EXAMPLES.

2. How much barley at 45 cents a bushel, rye at 75 cents, and wheat at \$1.00, must be mixed, that the composition may be worth 80 cents a bushel?

Ans. 1 bushel of rye, 1 of barley, and 2 of wheat.

3. A goldsmith would mix gold of 19 carats fine with some of 15, 23, and 24 carats fine, that the compound may be 20 carats fine. What quantity of each must be take?

Ans. 1oz. of 15 carats, 2oz. of 19, 1oz. of 23, and 1oz. of 24.

4. It is required to mix several sorts of wine at 60 cents, 80 cents, and \$1.20, with water, that the mixture may be worth 75 cents per gallon; how much of each sort must be taken?

Ans. 1gal. of water, 1gal. of 60 cents, 9gal. of 80 cents, and 1gal. of \$1.20.

- 510. When the quantity of one or more of the articles composing a mixture of a given mean value is given, to find the quantity of each of the others.
  - Ex. 1. How much gold of 15, 17, and 22 carats fine must be mixed with 5 ounces of 18 carats fine, so that the composition may be 20 carats fine?

Ans. loz. at 15 carats, loz. at 17 carats, 9oz. at 22 carats.

and loss we take 80z. additional, at 22 carats, a loss of 16. We have then for the result 10z. at 15 carats, 10z. at 17 carats, and 90z. at 22 carats.

RULE. — Take of the limited article or articles the quantity or quantities given, with a unit of each of the other articles of the proposed mixture, and note the gain or loss; and then take, if required, such additional quantity or quantities of the articles not limited, as shall equalize the gain and loss.

### EXAMPLES.

- 2. How much wine at \$1.75 and at \$1.25 per gallon must be mixed with 20 gallons of water, that the whole may be sold at \$1.00 per gallon?

  Ans. 20 gallons of each.
- 3. How much wheat at \$2.00 per bushel and at \$1.80 per bushel must be mixed with 4 bushels at \$2.20 per bushel and 10 bushels at \$1.70 per bushel to make a mixture worth \$1.90 per bushel? Ans. 9 bushels at \$2.00; 1 bushel at \$1.80.
- 4. How many pounds of sugar, at 8, 14, and 13 cents a pound, must be mixed with three pounds at  $9\frac{1}{4}$  cents, 4 pounds at  $10\frac{1}{2}$  cents, and 6 pounds at  $13\frac{1}{2}$  cents a pound, so that the mixture may be worth  $12\frac{1}{2}$  cents a pound?

Ans. 1lb. at 8cts.; 51lb. at 13cts.; and 9lb. at 14cts.

5. How much barley at 45 cents a bushel must be mixed with 10 bushels of oats at 58 cents a bushel, to make a mixture worth 50 cents a bushel?

- 511. When the quantity and rate of a mixture, with the rates of the articles composing it, are given, to find the quantity of each article which is not limited.
- How many gallons of water must be mixed with wine at \$1.50 a gallon so as to make a mixture of 100 gallons Ans. 20 gallons. worth \$1.20 a gallon?

rate of the water by 0.00, we then find, as in Art. 509, the quantity required of each article, in comthe given mean, to

Representing the

be 1 gallon of water and 4 gallons of the wine. Therefore, the quantity of water is to the whole quantity of the mixture as 1 to 5. Hence, in a mixture of 100 gallons at the mean rate given, the water must be 4 of 100 gallons, or 20 gallons.

RULE. - Find the proportional quantities of the several articles, as in Art. 509, or 510, as though the quantity of the mixture were not

Then take such a part of the given quantity of the mixture, as each of these proportional quantities is of their sum.

#### EXAMPLES.

- 2. A merchant has sugar at 8 cents, 10 cents, 12 cents, and 20 cents a pound; with these he would fill a hogshead that would contain 200 pounds. How much of each kind must he take, so that the mixture may be worth 15 cents a pound?
  - Ans. 334lb. of 8, 10, and 12cts., and 100lb. of 20cts.
- 3. How much wheat at \$2.00 and \$1.80 a bushel must be mixed with 4 bushels at \$2.20, and 10 bushels at \$1.70, so as to make a mixture of 48 bushels, worth \$ 1.90 per bushel?

Ans. 21 bushels \$ 2.00; 13 bushels at \$ 1.80.

- 4. How much gold of 15, 17, and 22 carats fine must be mixed with five ounces of 18 carats fine, to make a composition of 5 pounds, that shall be 20 carats fine?
- 5. A gentleman's servant having been ordered to purchase 20 animals for \$20, brought home sheep at \$4.00, lambs at \$0.50, and kids at \$0.25 each. Required the number of each kind. Ans. 3 sheep; 15 lambs; and 2 kids.

## MISCELLANEOUS EXAMPLES.

1. A manufacturer employs a number of men at \$1.20, and a number of boys at \$0.80, per day; and the amount of the wages of the whole is the same as if each had \$0.97½ per day. Required the number of men, that of the boys being 9.

Ans. 7 men.

- 2. What is the value of 5000 specie rix dollars 12 skillings of Sweden in United States money?

  Ans. \$5300.265.
- 3. Exchange between New Orleans and England being in New Orleans at 8 per cent. premium, and in Liverpool at 10 per cent. premium, if L. Sandford of Liverpool owes M. Lassale of New Orleans for cotton to the amount of 1500£. 15s. sterling, what will be the difference between Lassale drawing or Sandford remitting the amount?
- 4. If 17 gallons of spirits at \$1.26 per gallon be mixed with 7 gallons at a different price, and 20 per cent. be made by selling the mixture at \$1.56, what was the price of the latter kind per gallon?

  Ans. \$1.39\$ per gallon.
- 5. What is the value of 100 ounces 20 tari 10 grani of Naples in lire and centesimi of Leghorn?
- 6. If 20 United States gallons equal 1 eimer of Sweden, 3 eimers of Sweden equal 4 eimers of Trieste, 24 eimers of Trieste equal 9 ahms Danish, and 33 ahms Danish equal 5 carri of Naples, which will cost the most in United States money, 170 eimers of Trieste of wine at 1 florin 45 kreutzers per gallon, or 12 carri of wine at 1 ducat of Naples a gallon?
- 7. When exchange on England is at 8 per cent premium, and freight at 12d. per United States bushel, how much can be paid per bushel for wheat in Baltimore, in answering an order from Liverpool limited to 60s. per imperial quarter?

Ans. \$ 1.50 per bushel.

Ans. 1510 lire 25 centesimi.

8. A merchant mixes 11 pounds of tea with  $\frac{5}{5}$  pounds of an inferior quality, and gains 16 per cent. by selling the mixture at 87 cents per pound. Allowing that a pound of the one cost 12 cents more than a pound of the other, what was the cost of each kind per pound?

Ans. The one  $78\frac{3}{4}$ cts.; the other  $66\frac{3}{4}$ cts. per lb.

## INVOLUTION.

512. Involution is the process of finding the powers of quantities.

A power of a number or quantity is the result obtained by taking that quantity a certain number of times as a factor.

513. The number from which a power is derived is called the root of that power.

The first power is the root, or the number involved.

The second power is the product of the root multiplied by itself once, or used twice as a factor.

The third power is the root used three times as a factor; &c.

514. The *index* or *exponent* of a power is a small figure written at the right, above the root, indicating the number of times it is employed as a factor. Thus, the second power of 4 is written  $4^2$ , the third power of 9 is written  $9^3$ , and the fourth power of  $\frac{3}{4}$  is written  $(\frac{3}{4})^4$ .

NOTE. — In denoting the power of a fraction, the fraction is included in a parenthesis, in order that the exponent may be regarded as applying to the whole expression, and not to the numerator alone. When no index is written, the number itself is to be considered the first power. The second power is sometimes called the square of a number, the third power the cube, and the fourth power the biquadrate.

515. To raise a number to any required power.

2 = 2, the first power of 2, is written  $2^1$  or 2.  $2 \times 2 = 4$ , the second power of 2, is written  $2^2$ .  $2 \times 2 \times 2 = 8$ , the third power of 2, " "  $2^3$ .  $2 \times 2 \times 2 \times 2 = 16$ , the fourth power of 2, " "  $2^4$ .

 $2\times2\times2\times2\times2=32$ , the fifth power of 2, " "  $2^5$ .

By examining the several powers of 2 in the examples, it is seen that each has been produced by taking the 2 as a factor as many times as there are units in the exponent of each power raised. Hence the

RULE. — Multiply the given number into itself, till it has been used as a factor as many times as there are units in the exponent of the power to which the number is to be raised.

Note 1. — The number of multiplications will always be one less than the number of units in the exponent of the power to be raised, since in the first

multiplication the root is used twice; once by being taken as the multiplicand, and once more as the multiplier.

Note 2. — A fraction is involved by involving both its numerator and its denominator.

### EXAMPLES.

1. What is the 3d power of 8?	Ans. 512.
2. What is the 5th power of 4?	Ans. 1024.
3. What is the 3d power of ??	Ans. <del>27</del> .
4. What is the 4th power of 23?	Ans. 50 § §.
5. What is the 5th power of $\frac{1}{3}$ ?	Ans. $\frac{1}{2}$
6. What is the 6th power of 5?	Ans. 15625.
7. What is the 6th power of 13?	Ans. 16 <del>12122</del> .
8. What is the value of 710?	Ans. 282475249.
9. What is the value of .0454?	Ans000004100625.

516. To raise a number to any required higher power, without producing all the intermediate powers.

Ex. 1. What is the 7th power of 5? Ans. 78125.

We raise the 5 to the 2d and to the 3d power, and write above each power its exponent. Then, by adding the exponent 2 to itself, and increasing the sum by the exponent 3, we obtain 7, a number equal to the exponent of the required power; and by multiplying 25, the power belonging to the exponent 2, into itself, and the product thence arising by 125, the power belonging to the exponent 3, we obtain 78125, the required 7th power. Therefore,

The product of two or more powers of the same number is that power which is denoted by the sum of their exponents. Hence, the

Rule. — Multiply together two or more powers of the given number, the sum of whose exponents is equal to the exponent of the power required, and the product will be that power.

Note. — When the number to be involved contains a decimal, it is generally sufficient to retain in the result not more than six places of decimals; and the work may be accordingly contracted as in the multiplication of decimals (Art. 273).

## EXAMPLES.

2. What is the 7th power of 8?	Ans. 2097152.
3. What is the 9th power of 7?	Ans. 40353607.
4. What is the 10th power of 6?	• `

- 5. What is the 5th power of 195? Ans. 281950621875.
- 6. What is the 6th power of  $\frac{2}{3}$ ? Ans.  $\frac{64}{728}$ .
- 7. Required the 2d power of 4698.
- 8. Required the 2d power of 6031. Ans. 36372961.
- 9. What is the 13th power of 7? Ans. 96889010407.
- 10. What is the 12th power of 6?
- 11. What is the 15th power of 9? Ans. 205891132094649.
- 12. What is the 4th power of 4.367? Ans. 363.691179+.
- 13. Involve the following numbers to the powers denoted by their respective exponents:  $(2\frac{3}{4})^5$ ,  $1.04^{15}$ , and  $(3\frac{2}{4})^4$ .

Ans.  $157_{1034}^{283}$ ; 1.800943+;  $116\frac{1325}{466}$ .

### EVOLUTION.

- 517. EVOLUTION, or the extraction of roots, is the process of finding the roots of quantities. It is the reverse of involution.
- 518. The root of a quantity or number is such a factor as, being multiplied into itself a certain number of times, will produce that quantity or number.

The root takes the name of the power of which it is the correlative term. Thus, if the number is a second power, the root is called the second or square root; if it is the third power, the root is called the third or cube root; if it is the fourth power, its root is called the fourth or biquadrate root; and so on.

Rational roots are such as can be exactly obtained. Surd roots are such as cannot be exactly obtained.

519. Roots are usually denoted by writing the radical sign,  $\checkmark$ , before the power, with the index of the root over it; in case, however, of the second or square root, the index 2 is omitted. Thus, the third root of 27 is denoted by  $\sqrt[3]{27}$ , the second root of 16 is denoted by  $\sqrt[4]{3}$ .

Roots are sometimes denoted by a fractional index or exponent, of which the numerator indicates the power, or the number of times the number is to be taken as a factor, and the denominator indicates the root, or the number of equal factors into which that product is to be divided. Thus the square or second root of 12 is denoted by  $12^{\frac{1}{2}}$ , the fourth root of  $\frac{2}{5}$  by  $(\frac{2}{5})^{\frac{1}{5}}$ , and the square of the cube root of 27, or the cube root of the square of 27, is denoted by  $27^{\frac{1}{5}}$ .

- 520. All the rational roots of whole numbers are also whole numbers, since every power of a fractional number is also a fractional number.
  - 521. Prime numbers have no rational roots.

A composite number, to have a given rational root, must have the exponent of the power of each of its prime factors exactly divisible by the exponent of that root.

Note. — The number of composite numbers that have rational roots is comparatively small. The number of rational square roots of whole numbers from 1 to 250000 inclusive is only 500, and the number of rational cube roots of whole numbers from 1 to 8000000 inclusive is only 200.

522. The roots represented by the first ten numbers and their first six corresponding powers are shown in the following

_	_			
•	٠.	DI	T	٠.

1st Power, 2d Power, 8d Power, 4th Power,	111111	2 4 8 16	8 9 27 81	16 64 <b>2</b> 56	5 25 125 625	6 86 216 1296 7776	843 2401	512 4096	6561	10000
5th Power, 6th Power,	1 1	32 64	243	1024 4096	8125 15625	7776	16807	32768	59049	

Note. — It will be observed by the table, that a rational square root can only be obtained from numbers ending in 1, 4, 5, 6, or 9; or in an even number of ciphers, preceded by one of these figures. It is true, also, that, if the square number ends in 1, its square root ends in 1 or 9; if in 4, its square root ends in 2 or 8; if in 9, its square root ends in 3 or 7; if in 6, its square root ends in 4 or 6; and if in 5, its square root ends in 5.

A perfect cube, however, may end in either of the nine digits, and in ciphers if the number of them is three or any multiple of three; also if the cube number ends in 1, its cube root will end in 1; if in 2, its cube root ends in 8; if in 3, its cube root ends in 7; if in 4, its cube root ends in 4; if in 5, its cube root ends in 5; if in 6, its cube root ends in 6; if in 7, its cube root ends in 3; if in 8, its cube root ends in 2; and if in 9, its cube root ends in 9.

## EXTRACTION OF THE SQUARE ROOT.

- 523. The extraction of the square root of a number is the process of finding one of its two equal factors; or of finding such a factor as, when multiplied by itself, will produce the given number.
- 524. The method generally adopted for extracting the square root depends upon the following principles:—
- 1. The square of any number has, at most, only twice as many figures as its root, and, at least, only one less than twice as many. For the square of any number of a single figure consists of either one or two places of figures, as  $1^2 = 1$ , and  $9^2 = 81$ ; the square of any number of two figures consists of either three or four places, as  $10^2 = 100$ , and  $99^2 = 9801$ ; and the same law holds in regard to numbers of three or more figures. Therefore, when the square number consists of one or two figures, its root will consist of one figure; when of three or four figures, its root will consist of two figures; when of five or six figures, its root will consist of three figures; and so on. Hence, if a number be separated into as many periods as possible of two figures each, commencing at the right, to these periods respectively will correspond the units, tens, hundreds, &c. of the square root of the number.
- 2. The square of a number consisting of TENS and UNITS is equal to the square of the tens, plus twice the product of the tens into the units, plus the square of the units. Thus, if the tens of a number be denoted by a and the units by b, the square of the number will be denoted by  $(a + b)^2 = a^2 + 2 a b + b^2$ . Then, by this formula, if a = 3, and b = 6, we have a = 3 tens a = 3 units a = 3 and a = 3.

$$36^2 = (30 + 6)^2 = 30^2 + 2 \times (30 \times 6) \times 6^2 = 1296$$
. Or, analytically,

It is evident, as evolution is the reverse of involution (Art. 517), that from the process now given of obtaining a square may be deduced a method of extracting its root. Since the square of (a+b) is  $a^2+2$  a  $b+b^2$ , the square root of  $a^2+2$  a  $b+b^2$ must be a + b. Now it will be observed that a, the first term of the root, is the square root of a2, the first term of the square; and if  $a^2$  be subtracted, there will remain  $2ab + b^2$ . from which b, the second term of the root, is to be obtained. But  $2ab+b^2$  is the same as  $(2a+b)\times b$ , therefore the remainder equals  $(2a + b) \times b$ . But as b, the units, is alwavs much less than 2 a, twice the tens, we consider that 2 a  $\times$  b is about equal to the whole remainder, and taking 2 a (which we know) as the trial divisor, we obtain b, the units. But as the true divisor is 2a + b, we add the units to twice the tens and multiply the sum by the units, which gives a product equal to the whole remainder, or  $2 a b + b^2$ .

Since every number of more than one figure may be considered as composed of tens and units, we may have tens and units of units, tens and units of tens, tens and units of hundreds, &c. Hence, the principle just explained applies equally whether the root contains two or more than two figures.

525. To extract the square or second root of numbers.

Ex. 1. What is the square root of 1296? . Ans. 36.

Beginning at the right, we separate the number into periods of two figures each, by placing a point (·) over the right-hand figure of each period. Since the number of periods is two, the root will consist of two figures, tens and units. Then 1296 = the square of the tens plus twice the product of the tens into the units, plus the square of the units. The square of tens is hundreds, and must

therefore be found in the hundreds of the number. The greatest number of tens whose square does not exceed 12 hundreds is 3, which we write as the tens figure of the root. We subtract the 9 hundreds, the square of the 3 tens, from the 12 hundreds, and there remain 3 hundreds; after which we write the figures of the next period, and the remainder is 396 = twice the product of the tens into the units plus the square of the units. We have then next to find a number which, added to twice the 3 tens of the root, and multiplied into their sum, shall equal 396. By dividing this remainder

by twice the three tens of the root, we may obtain the units, a number somewhat too large. But though it may be too large, it cannot be too small, since the remainder 396 contains twice the product of the tens into the units, and also the square of the units. We therefore make twice the three tens of the root = 6 tens, a trial divisor, with which we divide the 39 tens, exclusive of the 6 units, which cannot form any part of the product of the tens by the units. The quotient figure obtained, 6, must be the units figure of the root, or a number somewhat larger. To determine whether it expresses the real number of the units in the root, we annex it to the 6 tens, and multiply the number 66, thus formed, by it. The product is 396, which being subtracted, there is no remainder. Therefore 1296 is a perfect square, and 36 the root sought.

## 2. What is the square root of 278784?

Ans. 528.

Since there are three periods, the root will contain three figures; the first two may be considered as tens and units of TENS. As the square of tens cannot give less than hundreds, we must find that square in the two left-hand periods; and as we have tens and units of tens, their square = the square of the tens plus twice the tens into the units, plus the square

of the units = 2787 (nearly). We proceed then with the first two periods exactly the same as when the root consists of but two figures, and thus take from the given number the square of the 52 tens, which leaves a remainder of 8384. We now consider the given number 278784, as the square of a number consisting of 52 tens and a certain number of units, which square will of course equal the square of the tens plus twice the tens into the units, plus the square of the units. But the square of the tens, or (52)<sup>2</sup> has already been taken from the given number, leaving a remainder, 8384, which must equal twice the tens into the units plus the square of the units. From this we readily obtain the units, just as when we had but two figures in the root.

RULE. — Separate the given number into as many periods as possible of two figures each, by placing a point over the place of units, another over the place of hundreds, and so on.

Find the greatest square in the left-hand period; write the root of it at the right of the given number after the manner of a quotient in di-

vision, and subtract the second power from the left-hand period.

Bring down the next period to the right of the remainder for a dividend, and double the root already found for a trial divisor. Find how often this divisor is contained in the dividend, exclusive of the right-hand figure, and write the quotient as the next figure of the root.



Annex the last root figure to the trial divisor for the true divisor, which multiply by the last root figure and subtract the product from the dividend. To the remainder bring down the next period for a new div-

Double the root already found for a new trial divisor, and continue the operation as before, till all the periods have been brought down-

NOTE 1. - When the product of any trial divisor exceeds its corresponding dividend, the last root figure must be made less.

If a dividend does not, contain its corresponding divisor, a cipher must be placed in the root, and also at the right of the divisor; then, after bringing down the next period, this last divisor must be used as the divisor of the new dividend.

Note 2. — When there is a remainder after extracting the root of a number, periods of ciphers may be annexed, and the figures of the root thus obtained will be decimals.

NOTE 8. - If the given number is a decimal, or a whole number and a decimal, the root is extracted in the same manner as in whole numbers, except, in pointing off the decimals, either alone or in connection with the whole number, we place a point over every second figure toward the right, from the separatrix, filling the last period, if incomplete, with a eipher. The number of decimal places in the root will always equal the number of periods of decimals in the power.

NOTE 4. - If the given number is a common fraction reduce it to its simplest form, if it is not so already, and extract the root of both terms, if they are perfect powers; otherwise, either find their product, extract its root, and divide the result by the denominator, or reduce the fraction to a decimal, and extract the root of the decimal.

Note 5. - When the given number is a mixed number, it may be changed to the form of a common fraction, or the fractional part may be reduced to a decimal, before attempting to extract the root.

### EXAMPLES.

3. What is the square root of -144a?

Ans. 4.

$$\sqrt{\frac{1}{15129}} = \frac{12}{123} = \frac{4}{11}$$

4. What is the square root of  $\frac{3}{80}$ ?

Ans. .1936--

$$3 \times 80 = 240$$
;  $\frac{\sqrt{240}}{80} = .1936 + .$ 

Or 
$$\frac{3}{80} = .0375$$
;  $\sqrt{.0375} = .1936 + .$ 

5. What is the square root of 3444736?

Ans. 1856.

6. What is the square root of 998001?

Ans. 999.

7. What is the square root of  $\frac{34.3}{1008}$ ?

8. Extract the square root of 234.09?

Ans. 15.3.

9. What is the square root of 421?

Ans. 64.

10. What is the square root of .000729?	Ans027.
11. What is the square root of 17.3056?	Ans. 4.16.
12. What is the square root of $52\frac{9}{16}$ ?	Ans. 71.
13. What is the square root of $95\frac{1}{16}$ ?	Ans. $9\frac{3}{4}$ .
14. What is the square root of $363\frac{1}{361}$ ?	Ans. $19\frac{1}{19}$ .
15. How much is $\sqrt{1.96}$ ?	Ans. 1.4.
16. How much is 6561½?	Ans. 81.
17. How much is $\sqrt{9^3}$ ?	Ans. 27.
18. How much is 8 <sup>‡</sup> ?	Ans. 64.
19. How much is one of the two 9645192360241?	equal factors of Ans. 3105671.

526. When the square root is to be extracted to many places of figures, the work may be contracted thus:—

Having found in the usual way one more than half of the root figures required, the rest may be found by dividing the last remainder, with a single figure annexed instead of two, by the last divisor, and proceeding as in contracted division of decimals. (Art. 276.)

### EXAMPLES.

What is the square root of 785 to five places of decimals?
 Ans. 28.01785.

CONTRACTED METHOD.	COMMON METHOD.
$\begin{vmatrix} 7 & 8 & 5 \\ 4 \end{vmatrix}$ 2 8.0 1 7 8 5 +	$\begin{vmatrix} 7 & 8 & 5 \\ 4 & 1 \end{vmatrix}$ 2 8.0 1 7 8 5 +
48 385 384	$\begin{array}{c c} 48 & 385 \\ \hline 384 & \end{array}$
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	$\begin{array}{c ccccccccccccccccccccccccccccccccccc$
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	$5602\overline{7}   439900 \\   392189$
$\begin{array}{r} \overline{4771} \\ \overline{4482} \end{array}$	$\begin{array}{r rrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrr$
$\begin{array}{r} \hline 289 \\ 280 \\ \hline \end{array}$	$\begin{array}{c ccccccccccccccccccccccccccccccccccc$
9	813775

The nature and extent of the contraction will be seen by comparing the contracted method with the common method.

- 2. Extract the square root of 62 to four places of decimals.

  Ans. 2.5298+.
- 3. Required the square root of 2 to five places of decimals.

  Ans. 1.41421+.
- 4. Required the square root of 3.15 to eight places of decimals.

  Ans. 1.77482393+.
- 5. Required the square root of 373 to seven places of decimals.

  Ans. 19.31320794.
- 6. Extract the square root of 8.93 to eight places of decimals.

  Ans. 2.98831055+.

## EXTRACTION OF THE CUBE ROOT.

- 527. The extraction of the cube root of a number is the process of finding one of its three equal factors; or, of finding a factor which, being multiplied into itself twice, will produce the given number.
  - 528. The common method of extracting the cube root depends upon the following principles:—
- 1. The cube of any number has, at most, only three times as many figures as its root, and, at least, only two less than three times as many. For the cube of a number of a single figure consists of, at most, three figures, and, at least, two less than that number, as  $1^3 = 1$ , and  $9^3 = 729$ ; the cube of a number of two figures consists of, at most, six figures, and, at least, two figures less than that number, as  $10^3 = 1000$ , and  $99^3 = 970299$ ; and so on. Therefore, when a cube number consists of one, two, or three figures, its root will consist of one figure; when of four, five, or six figures, its root will consist of two figures, and so on; and if a number be separated into as many periods as possible of three figures, each commencing at the right, to these periods respectively will correspond the units, tens, hundreds, &c. of the cube root of that number.
- 2. The cube of a number consisting of TENS and UNITS is equal to the cube of the tens, plus three times the square of the tens into the units, plus three times the tens into the square of the units, plus the cube of the units. Thus, if the tens of a number be denoted by a, and the units by b, the cube of the number

It is evident, as evolution is the reverse of involution, that from this process of obtaining a cube may be deduced a method of extracting the cube root. Since the cube of a + bis  $a^3 + 3a^2b + 3ab^2 + b^3$ , the cube root of  $a^3 + 3a^2b +$  $3 a b^2 + b^3$  must be a + b. Now a, the first term of the cube root, is the cube root of  $a^3$ , the first term of the cube; and if  $a^3$  be subtracted, there will remain  $3 a^2 b + 3 a b^2 + b^3$ , from which b, the second term of the root, is to be obtained. But  $3 a^2 b + 3 a b^2 + b^3$  is the same as  $(3 a^2 + 3 a b + b^2) \times b$ ; therefore the remainder equals  $(3 a^2 + 3 a b + b^2) \times b$ . But as 3ab, three times the tens into the units, plus  $b^2$ , the square of the units, is generally much less than  $3a^2$ , three times the square of the tens, we consider that  $3a^2 \times b$  is about equal to the whole remainder, and taking  $3 a^2$  (which we know) as the trial divisor, we obtain b, the units. But as the true divisor is  $3a^2 + 3ab + b^2$  we add three times the tens by the units plus the square of the units, and multiply the sum by the units, which gives a product equal the whole remainder, or  $3 a^2 b + 3 a b^2 + b^3$ .

Since every number of more than one figure may be considered as composed of tens and units, we may have tens and units of units, tens and units of tens, tens and units of hundreds, &c. Hence, the principle just explained applies equally whether the root contains two or more than two figures.

# 529. To extract the cube or third root of numbers.

## Ex. 1. What is the cube root of 46656?

Ans. 36.

Trial div., 
$$3 \times 30^2 = 2700$$
 $3 \times 30 \times 6 = 540$ 
 $6^2 = 36$ 
True divisor,
$$3^3 = 2700$$

$$3 \times 30 \times 6 = 540$$

$$6^3 = 36$$

$$3276 \times 6 = 19656$$

 $36 \times 36 \times 36 = 46656$ 

Beginning at the right, we separate the given number into periods, by placing a point over the units figure and over the third figure to the left. Since the number of periods is two, the root will consist of two figures.

tens and units. Then 46656 = the cube of tens, plus three times the square of the tens into the units, plus three times the tens into the square of the units, plus the cube of the units. The cube of tens is thousands, and must therefore be found in the thousands of the num-The greatest number of tens whose cube does not exceed 46 thousands is 3, which we write as the tens figure of the root. We then subtract the 27 thousands, the cube of the 3 tens, from the 46 thousands, and there remain 19 thousands; and, annexing the next period, we have as the entire remainder, 19656, equal three times the square of the tens into the units, plus three times the tens into the square of the units, plus the cube of the units, or the product of three times the square of the tens, plus three times the tens into the units, plus the square of the units, multiplied by the units. By dividing this remainder by three times the square of the tens of the root, we obtain the units, or a number somewhat too large. Although it may be too large, it cannot be too small, since the remainder 19656 contains not only three times the square of the tens into the units, but three times the tens into the square of the units, plus the cube of the units. We therefore make three times the square of the tens of the root, = 27 hundreds, a trial divisor, with which we divide the 196 hundreds of the remainder, disregarding the 56 units, since they cannot form any part of the product of the square of the tens by the The quotient figure obtained, 7, must be the units figure of the root, or a number somewhat larger.

But on undertaking to complete the divisor on the supposition that 7 is the true units figure of the root, we find a divisor too large for the remainder. We therefore take 6, a number one less, and to determine whether it expresses the real number of units in the root, we add to the 27 hundreds of the trial divisor three times the 3 tens of the root into the 6 units, plus the square of the 6 units; and multiplying the true divisor, 3276, thus formed, by the units, and subtracting the product, 19656, from the remainder, there is nothing left.

Hence, 46656 is a perfect cube, and 36 its-cube root.

## 2. What is the cube root of 12326391?

Ans. 231.

Trial divisor, 
$$3 \times 20^2 = 1200$$
 $3 \times 20 \times 3 = 180$ 
 $3^2 = 9$ 

True divisor,  $3 \times 230^2 = 158700$ 
 $3 \times 230 \times 1 = 690$ 
 $1^2 = 1$ 

True divisor,  $159391 \times 1 = 159391$ 

Since there are three periods the root will contain three figures. the first two of which may be considered as tens and units of tens. As the cube of tens cannot give less than thousands, we must find that cube in the two left-hand periods; and as we have tens and units of tens, their cube will equal the cube of the tens, plus three times the square of the tens into the units, plus three times the tens into the square of the units, plus the cube of the units. This we apply to the first two periods exactly the same as when the root consists of but two figures, and thus take from the given number the cube of the 23 tens, which leaves a remainder of 159391. We now consider the given number, 159391, as the cube of a number consisting of 23 tens and a certain number of units, which cube will of course equal the cube of the tens, plus three times the square of the tens into the units, plus three times the tens into the square of the units, plus the cube of the units. But the cube of the tens, or (23)3, has already been taken from the given number, leaving a remainder, 159391, which must equal three times the square of the tens into the units, plus three times the tens into the square of the units, plus the cube of the units; or equal the product of three times the square of 23, plus three times 23 into the units, plus the square of the units, multiplied by the units. From this we readily obtain the units, just as when we had but two figures in the required root.

In the second operation the work is somewhat abridged by rendering shorter the method of finding each divisor after the first, of both kinds. Thus, the second true divisor is obtained by prefixing to the second root figure, 3, three times the part of the root preceding it, or  $8 \times 2 = 6$ , and adding 189, the product of the number 63 thus formed by the last root figure, 3, to the preceding trial divisor, or 1200 + 189 = 1389, second true divisor. This is equivalent to adding to the trial divisor, in forming the true divisor, three times the tens into the units, plus the square of the units, or  $3 \times 20 \times 3 + 3^2$ . For the second trial divisor we annex two ciphers to the sum found by adding together the square of the last root figure, the last true divisor, and the number standing over it, or 9 + 1389 + 189 = 1587, with two ciphers annexed = 158700, the second trial divisor. is equivalent to taking for the trial divisor three times the square of the tens, or the part of the root already found, or  $3 \times 230^{2}$ . next true divisor is found in like manner as was the second true divisor.

Rule 1. — Separate the given number into as many periods as pos-

sible of three figures each, beginning at the units' place.

Find the greatest cube in the left-hand period, and write its root as the first figure of the required root. From that period subtract the cube, and to the remainder bring down the next period for a dividend.

Multiply the square of the root figure by 3, and to the product annex two ciphers for a trial divisor, and see how often it is contained in the

dividend, and write the result as the next figure of the root.

Add to the trial divisor three times the product of the tens figure of the root by the units figure with a cipher annexed, and the square of the last figure, for a true divisor.

Multiply the true divisor by the last figure of the root; subtract the product from the dividend, and to the remainder bring down the next

period for a new dividend.

Multiply the square of the root figures already found by 3, and to the product annex two ciphers for a new trial divisor; and proceed as before until all the periods are brought down. Or,

RULE 2. — Having found the first trial divisor and determined the

second root figure as by the preceding rule, -

Take three times the part of the root already found, except the last figure, to it annex the last figure of the root, multiply the result by the figure annexed, and write the product below the trial divisor, and add it to the same for a true divisor.

Multiply the true divisor by the last figure of the root: subtract the product from the dividend, and to the remainder bring down the next

period for a new dividend.

To the last true divisor and the number immediately over it, add the square of the last root figure, and to the sum annex two ciphers for a new trial divisor; then proceed as before.

Note 1. — The observations made in Notes 1, 2, 3, and 5, under the rule for the extraction of the square root (Art. 525), are equally applicable to the extraction of the cube root, except that two ciphers must be placed at the right of a true divisor when it is not contained in its corresponding dividend, and in

pointing off decimals each period must contain three figures.

Note 2.—If the given number is a common fraction, reduce it to its simplest form, if it is not so already, and extract the root of both terms, if they be perfect powers; otherwise, either find the product of the numerator by the square of the denominator, extract its root, and divide the result by the denominator; or reduce the fraction to a decimal, and extract the root of the decimal.

### EXAMPLES.

3.	What is the cube root of 77308776?	Ans. 426.
4.	What is the cube root of $\frac{1}{3}$ ?	Ans. 5.
<b>5.</b>	What is the cube root of 84.604519?	Ans. 4.39.
6.	What is the cube root of 54439939?	Ans. 379.
7.	What is the cube root of 60236288?	Ans. 392.
8.	Required the cube root of .726572699.	Ans899.
9.	Extract the third root of 109215352.	Ans. 478.
10.	What is the third root of $\frac{1331}{913}$ ?	Ans. 11.
11.	What is the value of $\sqrt[8]{\frac{5}{6}} \frac{93}{8} \frac{19}{2} \frac{9}{1}$ ?	Ans. 39.
12.	What is the value expressed by $\sqrt[3]{3496}$	5783?
	<u>.</u>	Ans. 327.
13.	What is the value of 1226153272321?	Ans. 4968.

530. When the cube root is to be extracted to many places

14. What is the value of  $436036824287^{\frac{1}{2}}$ ?

of decimals, the work may be contracted thus: -

Having found in the usual way one more than half of the root figures required, the rest may be found by dividing the last remainder by its corresponding true divisor, as in contracted division of decimals (Art. 276), observing however at each step to reject two figures from the right of the divisor and one from the right of the remainder.

### EXAMPLE.

- 1. Required the cube root of 2 to four places of decimals.

  Ans. 1.2599+.
- 2. Find the third root of 11 to four places of decimals.

  Ans. 2.2239.
- 3. Extract the cube root of 3 to six places of decimals.
- Ans. 1.442249+.

  4. Extract the cube root of 9 to fifteen places of decimals.

  Ans. 2.08008382301904.

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Ans. 7583.

#### EXTRACTION OF ANY ROOT.

- 581. The root corresponding to any perfect power may be obtained by resolving that power into its prime factors, and multiplying together one of each number of equal factors denoted by the exponent of the required root. Thus one of each two equal factors of the power will give the second or square root; one of each three equal factors will give the third or cube root; one of each four equal factors will give the fourth root; and so on.
- 582. When the index or exponent of the root to be extracted is a composite number, the root may be obtained by successive extractions of the simpler roots denoted by the several factors of that exponent. Thus the fourth root may be obtained by extracting the square root twice in succession; the sixth root by extracting the square root and then the cube root; and so on.

## EXAMPLES.

1. Required the fourth root of 50625? Ans. 15. BY FACTORS. BY SUCCESSIVE EXTRACTIONS. 5|50625 50625.225 5 10125 4 42 106 5!2025 84  $\times$  5 405 225|15, Ans. 445|2225 3|81 2225 3 27 25 125 O 125 3 9

 $5 \times 3 = 15$ , Ans.

 $\times 3$ 

2. What is the square root of 998001? Ans. 999.

3. What is the cube root of 262144? Ans. 64.

4. What is the fourth root of 43046721?

5. What is the fifth root of 14348907? Ans. 27.

6. What is the sixth root of 11390625?

Ans. 15.

533. When the given number is an imperfect power, or otherwise, or the exponent denoting the root is prime, or otherwise, the required root may be found by an elegant process, perfect in principle, called from its inventor,

### HORNER'S METHOD.

Ex. 1. Required the cube root of 9295677. Ans. 453.

	OPERATIO	on.	The greatest cube
<b>o</b> .	0 •	929596771453	contained in the left-
4	16	64	hand period we find to
4	<del>16</del>	28959	be 64, whose root, 4, we write as the first
4	32	27125	figure of the required
4 4 8 4	4800	1834677	root. This figure, 4,
9			we write under the ci-
4.	625	1834677	pher of the first column;
$\overline{120}$	5425	0	and adding it to the
5	650	sum multiplied	cipher obtain 4, which by the 4 gives 16; and
$\overline{125}$	607500	the result, 16, v	ve write under the cipher
5	4059		column, and by addition
	011770		sum, multiplied by the 4,
<b>130</b>	611559		he result, 64, we write in
5		the last column i	ander the left-hand period,
	•	92, of the give	n number. The 64 sub-
<b>1350</b>	tracted from		gives for a remainder 28.
3	We next a	dd the 4 to the	last term, 4, of the first
	column, obta	ining 8: and the r	esult, 8, multiplied by the
<b>1353</b>			nder the last term, 16, of
10 377			the same together, obtain

48. We next add the 4 to the last term, 8, of the first column, obtaining 12; and, annexing one cipher to the last term, 12, of the first column, obtaining 120, two ciphers to the last term of the second column, obtaining 4800, and to the remainder in the last column bringing down the next period, 959, obtaining 28959, we complete the work preparatory to the finding of the second root figure.

To determine that root figure, we take the last term, 4800, of the second column, for a trial divisor, and the last term, 28959, of the last column, for a dividend; and, dividing, 6 would appear to be the second figure of the root. This, on trial, however, is found to be too large; we therefore take 5, which answers. This 5 we add to the last term, 120, of the first column, obtaining 125; which sum, 125, multiplied by the 5, gives 625, and that product, added to the last term, 4800, of the second column, gives 5425; and this result, 5425, multiplied by the 5, gives 27125, which, written in the last column and subtracted from the figures above it, gives a remainder 1834. we add the 5 to the last term, 125, of the first column, obtaining 130; and the result, 130, multiplied by the 5, gives 650, which we write under the last term, 5425, of the second column, and by addition ob-We next add the 5 to the last term, 130, of the first column, obtaining 135; and, annexing one cipher to the last term, 135, of the first column, obtaining 1350, two ciphers to the last term, 6075, of the second column, obtaining 607500, and to the remainder in the last column bringing down the next period, 677, obtaining 1834677, the work is completed preparatory to finding the third root

figure.

To determine that figure, as before, we divide the last term of the third column by the last term of the second. We thus obtain 3, which, added to the last term of the first column, gives 1353, which sum, multiplied by the 3, gives 4059; and that product, being added to the last term of the second column, gives 611559; and that sum, multiplied by the 3, gives 1834677, which being exactly as large as the last term of the third column, on being written under it and subtracted there is no remainder. The given number is therefore a perfect power, and the cube root sought is 453.

In practice, the work may be performed with less figures, by writ-

ing down in the several columns only the results.

Rule.— Commence as many columns as there are units in the exponent of the root to be extracted, by writing the given number as the head of the right-hand column, and a cipher as the head of each of the others.

Separate the given number into as many periods as possible of as many figures each as the exponent of the root requires; and having found the nearest root of the left-hand period, write it as the first figure

of the required root.

Write this figure in the first column, and, having added it to what stands above it, multiply the sum by the same figure, and write the product in the second column; add, in like manner, in the second column, and multiply the sum by the same figure, writing the product in the third column; and so proceed, writing the last product in the last column, and subtracting it from what stands above it.

Then add the same figure to the last term of the first column, multiply the sum by the same figure, and add the product to the last term of the second column; and so on, writing the last product in the last column but one. Repeat the process, stopping each time with one column farther to the left, till the last product shall fall in the second

column.

Add the figure found for the root to the last term of the first column; annex one cipher to the last number in the first column, two ciphers to the last number in the second column, and so on; and to the last number in the last column bring down the next period for a dividend.

Take the last term of the column next to the last for a trial divisor, and see how often it is contained in the dividend, and write the result as

the next figure of the root.

Add this figure to the last term of the first column, multiplying the sum by the same figure, add the product to the second column, and so on; proceed as before, till all the periods have been brought down, or an answer sufficiently exact has been obtained.

NOTE 1. — When any dividend will not contain its corresponding trial divisor, write a cipher in the root, bring down to the dividend another period. annex an additional cipher to the last term of the first column, two additional

ciphers to the last term of the second column, and so on; and use the same trial divisor as before, increased however by the additional ciphers.

NOTE 2. — When the given number does not have an exact root, periods of ciphers may be annexed.

Note 3. — When the root is required to many places of decimals, the work may be contracted by rejecting one figure at the right from the number in the column next to the last, two from the number in the column next farther to the left, and so on, and otherwise proceeding as directed in the rule, except that the new figure of the root is not added to the first column. As soon as all the figures are rejected in the number of the first column, the remainder of the work may be performed as in contracted division of decimals (Art. 276).

### EXAMPLES.

2. Required the fourth root of 1.016397 to eight places of decimals.

Ans. 1.00407427.

OPERATION.			
0	0	0 .	i.01639700 1.00407427
1	1	1	1
<u>1</u>	1	<u> </u>	1639700
1	2	<b>3</b> .	1609632
$\overline{2}$	. <b>3</b>	4000000	30068
1	3	2408	<b>2</b> 8 <b>338</b>
3	60000	402408	1730
1	2·	2416	1619
<b>4</b> 00	602	404824	111
•	2		81
	604	·	30
	2	•	28
	606		2

- 3. What is the cube root of 41673648563? Ans. 3467.
- 4. What is the cube root of 43614208?
- 5. What is the cube root of 1.05 to six places of decimals?

  Ans. 1.016397.
  - 6. What is the fifth root of 184528125?

Ans. 45.

- 7. Required the fourth root of 100 to six places of decimals?

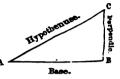
  Ans. 3.162278.
- 8. Required the fifth root of the fourth power of 9 to seven places of decimals?

  Ans. 5.7995466.

#### APPLICATIONS OF POWERS AND ROOTS.

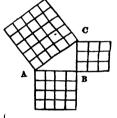
534. A TRIANGLE is a figure having three sides and three angles. When one of the sides of a triangle is perpendicular to another side, the opening between them is called a *right angle*, and the triangle is called a *right-angle* triangle.

The lowest side, A B, is called the base of the triangle A B C, the side B C the perpendicular, the longest side, A C, the hypothenuse, and the angle at B is a right angle. Also, the line B C, being perpendicular to the base, is the altitude.



535. The square described upon the hypothenuse of a rightangled triangle is equivalent to the sum of the squares described upon the other two sides.

Thus, if the hypothenuse A C be 5 feet, the base A B 4 feet, and the perpendicular B C 3 feet, then  $5^2 = 4^2 + 3^2$ , or 25 = 16 + 9.



**536.** To find the *hypothenuse*, the base and perpendicular being given.

Add together the square of the base and the square of the perpendicular, and extract the square root of the sum.

Thus, if the base be 4 and the perpendicular 3, the hypothenuse will equal  $\sqrt{4^2 + 3^2} = \sqrt{25} = 5$ .

**537.** To find the *perpendicular*, the base and hypothenuse being given.

Subtract the square of the base from the square of the hypothenuse, and extract the square root of the remainder.

Thus, if the base be 4 and the hypothenuse 5, the perpendicular will equal  $\sqrt{5^2-4^2} = \sqrt{9} = 3$ .

538. To find the base, the hypothenuse and perpendicular being given.

Subtract the square of the perpendicular from the square of the hynotherwise, and extract the square root of the remainder.

Thus, if the perpendicular be 3 and the hypothenuse 5, the base will equal  $\sqrt{5^2 - 3^2} = \sqrt{16} = 4$ .

539. All triangles having the same base are to each other as their altitudes.

All similar triangles, and other similar rectilineal figures, are

to each other as the squares of their homologous or corresponding sides.

Thus, the triangles A C E and A C D, having the same base, A C, are to each other as the altitude E C of the one is to the altitude D C of the other.



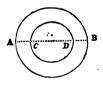
Also, the triangles A C E and B C D,

having their corresponding angles the same, and their sides in direct proportion, are said to be similar, and are to each other as the squares of their corresponding sides, or as (A E)2 is to (B D)<sup>2</sup>, (A C)<sup>2</sup> is to (B C)<sup>2</sup>, and (C E)<sup>2</sup> is to (C D)<sup>2</sup>. Likewise the larger square, of which A C is one of the equal sides. is to the smaller square, of which B C is one of the equal sides. as (A C)2 is to (B C)2.

540. All circles (Art. 143) are to each other as the squares of their diameters, semidiameters, or circumferences.

The circumference of a circle is the line which bounds it; and the diameter is a line drawn through the centre, and terminated by the circumference; as A B and C D.

Then, the larger circle, of which A B is the diameter, is to the smaller, of which C D is the diameter, as (AB)<sup>2</sup> is to (CD)<sup>2</sup>, &c.



541. To find the side, diameter, or circumference of any surface, which is similar to a given surface.

State the question as in Proportion, and square the given sides, diameters, or circumferences, and the square root of the fourth term of the proportion will be the answer required.

Thus, if 12 feet be the length of a side of a triangle whose area is 72 square feet, the length of the corresponding side of a 33 \*

similar triangle whose area is 32 square feet would be found as follows:

 $72:32::12^{2}=144:64; \sqrt{64}=8$  feet, length required.

542. To find the area of any surface which is similar to a given surface.

State the question as in Proportion, and square the given sides, diameters, or circumferences, and the fourth term of the proportion will be the answer required.

Thus, if 72 square feet be the area of a triangle of which 12 feet is one of the sides, the area of a similar triangle of which the corresponding side is 8 feet would be found as follows:

 $12^2 = 144 : 8^2 = 64 : 72 \text{ sq. ft.} : 32 \text{ sq. ft.}$ , area required.

543. To find the side of a square equal in area to any given surface.

Find the square root of the given area, and that root will be the side of the area required.

544. A sphere is a solid bounded by a continued convex surface, every part of which is equally distant from the point within called the centre.

The diameter of a sphere is a straight line passing through the centre, and terminated by the surface; as A B.



545. A CONE is a solid having a circle for its base, and tapering uniformly to a point, called the vertex.

The altitude of a cone is its perpendicular height, or a line drawn from the vertex perpendicular to the plane of the base, as B C. The diameter of its base is a straight line drawn through the centre of the plane of the base from one side of the circle to the other; as A D.



**546.** Spheres are to each other as the cubes of their diameters, or of their circumferences.

Similar cones are to each other as the cubes of their altitudes, or of the diameters of their bases.

All similar solids are to each other as the cubes of their homologous or corresponding sides, or of their diameters.

NOTE. — Cones and other solids are said to be similar when their corresponding parts are in direct proportion to each other.

547. To find the *contents* of any solid which is similar to a given solid.

State the question as in Proportion, and cube the given sides, diameters, altitudes, or circumferences, and the fourth term of the proportion is the answer required.

548. To find the side, diameter, altitude, or circumference of any solid, which is similar to a given solid.

State the question as in Proportion, and cube the given sides, diameters, altitudes, or circumferences, and the cube root of the fourth term of the proportion is the answer required.

549. To find the side of a cube that shall be equal in solidity to any given solid.

Find the cube root of the contents of the given solid, and that root will be the side of the cube required.

550. To find a mean proportional (Art. 333) between any two numbers.

Find the square root of the product of the two numbers, and that root will be the mean proportional required.

551. To find two mean proportionals between two given numbers.

Find the cube root of the quotient of the greater of the two numbers divided by the less. The product of the less number by that root will be the least mean proportional; and the quotient of the greater number by the same root will be the other mean proportional.

552. To find any two numbers, whose sum and product are given.

From the square of half the sum of the two numbers subtract their product, and the square root of the remainder will equal half the difference of the two numbers, which added to half their sum will give the larger, and subtracted from half their sum will give the smaller, of the numbers required.

553. To find any two numbers, when their sum and the difference of their squares are given.

The difference of their squares divided by the sum of the numbers will give their difference; and half of their difference added to half of their sum will give the larger, and half of their difference subtracted from half of their sum will give the smaller, of the numbers required.

## EXAMPLES.

- 1. A certain general has an army of 141376 men. How many must he place in rank and file to form them into a square?

  Ans. 376.
- 2. If the area of a circle be 1760 yards, how many feet must the side of a square measure to contain that quantity?

Ans. 125.857+ feet.

- 3. If a line 144 feet long will reach from the top of a fort to the opposite side of a river 64 feet wide, on whose brink it stands, what is the height of the fort?

  Ans. 128.99—.
- 4. A certain room is 20 feet long, 16 feet wide, and 12 feet high; how long must a line be to extend from one of the lower corners to the upper corner farthest from it? Ans. 28.28ft.
- 5. A certain field is 40 rods square; what must be the length of one of the equal sides of another field that shall contain only one fourth as much area?

  Ans. 20 rods.
- 6. The areas of two similar triangular-shaped fields are 60 and 90 acres, and a side of the former is 66 rods. Required the corresponding side of the latter?
- 7. If a lead pipe 3 of an inch in diameter will fill a cistern in 3 hours, what should be its diameter to fill it in 2 hours?

Ans. .918+ inches.

- 8. If a pipe 1½ inches in diameter will fill a cistern in 50 minutes, how long would it require a pipe that is 2 inches in diameter to fill the same cistern?

  Ans. 28m. 7½s.
- 9. If a pipe 6 inches in diameter will draw off a certain quantity of water in 4 hours, in what time would it take 3 pipes of four inches in diameter to draw off twice the quantity?

Ans. 6 hours.

- 10. The first term of a proportion is 40, and the fourth term 90. Required a mean proportional between them. Ans. 60.
- 11. In a pair of scales a body weighed 31½ pounds in one scale, and only 20 pounds in the other scale. Required its true weight.

  Ans. 25 pounds.
- 12. I wish to set out an orchard of 2400 mulberry-trees, so that the length shall be to the breadth as 3 to 2, and the distance between any two adjacent trees 7 yards. How many trees must there be in the length, and how many in the

breadth; and on how many square yards of ground will they stand? Ans. 60 in length; 40 in breadth; 112749 sq. yd.

13. The sum of two persons' ages is 50 years, and their product is 600 years. What are their ages?

Ans. Of the one, 20 years; of the other, 30 years.

- 14. Two ships sail from the same port; one goes due north 128 miles, the other due east 72 miles; how far are the ships from each other?

  Ans. 146.86+.
- 15. There are two columns in the ruins of Persepolis left standing upright; one is 70 feet above the plane, and the other 50; in a straight line between these stands a small statue, 5 feet in height, the head of which is 100 feet from the summit of the higher, and 80 feet from the top of the lower column. Required the distance between the tops of the two columns.
- 16. The sum of two numbers is 44, and the square of their difference is 16. Required the numbers.
- Ans. 24 the larger number; 20 the smaller.

  17. A tree 80 feet in height stands on a horizontal plane; at what height from the ground must it be broken off, so that the top of it may fall on a point 40 feet from the bottom of the tree, the end where it was broken off resting on the stump?

Ans. 30 feet.

- 18. The height of a tree, growing in the centre of a circular island, 100 feet in diameter, is 160 feet; and a line extending from the top of it to the farther shore is 400 feet. What is the breadth of the stream, provided the land on each side of the water be level?

  Ans. 316.6+ feet.
- 19. A ladder 70 feet long is so planted as to reach a window 40 feet from the ground, on one side of the street, and without moving it at the foot it will reach a window 30 feet high on the other side; what is the breadth of the street?
- 20. If an iron wire  $\frac{1}{10}$  of an inch in diameter will sustain a weight of 450 pounds, what weight might be sustained by a wire an inch in diameter?

  Ans. 45000lb.
- 21. A gentleman proposes to plant a vineyard of 10 acres. If he places the vines 6 feet apart, how many more can he plant by setting them in the quincunx order than in the square order, allowing the plat to lie in the form of a square, and no vine to be set nearer its edge than 1 foot in either case?

Ans. 1870 more in the quincunx order.

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- 22. Four men, A, B, C, and D, bought a grindstone, the diameter of which was 40 inches and the place for the shaft 4 inches in diameter. It was agreed that A should grind off his share first, then in turn B, C, and D. Required how many inches each man will grind off from the semidiameter, providing they each paid the same sum.
- Ans. A, 2.651in.; B, 3.137in.; C, 4.064in.; and D, 8.148in. 23. I have a board whose surface contains 49 square feet; the board is 1; inches thick, and I wish to make a cubical box

of it. Required the length of one of its equal sides.

Ans. 36 inches.

24. A carpenter has a plank 1 foot wide, 2237 feet long, and 21 inches thick; and he wishes to make a box whose width shall be twice its height, and whose length shall be twice its width. Required the contents of the box.

Ans. 5719 cubic inches.

25. If a ball, 3 inches in diameter, weigh 4 pounds, what will be the weight of a ball that is 6 inches in diameter?

Ans. 32lbs.

- 26. If a globe of gold, one inch in diameter, be worth \$120, what is the value of a globe 31 inches in diameter?
- 27. If the weight of a well-proportioned man, 5 feet 10. inches in height, be 180 pounds, what must have been the weight of Goliath of Gath, who was 10 feet 43 inches in height?

  Ans. 1015.1—lb.
  - 28. If a bell, 4 inches in height, 3 inches in width, and ‡ of an inch in thickness, weigh 2 pounds, what should be the dimensions of a similar bell that would weigh 2000 pounds?

Ans. 3ft. 4in. high, 2ft. 6in. wide, and 21in. thick.

- 29. What are the two mean proportionals between 56 and 12096?

  Ans. 336 and 2016.
- 30. Having a small stack of hay, 5 feet in height, weighing 1cwt., I wish to know the weight of a similar stack that is 20 feet in height.

  Ans. 64cwt.
- 31. If a man dig a small square cellar, which will measure 6 feet each way, in one day, how long would it take him to dig a similar one that measured 10 feet each way?

Ans. 4.629+ days.

32. If an ox, whose girth is 6 feet, weighs 600lb., what is the weight of an ox whose girth is 8 feet? Ans. 1422.2+lb.

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33. Four women own a ball of yarn, 5 inches in diameter. It is agreed that each shall wind off her share from the ball. How many inches of its diameter shall each wind off?

Ans. First, .45+ inches; second, .57+ inches; third, .82+ inches; fourth, 3.149+ inches.

34. John Jones has a stack of hay in the form of a quadrangular pyramid. It is 16 feet in height, and 12 feet wide at its base. It contains 5 tons of hay, worth \$17.50 per ton. Mr. Jones has sold this hay to Messrs. Pierce, Row, Wells, and Northend. As the upper part of the stack has been injured, it is agreed that Mr. Pierce, who takes the upper part, shall have 10 per cent. more of the hay than Mr. Rowe; and Mr. Rowe, who takes his share next, shall have 8 per cent. more than Mr. Wells; and Mr. Northend, who has the bottom of the stack, that has been much injured, shall have 10 per cent. more than Mr. Wells. Required the quantity of hay, and how many feet of the height of the stack, beginning at the top, each receives.

Ans. Pierce receives  $27\frac{54}{273}$ cwt. and 10.366 feet in height; Rowe,  $24\frac{1}{2}\frac{9}{3}$ cwt. and 2.493 feet; Wells,  $22\frac{3}{2}\frac{4}{3}$ cwt. and 1.666 feet; Northend,  $25\frac{5}{2}\frac{5}{13}$ cwt. and 1.474 feet.

# PROGRESSION, OR SERIES.

554. A SERIES is a succession of numbers that depend on one another by some fixed law.

The numbers constituting a series are called its terms; of which the first and last are called extremes, and the other terms the means.

#### ARITHMETICAL PROGRESSION.

555. ARITHMETICAL PROGRESSION, or PROGRESSION BY DIFFERENCE, is a series that increases or decreases by a constant number, called the *common* DIFFERENCE.

The series is said to be an ascending one when each term

after the first exceeds the one before it; and a descending one when each term after the first is less than the one before it.

Thus, 1, 5, 9, 13, 17, 21, 25, 29, 33, is an ascending series, in which each term after the first is derived from the one preceding it by the addition of the common difference 4; and 25, 22, 19, 16, 13, 10, 7, 4, 1, is a descending series, in which each term after the first is derived from the one preceding it by the subtraction of the common difference 3.

- 556. In arithmetical progression, the first term, the last term, the number of terms, the common difference, and the sum of the terms, are so related to each other, that, three of these being given, the two others may be readily determined.
- 557. To find the common difference, the extremes and number of terms being given.
- Ex. 1. The extremes of an arithmetical series are 3 and 45, and the number of terms is 22. Required the common difference.

  Ans. 2.

 $\frac{45 - 3}{22 - 1} = 2.$ 

It is evident that the number of common differences in any series must be 1 less than the number of terms. Therefore, since the number of terms in this series is 22, the number of terms in this series is 22.

ber of common differences will be 22 - 1 = 21, and their sum will be equal to the difference of the extremes; hence the difference of the extremes, 45 - 3 = 42, divided by the number of common differences, 21, gives 2 as the common difference required.

Rule. — Divide the difference of the extremes by the number of terms less one, and the quotient will be the common difference.

## EXAMPLES.

- 2. A certain school consists of 19 teachers and scholars, whose ages form an arithmetical series; the youngest is 3 years old, and the oldest 39. What is the common difference of their ages?

  Ans. 2 years.
- 3. A man is to travel from Albany to a certain place in 11 days, and to go but 5 miles the first day, increasing the distance equally each day, so that the last day's journey may be 45 miles. Required the daily increase. Ans. 4 miles.

- To find the number of terms, the extremes and common difference being given.
- Ex. 1. If the extremes of an arithmetical series are 3 and 19, and the common difference is 2, what is the number of terms? Ans. 9.

$$\frac{19 - 3}{2} + 1 = 9$$

It is evident, that, if the difference of the  $\frac{19-3}{2}+1=9$ . extremes be divided by the common difference, the result will be the number of common differences; thus 19 - 3 = 16:  $16 \div 2 = 8$ . Then, as the number of

terms must be 1 more than the number of common differences, 8 + 1 = 9 is the number of terms in the series.

Rule. - Divide the difference of the extremes by the common difference, and the quotient increased by 1 will be the required number of terms.

### EXAMPLES.

- 2. A man going a journey travelled the first day 7 miles, the last day 51 miles, and each day increased his journey by 4 How many days did he travel? Ans. 12.
- 3. In what time can a debt be discharged, supposing the first week's payment to be \$1, and the payment of every succeeding week to increase by \$2, till the last payment shall be \$ 103? Ans. 52 weeks.
- To find the sum of all the terms, the extremes and number of terms being given.
- The extremes of an arithmetical series are 3 and 19, and the number of terms 9. Required the sum of the Ans. 99. series.

$$\frac{3+19}{2} \times 9 = 99, \text{Ans.}$$

Or, 3 + 19 = 22;  $22 \times 4\frac{1}{2} = 99$ , Ans.

In an arithmetical series the sum of the extremes is equal to the sum of two terms that are equally distant from them, or to

double the middle term, if the number of terms be odd. Thus, in the series, 3, 5, 7, 9, 11, 13, 15, 17, 19, the sum of 3 and 19 is equal to the sum of 5 and 17, or of 7 and 15, and is double the middle term, The reason of this is evident, since 5 and 7 exceed the less ex-· treme by the same quantities by which 17 and 15 are respectively less than the other extreme.

Hence, in this latter series, it is evident that, if each term were made 11, half the sum of the extremes, the sum of the whole would remain the same; therefore the sum of the series must equal half the sum of the extremes multiplied by the number of terms, or the sum of the extremes multiplied by half the number of terms.

Rule. — Multiply half the sum of the extremes by the number of terms. Or,

Multiply the sum of the extremes by half the number of terms.

## EXAMPLES.

- 2. If the least term of a series of numbers in arithmetical progression be 4, the greatest 100, and the number of terms 17, what is the the sum of the terms?

  Ans. 884.
- 3. Suppose a number of stones were laid a rod distant from each other, for thirty miles, the first stone being a rod from a basket. What distance will that man travel who gathers them up singly, returning with them one by one to the basket?

Ans. 288090 miles 2 rods.

- 560. To find the sum of the terms, the extremes and common difference being given.
- Ex. 1. If the two extremes are 3 and 19, and the common difference is 2, what is the sum of the series?

$$\frac{19 - 3}{2} + 1 = 9;$$

$$\frac{19 + 3}{2} \times 9 = 99, \text{ Ans.}$$

It has been shown (Art. 558) that, if the difference of the extremes be divided by the common difference, the quotient will be the number of terms less one. Therefore the number of terms less one will be  $\frac{19-3}{2}=8$ ; and 8+1 will equal the

number of terms. It has also been shown (Art. 559) that, if the number of terms be multiplied by the sum of the extremes, and the product divided by 2, the quotient will be the sum of the series; therefore  $\frac{19+3}{2} \times 9 = 99$ , the answer required.

RULE. — Divide the difference of the extremes by the common difference, and to the quotient add 1; by this sum multiply half the sum of the extremes, and the product will be the sum required.

## EXAMPLES.

- 2. If the extremes are 3 and 45, and the common difference 2, what is the sum of the series?

  Ans. 528.
  - 3. A owes B a certain sum, to be discharged in a year, by

paying 6 cents the first week, 18 cents the second week, and thus to increase every week by 12 cents, till the last payment should be \$6.18. What is the debt?

Ans. \$162.24.

- 561. To find one of the extremes, when the other extreme and the number and sum of the terms are given.
- Ex. 1. If 3 be the first term of a series, 9 the number of terms, and 99 the sum of the series, what is the last term?

It has been shown (Art. 559) that, if the sum of the extremes be multiplied by the number of terms, the product will be twice the sum of the series; therefore, if twice the will be the sum of the extremes. If from this we subtract the given extreme, the remainder must be the other extreme.

RULE. — Divide twice the sum of the series by the number of terms; from the quotient take the given term, and the remainder will be the term required.

## EXAMPLES.

- 2. The sum of a series of ten thousand even numbers is 100010000, and the last term of the series is 20000. Required the first term.

  Ans. 2.
- 3. A merchant, being indebted to 22 creditors \$528, ordered his clerk to pay the first \$3, and the rest sums increasing in arithmetical progression. What is the difference of the payments, and the last payment?

Ans. Difference 2; last payment \$45.

- 562. To find any number of arithmetical means, the extremes and the number of terms being given.
- Ex. 1. If the first term of an arithmetical series is 1, the last term 99, and the number of terms 8, what are the second and seventh terms of the series?

Ans. The second term, 15; the seventh, 85.

$$\frac{99-1}{8-1}=14;$$
 $1+14=15; 99-14=85.$ 

We find the common difference, 14, as in Art. 557, the first term, 1, plus the common difference, 14, gives 15 for the second term, and the last term, 99, minus the common difference, 14, gives the seventh term.

RULE. — Find the common difference, which, added to the less extreme, or subtracted from the greater, will give one mean. From that mean derive others in the same way, till those required are found.

#### EXAMPLES.

2. The extremes of a series are 4 and 49, and the number of terms 6. Required the middle two terms.

Ans. 22 and 31.

3. Insert five arithmetical means between 20 and 30.

Ans. 213, 231, 25, 263, and 281.

### GEOMETRICAL PROGRESSION.

563. Geometrical Progression, or progression by quotients, is a series of numbers that increase or decrease by a constant multiplier or divisor, called the common ratio.

The series is an ascending one, when each term after the first increases by a constant ratio; and a descending one, when each term after the first decreases by a constant ratio.

- Thus 2, 6, 18, 54, 162, 486 is an ascending geometrical series; and 64, 32, 16, 8, 4, 2 is a descending geometrical series. Of the former, 3 is the common ratio, and of the latter, 2.
- 564. In geometrical progression the first term, the last term, the number of terms, the common ratio, and the sum of the terms are so related to each other, that, any three of these being given, the other two may be readily determined.
- 565. To find any proposed term, one of the extremes, the ratio, and the number of terms being given.
- Ex. 1. If the first term of a geometrical series be 3, the ratio 2, and the number of terms 8, what is the last term?

Ans. 384.

 $3 \times 2^7 = 3 \times 128 = 384$ . It is evident that the successive terms are the result of repeated multiplications by the ratio; thus the second term must be the product of the first term by the ratio, the third term the product of the second term by the ratio, and so on. The eighth, or last term, therefore, must be the result of seven

such multiplications, or the product of the first term, 3, by  $2^7$ , or  $3 \times 128 = 384$ .

If the last term had been given and the first required, the process would evidently have been by division, since every less term is the result of a division of the term next larger by the ratio.

RULE. — Raise the ratio to the power whose index is one less than the number of terms; by which multiply the least term to find the greatest, or divide the greatest to find the least.

Note 1. — When the ratio requires to be raised to a high power, the process may be abridged, as in Art 516.

Note 2. — The rule may be applied in computing compound interest, the principal being the first term, the amount of one dollar for one year the ratio, the time, in years, one less than the number of terms, and the amount the last term.

### EXAMPLES.

- 2. If the first term be 5, and the ratio 3, what is the seventh term?

  Ans. 3645.
- 3. If the series be 72, 24, 8, &c., and the number of terms 6, what is the last term?

  Ans.  $\mathfrak{F}_7$ .
  - 4. If the larger extreme be 885735, the ratio 3, and the number of terms 12, what are the tenth and the eleventh terms?

    Ans. 15 and 45.
  - 5. If the seventh term is 5, and the ratio  $\frac{1}{3}$ , what is the first term?

    Ans. 3645.
  - 6. If the first term is 50, the ratio 1.06, and the number of terms 5, what is the last term?

    Ans. 63.123848.
  - 7. If I were to buy 30 oxen, giving 2 cents for the first ox, 4 cents for the second, 8 cents for the third, &c., what would be the price of the last ox?

    Ans. \$10737418.24.
  - 8. What is the amount of \$160.00 at compound interest for 6 years?

    Ans. \$226.96305796096.
  - 9. What is the amount of \$300.00 at compound interest at 5 per cent. for 8 years?

    Ans. \$443.236+.
  - 10. What is the amount of \$100.00 at compound interest at 6 per cent. for 30 years?

Ans. \$574.349117291325011626410633231080264584635-7252196069357387776.

566. To find the sum of a series, the first term, the ratio, and the number of terms being given.

34 \*

Ex. 1. If the first term be 1, the ratio 3, and the number of terms 5, required the sum of the terms.

Ans. 121.

 $\frac{(81 \times 3) - 1}{3 - 1} = 121.$ 

If we multiply the series 1, 3, 9, 27, 81 by the ratio 3, we shall obtain as a second series 3, 9, 27, 81, 243, whose sum is three times the sum of the first se-

ries, and the difference between whose sum and the sum of the first series is evidently twice the sum of the first series. Now it will be observed that the two series have their terms alike, with the exception of the first term in the first series, and the last in the second series. We have then only to subtract the first term in the first series from the last term in the second, and the remainder is twice the sum of the first series; and half this being taken gives the required sum of the series. Therefore the sum of the first series must be 242 ÷ 2 = 121.

RULE. — Find the last term as in Art. 565, multiply it by the ratio, and the product less the first term divide by the ratio less 1; the result will be the sum of the series.

NOTE 1. — If the ratio is less than a unit, the product of the last term mulplied by the ratio must be subtracted from the first term, and the remainder divided by unity or 1 decreased by the ratio.

NOTE 2. — When a descending series is continued to infinity, it becomes what is called an *infinite* series, whose last term must always be regarded as 0, and its ratio as a fraction. To find the sum of an infinite series, —

Divide the first term by 1 decreased by the fraction denoting the ratio, and the

quotient will be the sum required.

This process furnishes an expeditious way of finding the value of circulating decimals, since they are composed of numbers in geometrical progression, whose common ratios are  $\frac{1}{10}$ ,  $\frac{1}{100}$ ,  $\frac{1}{100}$ , &c. according to the number of factors contained in the repetend. Thus, 3333, &c. represents the geometrical series  $\frac{3}{10}$ ,  $\frac{3}{100}$ ,  $\frac{3}{100}$ ,  $\frac{3}{100}$ , &c. whose first term is  $\frac{3}{10}$  and common ratio  $\frac{1}{10}$ .

## EXAMPLES.

2. The first term of a series is 5, the ratio  $\frac{2}{3}$ , and the number of terms 6; required the sum of the series.

$$(\frac{2}{3})^5 \times 5 = \frac{169}{243}; \frac{169}{243} \times \frac{2}{3} = \frac{329}{729}; 5 - \frac{329}{729} = \frac{3325}{729}; \frac{3325}{729} \div (1 - \frac{2}{3}) = \frac{3325}{243} = 13\frac{166}{243}, \text{ Ans.}$$

3. Find the value of the circulating decimal .232323, &c.

$$\frac{23}{100} \div (1 - \frac{1}{100}) = \frac{23}{99}$$
, Ans.

4. What is the sum of the series 4, 1,  $\frac{1}{4}$ ,  $\frac{1}{16}$ , &c., continued to an infinite number of terms?

$$4 \div (1 - \frac{1}{4}) = 5\frac{1}{3}$$
, Ans.

5. If the first term is 50, the ratio 1.06, and the number of terms 4, what is the sum of the series?

Ans. 218.7308.

- 6. A gentleman offered a house for sale on the following terms; that for the first door he should charge 10 cents, for the second 20 cents, for the third 40 cents, and so on in a geometrical ratio. If there were 40 doors, what was the price of the house?

  Ans. \$ 109951162777.50.
- 7. If the series  $\frac{2}{3}$ ,  $\frac{1}{3}$ ,  $\frac{1}{6}$ ,  $\frac{1}{12}$ ,  $\frac{1}{24}$ , &c. were carried to infinity, what would be its sum?

  Ans.  $1\frac{1}{4}$ .
- 8. A gentleman deposited annually \$10 in a bank, from the time his son was born until he was 20 years old. Required the amount of the 21 deposits at 6 per cent., compound interest, when his son was 21 years old?

  Ans. \$423.92.
- 9. Find the value of .008497133, &c., continued to infinity.

  Ans. 8838.
- 10. If a body be put in motion by a force which moves it 10 miles in the first portion of time, 9 miles in the second equal portion, and so, in the ratio of  $\frac{9}{10}$ , for ever, how many miles will it pass over?

  Ans. 100 miles.
- 567. To find the ratio, the extremes and number of terms being given.
- Ex. 1. If the extremes of a series are 3 and 192, and the number of terms 7, what is the ratio?

OPERATION. It has been shown (Art.  $192 \div 3 = 64$ ;  $\sqrt[4]{64} = 2$ , Ans. 565) that the last term of a geometrical series is equal to the product of the ratio by the first term raised to a power whose index is one less than the number of terms; hence the ratio must equal the root of the quotient of the last term by the first whose index is one less than the number of terms.

RULE. — Divide the last extreme by the first, and extract that root of the quotient whose index is one less than the number of terms.

Note. — When the sum of the series and the extremes are given, the ratio may be found by dividing the sum of all the terms except the first, by the sum of all the terms except the last.

### EXAMPLES.

- 2. If the last term of a series is 1, the largest term 512, and the number of terms 10; what is the ratio?

  Ans. 2.
- 3. If the extremes are 5 and 885735, and the sum of the series 1328600, what is the ratio?

  Ans. 3.
  - 4. What debt can be discharged in a year by monthly pay-

ments, in geometrical progression, of which the first payment is \$ 1, and the last \$ 2048; and what will be the ratio of the series?

Ans. Ratio, 2; debt, \$ 4095.

- 568. To insert any number of geometrical means, or mean proportionals, between two given numbers.
  - Ex. 1. Insert three geometrical means between 4 and 324.

    Ans. 12, 36, and 108.

324 + 4 = 81;  $\sqrt[4]{81} = 3$ .  $4 \times 3 = 12$ ;  $12 \times 3 = 36$ ;  $36 \times 3 = 108$ . Since the series will include, beside the inserted

extremes, the number of terms will be 5. Then, having the number of terms and the extremes, we find the ratio, as in the last article, to be 3; and by multiplying the first term by the ratio, we obtain the first of the terms to be inserted. That term multiplied by the ratio gives the next, and that multiplied by the ratio gives the other required mean.

Rule. — Take the two given numbers as the extremes of a geometrical series, and consider the number of terms in the series greater by two than the required number of means. Then find the ratio, as in Art. 567, and the product of the ratio and the first extreme will give one of the means, and the product of this mean and the ratio will give another, and so on.

Note. — When only a single mean is required to be inserted, it may be found as in Art. 550; when only two, as in Art. 551.

### EXAMPLES.

- Insert three geometrical means between ½ and 128.
   Ans. 2, 8, and 32.
- 3. Required five mean proportionals between the numbers 3 and 2187. Ans. 9, 27, 81, 243, and 729.
- **569.** To find the *number of terms*, the extremes and ratio being given.
- Ex. 1. If the extremes are 5 and 3645, and the ratio 3, what is the number of terms?

 $3645 \div 729$ ;  $3^6 = 729$ . 6 + 1 = 7, Ans.

By Art. 565 it is seen that the ratio raised to the power whose index is one less than the number of terms, and multiplied by the least term, equals the

largest term; hence, the largest term divided by the least term will equal a power of the ratio whose index is one less than the number of terms.

RULE. — Divide the largest term by the least; involve the ratio to a power equal to the quotient; and the index of that power, increased by 1, will be the number of terms.

## EXAMPLES.

- 2. If the extremes are 5 and 20480, and the ratio 4, what is the number of terms?

  Ans. 7.
- 3. In what time will a certain debt be discharged by monthly payments in geometrical progression, if the first and last payments are \$1 and \$2048, and the ratio 2?

Ans. In 12 months.

## ANNUITIES.

570. Annuities are fixed sums of money payable at the ends of equal periods of time, such as years, or half-years.

Annuities in perpetuity are such as continue for ever.

Annuities certain are such as commence at a fixed time, and continue for a certain number of years.

Annuities contingent are those whose commencement or continuance, or both, depend on some contingent event, as the death of one or more individuals.

Annuities deferred, or in reversion, are such as do not commence till after a fixed number of years, or till after some particular event has taken place.

- 571. An annuity forborne, or in arrears, is one whose periodical payments, instead of being paid when due, have been allowed to accumulate.
- 572. The amount of an annuity at compound interest, at any time, is the sum to which it will amount, supposing it to have been improved at compound interest during the intervening period.
- 573. The present value of an annuity at compound interest, for any given period, is the sum of the present values of all the payments of that annuity.



TABLE,

SHOWING THE AMOUNT OF AN ANNUITY OF ONE DOLLAR PER ANNUM, IMPROVED AT COMPOUND INTEREST FOR ANY NUMBER OF YEARS NOT EXCEEDING FIFTY.

1         1.000 000         1.000 000         1.000 000         1.000 000         1.000 000         2.060 000           2         2.030 000         2.085 000         2.040 000         2.050 000         2.060 000           3         3.090 900         3.106 225         3.121 600         3.152 500         3.183 600           4         4.183 627         4.214 943         4.246 464         4.310 125         4.374 616           5         5.309 136         5.362 466         5.416 323         5.525 631         5.637 093           6         6.468 410         6.550 152         6.632 975         6.501 913         6.975 319           7         7.662 462         7.779 408         7.892 294         8.142 008         8.393 838           8         8.892 336         9.051 687         9.214 226         9.549 109         9.897 468           9         10.159 106         10.368 496         10.582 795         11 1.026 564         11.491 316           10         11.463 879         11.731 393         12.006 107         12.577 893         13.180 795           11         12.607 796         13.141 992         13.486 351         14.206 787         14.971 643           12         14.192 030         14.601 962 <t< th=""><th>2.070 000 8.214 900 4.439 943 5.750 739 7.153 291 8.654 021 10.259 803 11.977 989 13.816 448 15.783 599 17.888 451 20.140 643</th></t<>	2.070 000 8.214 900 4.439 943 5.750 739 7.153 291 8.654 021 10.259 803 11.977 989 13.816 448 15.783 599 17.888 451 20.140 643
2	2.070 000 8.214 900 4.439 943 5.750 739 7.153 291 8.654 021 10.259 803 11.977 989 13.816 448 15.783 599 17.888 451 20.140 643
4         4.183 627         4.214 943         4.246 464         4.310 125         4.374 616           5         5.309 136         5.362 466         5.416 323         5.525 631         5.637 093           6         6.48 410         6.550 152         6.632 975         6.501 191         6.975 319           7         7.662 462         7.779 408         7.898 294         8.142 008         8.393 838           8         8.892 336         9.051 687         9.214 226         9.549 109         9.897 468           9         10.159 106         10.368 496         10.582 795         51 10.26 564         11.41 316           10         11.463 879         11.731 393         12.006 107         12.577 893         13.180 795           11         12.807 796         13.141 992         13.486 351         14.206 787         14.971 643           12         14.192 030         14.601 962         15.025 805         15.917 127         127 16.869 941           13         15.617 790         16.113 030         16.626 838         17.712 983         18.882 138           14         17.086 324         17.676 986         18.291 911         19.598 632         21.015 66           15         18.598 914         19.295 681         20.023	4.439 943 5.750 739 7.153 291 8.654 021 10.259 803 11.977 989 13.816 448 15.783 599 17.888 451 20.140 643
6         5.309 136         5.862 468         5.416 323         5.525 631         5.637 093           6         6.468 410         6.550 152         6.632 975         6.801 913         6.975 319           7         7.662 462         7.779 408         7.898 294         8.142 008         8.393 838           8         8.892 336         9.051 687         9.214 226         9.549 109         9.897 468           9         10.159 106         10.368 496         10.583 795         11.026 564         11.491 316           10         11.463 S79         11.7131 393         12.006 107         12.577 893         13.180 795           11         12.807 796         13.141 992         13.486 351         14.206 787         14.971 643           12         14.192 030         14.601 962         15.025 805         15.917 127         16.869 941           13         15.617 790         16.113 803         16.202 888         17.172 983         18.828 138           14         17.086 324         17.676 986         18.291 911         19.598 632         21.015 066           16         20.156 881         20.971 030         21.824 531         23.674 549         25.767 528           17         21.688 24         22.705 016         23.	5.750 739 7.153 291 8.654 021 10.259 803 11.977 989 13.816 448 15.783 599 17.888 451 20.140 643
6         6.468 410         6.550 152         6.823 975         6.601 913         6.975 319           7         7.662 462         7.779 408         7.898 294         8.142 008         8.393 368           8         8.892 336         9.051 687         9.214 226         9.549 109         9.897 468           9         10.159 106         10.368 496         10.582 795         11.026 564         11.491 316           10         11.463 779         11.731 393         12.006 107         12.577 898         13.140 792           11         12.807 796         13.141 992         13.486 351         14.206 787         14.971 648           12         14.192 030         14.601 962         15.025 805         15.917 127         16.869 941           13         15.617 790         16.113 030         16.626 838         17.712 983         18.882 138           14         17.086 324         17.676 986         18.291 911         19.598 632         21.015 066           15         18.598 914         19.295 681         20.023 588         21.578 564         23.275 970           16         20.156 881         22.705 016         23.697 512         25.840 366         28.212 880           18         23.414 435         24.499 691         <	7.153 291 8.654 021 10.259 803 11.977 989 13.816 448 15.783 599 17.888 451 20.140 643
7         7.662 462         7.779 408         7.898 294         8.142 008         8.393 838           8         8.892 338         9.051 687         9.214 226         9.549 109         9.897 468           9         10.159 106         10.368 496         10.582 795         11.026 564         11.491 816           10         11.463 879         11.731 393         12.006 107         12.577 893         13.180 795           11         12.807 796         13.141 992         13.486 351         14.206 787         14.971 643           12         14.192 030         14.601 962         15.025 805         15.917 127         16.869 941           13         15.617 790         16.113 030         16.626 838         17.712 983         18.882 138           14         17.086 324         17.676 986         18.291 911         19.598 632         21.015 668           15         18.598 914         19.295 681         18.291 911         19.598 632         21.015 066           16         20.156 881         20.975 016         23.697 512         25.840 366         28.212 80           18         23.414 435         24.499 691         25.645 413         23.132 855         30.905 653           19         25.116 868         26.357 180	8.654 021 10.259 803 11.977 989 13.816 448 15.783 599 17.888 451 20.140 643
8         8.892         386         9.051         687         9.214         226         9.549         109         9.897         468           9         10.159         106         10.368         496         10.582         795         11.026         564         11.491         31.80         795           11         11.463         879         11.713         393         12.006         107         12.577         893         13.180         795           12         14.192         300         14.601         962         15.025         805         15.917         127         16.869         941           13         15.617         790         16.113         303         16.626         888         17.712         983         18.828         138           14         17.086         324         17.676         986         18.291         911         19.598         682         21.015         066           16         20.156         881         20.971         30         21.824         549         681         22.028         82         15.78         564         23.275         705         561         23.697         512         25.840         366         28.2	10.259 803 11.977 989 13.816 448 15.783 599 17.888 451 20.140 643
10.159   106   10.368   496   10.582   795   11.026   564   11.491   316   10   11.463   379   11.731   393   12.006   107   12.577   893   13.180   795   13.141   992   13.486   351   14.206   787   14.971   648   12   14.192   030   14.601   962   15.025   805   15.917   127   16.866   941   13   15.617   790   16.113   030   16.626   838   17.712   983   18.882   131   14   17.086   324   17.676   986   18.291   911   19.598   632   21.015   066   15   18.598   914   19.295   681   20.033   588   21.578   564   23.275   970   16   20.156   881   20.071   030   21.824   531   23.657   492   25.670   528   17   21.761   588   22.705   016   23.697   512   25.840   366   22.212   880   18   23.414   435   24.499   691   25.645   413   28.132   385   30.905   653   19   25.116   868   26.357   180   27.671   229   30.536   780   23.228   902   26.870   874   22.279   682   29.778   709   33.065   954   36.785   591   21   28.676   486   30.269   471   31.969   202   35.719   252   39.992   727   22   30.536   780   23.228   902   34.247   970   38.505   214   43.392   290   23   24.452   844   44.60   414   36.617   889   41.430   475   46.995   828   47.727   099   50.815   577   258   36.459   264   38.949   857   41.645   908   47.727   099   50.815   577   258   36.459   264   38.949   857   41.645   908   47.727   099   50.815   577   258   36.459   264   38.949   857   41.645   908   47.727   099   50.815   577   258   36.459   264   38.949   857   41.645   908   47.727   099   50.815   577   258   36.459   264   38.949   857   41.645   908   47.727   099   50.815   578   50.156   383   42.940   23   42.93	11.977 989 13.816 448 15.783 599 17.888 451 20.140 643
10	13.816 448 15.783 599 17.888 451 20.140 643
11   12.807 796   13.141 992   13.486 351   14.206 787   14.971 648   12   14.192 301   14.601 962   15.025 805   15.917 127   16.869 941   18   15.617 790   16.113 803   16.626 883   17.712 983   18.882   138   14   17.086 824   17.676 986   18.291 911   19.598 682   21.015 066   15   18.598 914   19.295 681   20.023 588   21.578 564   23.275 970   16   20.156 881   20.971 030   21.824 531   23.657 492   25.670 528   17   21.761 588   22.705 016   23.697 512   25.840 366   28.212 880   18   23.414 435   24.499 691   25.645 413   28.132 385   30.905 653   19   25.116 868   26.857 180   27.671 229   30.539 004   33.799 992   20   26.870 374   28.279 682   29.778 079   33.065 954   36.785 591   21   28.676 486   30.269 471   31.969 202   35.719 252   39.992 727   22   30.536 780   23.238 902   34.247 970   38.505 214   43.92 290   23   32.452 884   34.460 414   36.617 889   41.430 475   46.995 828   24.347 976   34.565 264   38.949 857   41.645 908   47.727 099   56.815 577   25   36.545 264   38.949 857   41.645 908   47.727 099   50.815 577   26   38.553 042   41.313 102   44.311 745   51.113 454   50.156 383   27   40.709 634   42.759 960   47.084 214   54.669 126   63.705 766   28   42.930 923   48.290 627   49.967 583   58.402 688   68.528 112   29   45.218 850   48.910 799   52.966 286   62.322 712   73.639 798	15.783 599 17.888 451 20.140 643
12	17.888 451 20.140 643
13	20.140 643
14	
18.598 914	
16	22.550 488
17   21.761 588   22.705 016   23.697 512   25.840 366   28.212 880   18   23.414 485   24.499 691   25.645 413   28.132 385   30.995 653   19   25.116 868   26.857 180   27.671 229   30.539 004   33.759 992   20   26.870 374   28.279 682   29.778 079   33.065 954   36.785 591   21   28.676 486   30.269 471   31.969 202   35.719 252   39.992 727   23   05.367 80   33.238 902   34.247 970   38.505 214   43.392 290   23   32.452 884   34.460 414   36.617 889   41.430 475   46.995 828   24   34.426 470   36.666 528   89.082 604   44.501 999   50.815 577   25   36.569 264   38.949 857   41.645 908   47.727 099   54.846 512   26   38.553 042   41.313 102   44.311 745   51.113 454   59.156 383   27   40.709 634   42.759 960   47.084 214   54.669 126   63.705 766   28   42.930 923   46.290 627   49.967 583   58.402 588   68.528 112   29   45.218 850   48.910 799   52.966 286   62.322 712   73.639 798	25.129 022
18         23.414 435         24.499 691         25.645 413         28.132 885         30.905 653           19         25.116 868         26.875 180         27.671 229         80.539 004         33.759 992           20         26.870 874         28.279 682         29.778 079         33.065 964         36.785 591           21         28.676 486         30.269 471         81.969 202         35.719 252         39.992 727           22         30.536 780         32.328 902         84.247 970         38.505 214         43.392 290           23         32.452 884         34.460 414         36.617 889         41.430 475         46.995 828           24         34.426 470         36.666 528         89.082 604         44.501 999         50.815 577           25         36.459 264         38.949 857         41.645 908         47.727 099         54.864 512           26         38.553 042         41.313 102         44.311 745         51.113 464         59.156 383           27         40.709 634         42.759 060         47.084 214         54.669 126         68.705 766           28         42.930 923         46.290 627         49.967 538         58.402 588         68.705 768           28         42.930 828         48.910	27.888 054
19	30.840 217
20   26.870 874   28.279 682   29.778 079   33.065 954   36.785 591   21   28.676 486   30.269 471   31.969 202   35.719 252   39.992 727   22   30.536 780   32.328 902   34.247 970   38.505 214   43.392 290   23   32.452 884   34.460 414   36.617 889   41.430 475   46.995 828   24   34.426 470   36.666 528   89.082 604   44.501 999   50.815 577   25   36.559 264   38.949 857   41.645 908   47.727 909   54.864 512   26   38.553 042   41.313 102   44.311 745   51.113 454   59.156 383   27   40.709 634   42.759 960   47.084 214   54.699 126   63.705 766   28   42.930 923   46.290 627   49.967 583   58.402 588   68.528 112   29   45.218 850   48.910 799   52.966 286   62.322 712   73.639 798	33.999 033
21         28.676         486         30.269         471         81.969         202         35.719         252         39.992         727           22         30.536         780         32.328         902         84.247         790         38.505         214         43.392         290           23         32.452         884         34.460         414         36.617         889         41.430         475         46.995         828           24         34.426         470         36.666         528         89.082         604         44.501         999         50.815         577           25         36.459         264         38.949         857         41.645         908         47.727         999         54.864         512           26         38.553         242         41.313         102         44.311         745         51.113         464         59.156         383           27         40.709         634         42.759         960         47.084         214         54.669         126         63.705         766           28         42.930         923         46.290         627         49.967         583         58.402	37.378 965 40.995 492
22         30.536 780         32.328 902         84.247 970         38.505 214         43.892 290           23         32.452 884         34.460 414         86.617 889         41.430 475         46.995 828           24         34.426 470         36.666 528         89.082 604         44.501 999         50.815 577           25         36.459 264         38.949 857         41.645 908         47.727 099         54.864 512           26         38.553 042         41.313 102         44.311 745         51.113 464         59.156 383           27         40.709 634         42.759 060         47.084 214         54.669 126         63.705 766           28         42.930 923         46.290 627         49.967 583         58.402 568         68.528 112           29         45.218 850         48.910 799         52.966 286         62.322 712         73.639 798	44.865 177
23         32.452 884         34.460 414         86.617 889         41.430 475         46.995 828           24         34.426 470         36.666 528         89.082 604         44.501 999         50.815 577           25         36.459 264         38.949 857         41.645 908         47.727 909         54.864 512           26         38.553 042         41.313 102         44.311 745         51.113 454         59.156 383           27         40.709 634         42.759 960         47.084 214         54.669 126         63.705 766           28         42.930 923         46.290 627         49.967 583         58.402 588         68.528 112           29         45.218 850         48.910 799         52.966 286         62.322 712         73.639 798	49.005 739
24         34.426 470         36.666 528         89.082 604         44.501 999         50.815 577           25         36.459 264         38.949 857         41.645 908         47.727 099         54.864 512           26         38.553 042         41.313 102         44.311 745         51.113 454         50.156 383           27         40.709 684         42.759 060         47.084 214         54.669 126         63.705 766           28         42.930 923         46.290 627         49.967 583         58.402 583         68.528 112           29         45.218 850         48.910 799         52.966 286         62.322 712         73.639 798	53.436 141
25         36.459 264         38.949 857         41.645 908         47.727 099         54.864 512           26         38.553 042         41.313 102         44.311 745         51.113 464         59.156 383           27         40.709 634         42.759 060         47.084 214         54.669 126         63.705 766           28         42.930 923         46.290 627         49.967 583         58.402 583         68.528 112           29         45.218 850         48.910 799         52.966 286         62.322 712         73.639 798	58.176 671
26     38.553 042     41.313 102     44.311 745     51.113 454     59.156 383       27     40.709 634     42.759 060     47.084 214     54.669 126     63.705 766       28     42.930 923     46.290 627     49.967 583     58.402 568     68.528 112       29     45.218 850     48.910 799     52.966 286     62.322 712     73.639 798	63.249 030
27     40.709 684     42.759 060     47.084 214     54.669 126     68.705 766       28     42.930 923     46.290 627     49.967 583     58.402 583     68.528 112       29     45.218 850     48.910 799     52.966 286     62.322 712     73.639 798	68.676 470
28   42.930 923   46.290 627   49.967 583   58.402 583   68.528 112   29   45.218 850   48.910 799   52.966 286   62.322 712   73.639 798	74.483 823
29   45.218 850   48.910 799   52.966 286   62.322 712   78.639 798	80.697 691
	87.846 529
OU   T  OIO TIO  OI OAA OII  OU OUT 700  OU TOO 040  /3.000 100	94.460 786
31   50.002 678   54.429 471   59.328 335   70.760 790   84.801 677	102.073 041
32   52.502 759   57.334 502   62.701 469   75.298 829   90.889 778	110.218 154
83   55.077 841   60.841 210   66.209 527   80.063 771   97.843 165	118.933 425
84   57.780 177   63.453 152   69.857 909   85.066 959 104.183 755	128.258 765
85   60.462 082   66.674 013   73.652 225   90.320 307   111.434 780	
36   63.271 944   70.007 603   77.598 314   95.836 323   119.120 867	
87   66.174 228   78.457 869   81.702 246 101.628 139 127.268 119	
88   69.159 449   77.028 895   85.970 836   107.709 546   135.904 206	
89 72.234 238 80.724 906 90.409 150 114.095 028 145.058 458	
40 75.401 260 84.550 278 95.025 516 120.799 774 154.761 966	
41 78.663 298 88.509 537 99.826 536 127.839 763 165.047 684	
42 82.023 196 92.607 371 104.819 598 135.231 751 175.950 645	
48   85.483 892   96.848 629   110.012 882   142.993 339   187.507 577   144   86.443 460   145   146	247.276 496
44 89.048 409 101.238 331 115.412 877 151.143 006 199.758 032	200.120 851
45   92.719 861 105.781 678 121.029 392 159.700 156 212.743 514 1	
46   96.501 457   110.484 031   126.870 568   168.685 164   226.508 125   47   100.396 501   115.350 973 132.945 390   178.119 422   241.098 612	285.749 311
47   100.396 501   115.350 973 132.945 390   178.119 422   241.098 612   48   104.408 396   120.388 297   139.263 206   188.025 398   256.564 529	285.749 311 306.751 763
49 108.540 648 125.601 846 145.833 734 198.426 663 272.958 401	285.749 311 306.751 763 329.224 386
50 112.796 867 130.999 910 152.667 084 209.347 976 290.335 905	285.749 311 306.751 763 329.224 386 353.270 093
	285.749 311 306.751 763 329.224 386 353.270 093 878.999 000

TABLE,

SHOWING THE PRESENT WORTH OF AN ANNUITY OF ONE DOLLAR PER ANNUM, TO CONTINUE FOR ANY NUMBER OF YEARS NOT EXCERDING FIFTY.

Years.	3 per cent.	3½ per cent.	4 per cent.	5 per cent.	6 per cent.	7 per cent.	Years.
1	0.970 874	0.966 184	0.961 538	0.952 881	0.943 896	0.934 579	1
2	1.913 470		1.886 095	1.859 410	1.833 398	1.808 017	2
3	2.828 611		2.775 091	2.723 248	2.673 012	2.624 314	3
4	3.717 098		8.629 895	3.545 951	3.465 106	3.387 209	4
5	4.579 707	4.515 052	4.451 822	4.329 477	4.212 364	4.100 195	5
6	5.417 191	5.328 553	5.242 137	5.075 692	4.917 324	4.766 537	6
7	6.230 283		6.002 055	5.786 373	5.582 381	5.389 286	7
8	7.019 692		6.732 745	6.463 213	6.209 744	5.971 295	8
9	7.786 109	7.607 687	7.435 832	7.107 822	6.801 692	6.515 228	9
10	8.530 203	8.316 605	8.110 896	7.721 735	7.360 087	7.023 577	10
îi	9.252 624	9.001 551	8.760 477	8.306 414	7.886 875	7.498 669	iil
12	9.954 004	9.663 834	9.385 074	8.863 252	8.383 844	7.942 671	12
	10.634 955	10.302 738	9.985 648	9.393 573	8.852 683	8.357 635	13
	11.296 073		10.563 123	9.898 641	9.294 984	8.745 452	14
:	11.937 935		11.118 387		9.712 249	9.107 898	15
	12.561 102	12.094 117	11.652 296	10.837 770	10.105 895	9.446 632	16
				11.274 066	10.477 260	9.763 206	17
	13,753 518	13,189 682	12.659 297	11.689 587	10.827 603	10.059 070	18
	14.323 799			12.085 321	11.158 116	10.335 578	19
						10.598 997	20
	15.415 024			12.821 153	11.764 077	10.835 527	21
	15.936 917					11.061 241	22
	16.443 608			13.488 574	12,303 379	11.272 187	23
				13.798 642		11.469 334	24
						11.653 583	25
	17.876 842				13.003 166	11.825 779	26
27	18.327 031				13.210 534	11.986 709	27
28	18.764 108	17.667 019	16.663 068	14.898 127	13.406 164	12.137 111	28
29	19.188 455	18.035 767	16.983 715	15.141 074	13.590 721	12.277 674	29
30	19.600 441	18.392 045	17.292 033	15.372 451	13.764 831	12.409 041	30
81	20.000 428	18.736 276	17.588 494	15.592 811	13.929 086	12.531 814	31
32	20.338 766	19.068 865	17.873 552	15.802 677	14.084 043	12.646 555	32
83	20.765 792	19 390 208	18.147 646	16.002 549	14.230 230	12.753 790	33
34	21.131 837	19.700 684	18.411 198			12.854 009	84
	21.487 220					12.947 672	85
	21.832 <b>2</b> 52			16.546 852		13.035 208	36
	<b>22.167 23</b> 5		19.142 579			13.117 017	37
	22.492 462				14.846 019		38
1					14.949 075		39
	23.114 772				15.046 297		40
1	23.412 400				15.138 016		41
					15.224 548		42
					15.306 173		43
1	24.254 274					13.557 908	44
						13.605 522	45
						13.650 020	46
	25.024 708					13.691 608	47
	25.266 707				15.650 027	13.780 474	48
	25.501 657		21.341 472			13.766 799	49
50	25.729 764	23.455 618	21.482 185	18.255 925	15.761 861	13.800 746	50

- 574. To find the amount of an annuity, at compound interest, forborne, or in arrears, for any number of years.
- Ex. 1. What will an annuity of \$60, unpaid, or in arrears, 4 years, amount to, at 6 per cent. compound interest?

Ans. \$ 262.476.

 $\frac{1.06^4 - 1}{1.06 - 1} \times 60 = 262.476.$ Or, 4.874616 × 60 = 262.476.

The amounts of the several payments form a geometrical series, of which the annuity is the first term, the amount of \$1 for one year the ratio, the years the number of the terms, and the amount required is the sum of the series. Hence,

Rule. — Find the sum of the series as in geometrical progression. Or.

Multiply the amount of \$1 for the given time, found in the table, by the annuity, and the product will be the required amount.

Note. — The amount of an annuity at simple interest corresponds to the sum of an arithmetical series, of which the annuity is the first term, the interest on the annuity for one term the common difference, and the time in years the number of terms.

- 2. What will an annuity of \$500 amount to for 5 years, at 6 per cent. compound interest?

  Ans. \$2818.546.
- 3. What is the amount of an annuity of \$80, unpaid, or in arrears, for 9 years, at 5 per cent. compound interest?

Ans. \$882.125.

4. What is the amount of an annuity of \$1000, forborne for 15 years, at 3½ per cent. compound interest?

Ans. \$19295.68.

- 5. What will an annuity of \$30, payable semiannually, amount to, in arrears for 3 years, at 7 per cent. compound interest?
- 6. Suppose a salary of \$600 per year, payable quarterly, to remain unpaid 5½ years; to what sum will it amount, at 6 per cent. compound interest?

  Ans. \$5404.295.
  - 575. To find the present worth of an annuity, at compound interest.
  - Ex. 1. What is the present worth of an annuity of \$60, to be continued 4 years, at 6 per cent. compound interest?

Ans. \$ 207.906.

\$3.465106 × 60 = \$207.906 \rightarrow. The present worth required evidently may be obtained by finding the amount of the given annuity, by the last articles, and then finding in the usual way the present worth of that amount. A more expeditious method, however, is to find, in the table, the present worth of an annuity of \$1 for the given time and rate, and take that sum as many times as there are dollars in the given annuity, as in the operation.

RULE. — Multiply the present worth of an annuity of \$1 for the given time and rate by the number denoting the given annuity.

- 2. What is the present worth of an annuity of \$100, for 9 years, at 6 per cent.?

  Ans. \$680.169.
- 3. What is the present worth of an annuity of \$ 200, for 7 years, at 5 per cent.?

  Ans. \$ 1157.27.
- 4. Required the present worth of an annuity of \$500, to continue 40 years, at 7 per cent.
- 5. A gentleman wishes to purchase an annuity, which shall afford him, at 6 per cent. compound interest, \$500 a year, for ten years. What sum must be deposit in the annuity office to produce it?

  Ans. \$3680.04.
- 6. If a widow be entitled to \$160 a year, payable semiannually, from a fund, for 8 years, what is its value at present, at 6 per cent. compound interest?

  Ans. \$1004.88.
  - 576. To find the present worth of an annuity in perpetuity.
- Ex. 1. What is the present worth of a perpetual lease, which yields an income of \$600, the rate of interest being that of 6 per cent.?

  Ans. \$10000.

\$  $600 \div .06 = $10000$ .

The question is evidently the same as one requiring what principal in one year, at 6 per cent. interest, will yield \$600.

Rule. — Divide the given annuity by the number denoting the interest of \$ 1 for one year.

NOTE. — When the annuity is payable quarterly, semiannually, or in any other periods less than a whole year, the annuity must be increased by the interest which may thus accrue on the parts of the annuity payable before the end of the year, before dividing by the interest of \$1 for one year.

2. A ground rent in the city of Philadelphia yields an annual income of \$963, at 6 per cent. interest. What is the value of the estate?

Ans. \$16050.

3. What is the present value of a perpetual lease, yielding an income of \$6335, interest being at 7 per cent.

Ans. \$ 90500.

- 4. What sum should be paid for a perpetual annuity of \$1200, payable semiannually, interest being at 5 per cent.?

  Ans. \$24000.
- 577. To find the present worth of an annuity in reversion.
- Ex. 1. What is the present worth of an annuity of \$300, to commence in 3 years, and to continue 5 years, allowing compound interest at 6 per cent.?

  Ans. \$1061.03.

\$ 6.209744 — \$ 2.673012 = \$ 3.536732; of an an at 6 per mencing

The present worth of an annuity of \$1, at 6 per cent., commencing at once,

and continuing till the termination of the annuity, or for 3+5=8 years, is \$6.209744; and the present worth of the same annuity up to the time of the commencement of the reversion is \$2.673012. The difference of these present worths multiplied by the number of dollars in the given annuity is the present worth of the reversion.

Rule. — Find the present worth of an annuity of \$1, commencing immediately and continuing till the reversion COMMENCES, and also till the reversion TERMINATES; and multiply the difference of these present worths by the number of dollars in the given annuity. The result will be the present worth required.

- 2. The reversion of a lease of \$350 per annum, to continue 11 years, which commences 9 years hence, is to be sold. What is its worth, allowing the purchaser 6 per cent. per annum for his ready money?

  Ans. \$1633.70.
- 3. A father presents to his daughter, for 8 years, a rental of \$70 per annum, payable yearly, and its reversion for the 12 years succeeding to his son. What is the present value of the gift to his son, allowing 4 per cent. compound interest?

Ans. \$480.03.

- 4. What is the present worth of the reversion of a perpetuity of \$240 per annum, payable yearly, but not to come into possession till the expiration of 100 years, compound interest being allowed at 6 per cent.?

  Ans. \$11.78.
- 578. To find the annuity, the present worth, time, and rate being given.

Ex. 1. What annuity, continued for 4 years, at 6 per cent. compound interest, is now worth \$207.90?

Ans. \$ 60.

\$ 207.90  $\div$  3.465 = \$ 60.

The present value represented by the debt, divided by the present worth of \$ 1 for the given-time and rate, gives the annuity required.

RULE. — Divide the given present worth by the present worth of an annuity of \$1 for the given time and rate, and the result will be the annuity required.

NOTE. — When the amount of an annuity, the time and rate, are given, the annuity may be found by dividing the given amount by the amount of \$1 for the given time and rate.

- 2. The present value of an annuity, to be continued 10 years, at 6 per cent. compound interest, payable annually, is \$3680.04; required the annuity.

  Ans. \$500.
- 3. An annuity, remaining unpaid for 9 years, at 5 per cent. compound interest, amounted to \$882.125; what was the annuity?
- 4. A yearly pension which has been forborne for 6 years, at 6 per cent., amounts to \$279; what was the pension?

Ans. \$40.

# PERMUTATIONS AND COMBINATIONS.

- 579. PERMUTATION is the process of finding the number of changes that can be made in the arrangement of any given number of things.
- 580. Combination shows how often a less number of things combined can be taken out of a greater, without respect to their order.
  - 581. To find the number of changes that can be made with any given number of things, taken all at once.
  - Ex. 1. How many changes of order do the first three letters of the alphabet admit of?

    Ans. 6.



By trial we shall find that two are all the possible permutations that can be made of the first two letters of the alphabet; as, ab and ba. If we take an additional letter, 6 are all the possible permutations; as, abc, acb, bca, bac, cab, cba. Now, the same result may be obtained, in the case of the two letters, by multiplying together the first two digits, and in case of the three letters by multiplying together the first three digits, as in the operation.

RULE. — Multiply together all the terms of the natural series of numbers, from 1 up to the given number, inclusive, and the product will be the number required.

2. How many changes may be rung on 6 bells?

Ans. 720 changes.

- 3. For how many days can 10 persons be placed in a different position at dinner?
- 4. How many changes may be rung on 12 bells, and how long would they be in ringing, supposing 10 changes to be rung in one minute, and the year to consist of 365 days 5 hours and 49 minutes?

  Ans. 479001600, and 91y. 26d. 22h. 41m.
- 5. How many changes do the letters of the alphabet admit of?

  Ans. 403291461126605635584000000.
- 582. To find how many changes may be made by taking each time any number of different things less than all.
- Ex. 1. How many sets of 4 letters each may be formed out of 8 different letters?

  Ans. 1680.

$$8 \times (8-1) \times (8-2) \times (8-3)$$
  
=  $8 \times 7 \times 6 \times 5 = 1680$ .

It is evident that each one of the 8 letters may be arranged before each of the others; therefore 2 out of

the 8 letters admit of  $8 \times 7$  permutations. By taking 3 out of the 8 letters, the third letter can be arranged as the first, second, and third, in each of the same permutations, giving  $8 \times 7 \times 6$  permutations. In like manner for 4 out of 8, we obtain  $8 \times 7 \times 6 \times 5$  permutations.

RULE. — Take a series of numbers, beginning with the number of things given, and decreasing by 1, until the number of terms equals the number of things to be taken at a time, and the product of all the terms will be the answer required.

2. How many changes can be rung with 4 bells out of 6?

Ans. 360.

- 3. How many words can be made out of the 26 letters of the alphabet, 6 being taken at once?

  Ans. 165765600.
- 583. To find the number of combinations that can be formed from a given number of different things, taken a given number at a time.
- Ex. 1. How many combinations can be made of 3 letters out of 4, the letters all being different?

  Ans. 4.

$$\frac{4 \times 3 \times 2}{1 \times 2 \times 3} = \frac{4}{1} = 4.$$

We find the number of permutations which may be made by taking 3 out of 4 letters, as in Art. 582, and the number of permutations by the three letters all at once, as by Art. 581,

and, dividing the first by the latter, obtain the number of combinations required.

- RULE. Take the series, 1, 2, 3, 4, 5, &c., up to the less number of things, and find the product of the terms. Take also a series of numbers beginning with the greater number of things, and decreasing by 1 until the number of terms equals the less number of things, and find the product of the terms. The latter result divided by the former will give the number required.
- 2. How many combinations can be made of 7 letters out of 10, the letters all being different?

  Ans. 120.
- 3. A successful general, being asked what reward would satisfy him for his services, demanded only a cent for every file of 10 men which he could make with a body of 100 men. What would his demand amount to? Ans. \$173103094564.40.

# ANALYSIS BY POSITION.

- 584. Analysis by Position is the process of solving analytical questions, by assuming or supposing one or more numbers, and reasoning from them, operated upon as if they were the number or numbers required to be found.
- 585. Questions in which the required number is in any way increased or diminished in any given ratio, or in which it is multiplied or divided by any number, may be solved by means of a *single* assumption.

586. Questions in which the required numbers, or their parts, or their multiples, are increased or diminished by some given number which is no known part or multiple of the required number, or when a power or root of the required number is either directly or indirectly contained in the result given in the question, may be solved by two assumptions.

NOTE. — When the answer is obtained by means of a single assumption, the process is called Position; and when obtained by means of two assumptions, it is called Double Position.

Analysis by Position affords often a very compendious method of working questions, whose solution otherwise, except by algebra, would be lengthy and difficult. It is even found very useful in shortening some of the processes of algebra.

- 587. When the answer may be obtained by means of a single assumption or supposition.
- Ex. 1. A schoolmaster, being asked how many scholars he had, replied, that if he had as many more as he now has, and half as many more, he should have 200. Of how many scholars did his school consist?

  Ans. 80 scholars.

Assumed number, 60
As many more, 60
Half as many more, 30

150:200::60:80, Ans.

By having as many more, and half as many more, he would have had  $2\frac{1}{4}$  times the original number; therefore, the required number must be as many as 200 contains times  $2\frac{1}{4}$ , or 80. The same result may be obtained, as in the operation, thus: We assume the number of scholars to be 60; if to 60 as

many more, and half as many more, are added, the sum is 150. As this result has the same ratio to the result in the question as the supposed number has to the number required, we find the answer by a proportion.

Rule. — Assume any convenient number, and proceed with it according to the nature of the question. Then, if the result be either too much or too little, as the result found is to the result given, so will be the number assumed to the number required.

- 2. A person, after spending  $\frac{1}{2}$  and  $\frac{1}{4}$  of his money, had \$60 left; what had he at first?

  Ans. \$144.
- 3. What number is that, which, being increased by ½, ½, and ½ of itself, equals 125?
- 4. A's age is double that of B, and B's is triple that of C, and the sum of all their ages is 140. What is each person's age?

  Ans. A's 84, B's 42, C's 14 years.

- 5. A person lent a sum of money at 6 per cent., and at the end of 10 years received the amount, \$560. What was the sum lent?

  Ans. \$350.
- 588. When two or more assumptions or suppositions are required in finding the answer.
- Ex. 1. A lady purchased a piece of silk at 80 cents per yard, and lining for it at 30 cents per yard; the silk and lining contained 15 yards, and the price of the whole was \$7. How, many yards were there of each?

Ans. 5 yards of silk; 10 yards of lining.

Assume 6 yards of silk, \$4.80 Assume 4 yards of silk, \$3.20 Lining would be 9yd., 2.70 Lining would be 11yd., 3.30 Their sum, \$7.50 Their sum, \$6.50 Sum in the question, 7.00 Sum in the question, 7.00 Second error, —\$0.50 + .50 : 6 - 4 : : .50 : 1; 6 yards — 1 yard = 5 yards; 15 yards — 5 yards = 10 yards.

Since the silk and lining contain 15 yards, cost \$ 7.00, the average price per yard is  $46\frac{2}{3}$  cents; and 80 cents —  $46\frac{2}{3}$  cents =  $33\frac{1}{3}$  cents;  $46\frac{2}{3}$  cents — 30 cents =  $16\frac{2}{3}$  cents; and the quantity of lining will therefore be to that of the silk as  $33\frac{1}{3}$  is to  $16\frac{2}{3}$ , or as to 2 is to 1. Hence, if the given number of yards, 15, be divided into 3 parts, two of those parts, or 10 yards, will be for the lining, and the other part, or 5 yards, will be for the silk.

The same result is obtained as in the operation, thus: We assume the quantity of silk to be 6 yards; then the lining would be 9 yards. We find the cost of each of these at the prices given, and, adding, have as the sum of their costs \$7.50. This is a result too large by \$0.50, when compared with the sum in question. By assuming the quantity of the silk to be 4 yards, and proceeding in a like manner, we obtain for the cost of the silk and the lining \$6.50, a sum too small by \$0.50. The first error, arising from a result too large, is marked with the sign plus (+), and the second error, arising from a result too small, is marked with the sign minus (—). Then, since one of the results is too large, and the other too small, we then say, as the sum of the errors, or .50 + .50, is to the difference of the assumptions, or 6 — 4, so is .50 the less error, to 1, the correction; which, being added to 4 (yards), the sum, 5, expresses the number of yards of silk; and consequently that of the lining must be 10 yards.

Rule. — Assume two different numbers, perform on them separately the operations indicated in the question, and note the ERRORS of the re-

sults. Then, as the difference of the errors, if the results be both too great or both too small, or as the sum of the errors, if one result be too great and the other too small, is to the difference of the assumed numbers, so is either error to the correction to be applied to the number which produced that error.

NOTE 1. — The rule usually given fails in an important class of questions; but the rule here given, if not the simplest in the resolution of some questions,

has the advantage of being applicable in every case.

NOTE 2. — In relation to all questions which in algebra would be resolved by equations of the first degree, the differences between the true and the assumed numbers are proportional to the differences between the result given in the question and the results arising from the assumed numbers. But the principle does not hold exactly in relation to other questions; hence, when applied to them, the above rule, or any other of the kind that can be given, will only produce approximations to the true results. In which case the assumed numbers should be taken as nearly true as possible. Then, to approximate more nearly to the required number, assume for a second operation the number found by the first, and that one of the first two assumptions which was nearer the true answer, or any other number that may appear to be still nearer to it. In this way, by repeating the operation as often as may be necessary, the true results may be approximated to any assigned degree of accuracy. This process is sometimes applied with advantage in extracting the higher roots, when approximate results, differing but slightly from entire correctness, will answer.

- 2. A and B invested equal sums in trade; A gained a sum equal to 1 of his stock, and B lost \$225; then A's money was double that of B's. What did each invest? Ans. \$600.
- 3. A person, being asked the age of each of his sons, replied, that his eldest son was 4 years older than the second, his second 4 years older than the third, his third 4 years older than the fourth, or youngest, and his youngest half the age of the oldest. What was the age of each of his sons?

Ans. 12, 16, 20, and 24 years.

- 4. A gentleman has two horses, and a saddle worth \$50. Now if the saddle be put on the first horse, it will make his value double that of the second horse; but if it be put on the second, it will make his value triple that of the first. What was the value of each horse? Ans. The first, \$30; second, \$40.
- 5. A gentleman was asked the time of day, and replied, that  $\frac{2}{3}$  of the time past from noon was equal to  $\frac{8}{33}$  of the time to midnight. What was the time? Ans. 12 minutes past 3.
- 6. A and B have the same income. A saves 12 of his, but B, by spending \$100 per annum more than A, at the end of 10 years finds himself \$600 in debt. What was their income?

  Ans. \$480.



7. A gentleman hired a laborer for 90 days on these conditions: that for every day he wrought he should receive 60 cents, and for every day he was absent he should forfeit 80 cents. At the expiration of the term he received \$33. How many days did he work, and how many days was he idle?

Ans. He labored 75 days, and was idle 15 days.

- 8. There is a fish whose head weighs 15 pounds, his tail weighs as much as his head and  $\frac{1}{6}$  of his body, and his body weighs as much as his head and tail. What is the weight of the fish?

  Ans. 72lb.
- 9. If 12 oxen eat 3\frac{1}{3} acres of grass in 4 weeks, and 21 oxen eat 10 acres in 9 weeks, how many oxen would it require to eat 24 acres in 18 weeks, the grass growing uniformly?

Ans. 36 oxen.

10. What number exceeds three times its square root by 11? (Note 2.)

Ans. 26.4201648.

## SCALES OF NOTATION.

- 589. The scale of any system of notation is the law of relation existing between its units of different orders.
- 590. The radix of any scale is the number of units it takes of one order to make a unit of the next higher. Thus, 10 is the radix of the decimal or denary system, 2 of the binary, 3 of the ternary, 4 of the quaternary, 5 of the quinary, 6 of the senary, 7 of the septenary, 8 of the octary, 9 of the nonary, 11 of the undenary or undecimal, 12 of the duodenary or duodecimal, 20 of the vigesimal, 30 of the trigesimal, 60 of the sexagesimal, and 100 of the centesimal.
- 591. In writing any number in a uniform scale, as many distinct characters or symbols are required as there are units in the radix of the given system. Thus, in the decimal or denary scale 10 characters are required, in the binary scale 2 characters, in the duodenary or duodecimal 12 characters, and so on. In the binary scale use is made of the characters 1 and 0, in



the ternary, 1, 2, and 0, &c., the cipher being one of the characters in each scale. In the duodenary scale, eleven characters being required beside the cipher, the first nine may be supplied by the nine digits, the tenth by t, the eleventh by t, and the twelfth by 0.

592. To change any number expressed in the decimal scale to any other required scale of notation.

Ex. 1. Express the common number 75432, in the senary and duodenary scales.

Ans. 1341120 and 377e0.

		•			
		OPERATION.			
6)75432		12)75432			
6)12572	0	12)6286	0		
6)2095	2	$12)\overline{523}$	11,	or	e
6)349	1	12)43	7		
6)58	1	3	7		
6)9	4				
1	3	•			

By dividing the given number by 6, it is distributed into 12572 classes, each containing 6, with 0 remainder. By the second division by 6, these classes are distributed into 2095 classes, each containing 6 times 6, or the second power of 6, with a remainder of 2 of the former class, each containing 6. By the third division the

classes last found are distributed into 349 classes, each containing 6 of the latter, which were each the second power of 6, and therefore these are the third power of 6 with a remainder 1 time the second power of 6. In like manner, the next quotient expresses 58 times the fourth power of 6, with a remainder 1 time the third power of 6; the next quotient expresses 9 times the fifth power of 6, with a remainder 4 times the fourth power of 6; and the last quotient expresses 1 time the sixth power of 6, with a remainder 3 times the fifth power of 6. Hence, the given number is found to be equal to  $1 \times 6^6 + 3 \times 6^5 + 4 \times 6^4 + 1 \times 6^3 + 1 \times 6^2 + 2 \times 6 + 0$ , or according to the senary system of notation 1341120.

By proceeding in like manner, we find the given number to be equal to  $3 \times 12^4 + 7 \times 12^3 + 7 \times 12^2 + 11 \times 12 + 0$ , or, ac-

cording to the duodenary scale, 377e0.

RULE. — Divide the given number by the radix of the required scale repeatedly, till the quotient is less than the radix; then the last quotient, with the several remainders in the retrograde order annexed, placing ciphers where there is no remainder, will be the given number expressed in the required scale.

2. Change 37 from the decimal to the binary scale.

Ans. 100101.

3. Reduce 1000000 in the decimal scale to the ternary and also to the nonary. Ans. 1212210202001, and 1783661.

- 4. How will 476897 in the decimal scale be expressed in The duodecimal scale? Ans. 1tee 95.
  - -593. To change any number into the decimal scale, when expressed in any other scale of notation.
  - Ex. 1. Change 377t0 from the duodecimal to the decimal scale. Ans. 75432.

OPERATION.	
$ \begin{array}{r} 377t0 \\ 12 \\ 43 \\ 12 \\ 523 \\ 12 \\ 6286 \\ 12 \\ 75432 \end{array} $	We multiply dix, and add to then we multiple add to the proproceed till all ployed; and we several figures cobtained in a mof compound nu

the left-hand figure by the rathe product the next figure; ly this sum by the radix, and duct the next figure, and so the figures have been emthus have, as the values of the collected into one sum, 75432, nanner similar to the reduction umbers.

RULE. - Multiply the left-hand figure of the given number by the given radix, and to the product add the next figure; then multiply this sum by the radix, and add to this product the next figure; and so proceed till all the figures of the given number have been added. result will be the given number in the decimal scale.

Note. - When it is required to change a number from a scale other than decimal to another scale also other than decimal, first change the number as given into the decimal scale, and then the result into the required scale.

2. Reduce 234 from the quinary to the decimal scale.

Ans. 69.

3. Change 21122 from the ternary to the decimal scale.

Ans. 206.

- 4. Change 100101 in the binary scale to a number in the Ans. 37. decimal scale.
- 5. Reduce 13579 in the duodecimal scale to the undecimal scale. Ans. 190t3.
- 6. How will 123454321 in the senary scale be expressed in Ans. 9873t1. the duodenary scale?
- 594. To perform addition, subtraction, multiplication, division, &c. in a scale of notation whose radix is other than 10, we may

Proceed as in the common scale of notation, except that the radix of the given scale must be used in the cases wherein the number 10 would be applied in the decimal system.

- Ex. 1. Required the sum and difference of 45324502 and 25405534 in the senary scale, or scale whose radix is 6.
  - Ans. Sum, 115134440; difference, 15514524.
- 2. Multiply 2483 by 589 in the undenary scale, or scale whose radix is 11. Ans. 13122t5.
- 3. Divide 1184323 by 589 in the duodenary scale, whose radix is 12. Ans. 2483.
- 4. Extract the square root of 11122441 in the senary scale. Ans. 2405.

## DUODECIMALS.

- 595. DUODECIMALS are numbers expressed in a scale whose radix is 12, so that 12 units of each lower order make a unit of the next higher.
- 596. In finding the contents of surfaces and solids, however, it is customary to apply the term duodecimal to a mixture of the decimal and duodecimal scales. Thus, in admeasurements in which the foot is the leading unit, though the different orders of units are expressed according to the duodecimal scale, the number of units in each order is usually expressed according to the decimal scale.
- 597. According to this mixed scale, the foot is divided into 12 equal parts, and each of these parts into 12 other equal parts, and so on indefinitely, giving 12, 144, &c. In writing these fractions without their denominations, to distinguish their orders, or denominations, accents, called indices, are written on the right of the numerators. Thus, inches are called primes, and are marked '; the next subdivision is called seconds, marked ": the next is thirds, marked "; and so on.

Note. - Numbers expressed by the mixed scale of feet, primes, seconds, &c. may be changed to the pure duodecimal scale, and the operations of addition, subtraction, multiplication, division, and so on, then be performed with them, as in Art. 594, observing to place a point between the unit and its lower duodecimal orders, and in the result changing the figures on the left of the point into the decimal scale, and marking those on the right as primes, seconds. &c., according to their places from the order of units.

But the operations of adding, subtracting, &c. are usually performed by

other methods, such as are given in the articles that follow.

## ADDITION AND SUBTRACTION OF DUODECIMALS.

598. Duodecimals may be added and subtracted in the same manner as compound numbers.

Ex. 1. Add together 121ft. 3' 9", 105ft. 11' 8", 80ft. 0' 6", and 15ft. 10' 0" 4".

Ans. 323ft. 1' 11" 4"".

2. From 462ft, 4' 9" take 307ft, 9' 1".

3. What is the value of 92ft. 0' 6" — 21ft. 9' 10" + 19ft. 10' 3" 6"?

Ans. 90ft. 0' 11" 6".

## MULTIPLICATION OF DUODECIMALS.

599. The index of the unit of a product of any two duodecimal orders is equal to the sum of the indices of those factors. That is, feet multiplied by a number denoting feet produces feet; feet by a number denoting primes produces primes; primes by a number denoting primes produces seconds, &c.

Note. —In multiplication of duodecimals, or in other multiplication, the multiplier is always regarded as an abstract number, though the notation of feet, primes, &c. is usually retained, in order the better to note the different orders of units. For the same reason, in division of duodecimals, the divisor usually retains the notation of feet, primes, &c.

# 600. To multiply one duodecimal by another.

Ex. 1. Required the number of square feet in a platform 6 feet 8 inches long, and 4 feet 5 inches wide.

Ans. 29 sq. ft. 5' 4".

FIRST OF	ERATION.
6ft.	8′
<b>4</b> ft.	5′
2	9' 4"
26	8′
29sq.ft	. 5' 4"

We first multiply each of the terms in the multiplicand by the 5' in the multiplier; thus,  $8' \times 5' = 40''$  = 3' and 4". Writing the 4" under the multiplier, we reserve the 3' to add to the next product. Then 6ft.  $\times$  5' = 30'; and 30' + 3' = 38' = 2ft. and 9', which we write in their order beneath the multiplier. We next multiply by the 4ft., thus:  $8' \times 4$  feet = 32' = 2ft. and 8'. We write the 8' under the primes in the other partial product, and

reserve the 2ft. to add to the next product; and 6ft. × 4ft. = 24ft.; 24ft. + 2ft. = 26ft., which we write under the feet in the other partial product. The two being added together, we have 29 sq. ft. 5' 4"; or (the primes and seconds being changed to a fraction of a foot), 29 \( \frac{64}{14} \) sq. ft.

 $\frac{6.8}{4.5}$   $\frac{2 9 4}{2 2 8}$  2 5.5 4 = 29 sq. ft. 5' 4''.

ARCOND OPERATION.

In the second operation the work is performed as in pure duodecimals (Art. 594). The point (·) separates lower duodecimal orders from those beginning with feet. As to the number of feet in the multiplicand and multiplier, no change is required in that part of either of the given numbers in expressing them according to the duo-

decimal scale. In performing the multiplication, we make the several reductions required according to the radix 12; and have, after pointing off, 25.54 square feet, expressed according to the duodecimal scale. The units, or feet, at the left of the point, are readily changed to the decimal scale by multiplying the left-hand figure, 2by 12, the number of units in the radix, and adding the right-hand figure, 5, and giving the figures to the right of the point their proper notation, we have then 29sq. ft. 5' 4" for the answer, as before.

RULE. — Write the multiplier under the multiplicand, so that units

of the same orders shall stand in the same column.

Beginning at the right, multiply each term in the multiplicand by each term of the multiplier, and write the first term of each partial product under its multiplier, observing to carry a unit for every twelve from each lower order to the next higher.

The sum of the partial product will be the product required.

2. How many square feet in a floor 48 feet 6 inches long, and 24 feet 3 inches broad?

Ans. 1176 sq. ft. 1' 6".

3. The length of a room being 20 feet, its breadth 14 feet 6 inches, and height 10 feet 4 inches, how many square yards of painting are in it, deducting a fireplace of 4 feet by 4 feet 4 inches, and 2 windows, each 6 feet by 3 feet 2 inches?

Ans. 73-7 square yards.

4. Required the solid contents of a wall 53 feet 6 inches long, 10 feet 3 inches high, and 2 feet thick.

5. There is a house with four tiers of windows, and four windows in a tier; the height of the first is 6 feet 8 inches; the second, 5 feet 9 inches; the third, 4 feet 6 inches; the fourth, 3 feet 10 inches; and the breadth is 3 feet 5 inches; how many square feet do they contain in the whole?

Ans. 283sq. ft. 7'.

6. How many cords in a pile of wood 97 feet 9 inches long, 4 feet wide, and 3 feet 6 inches high?

Ans. 10½76 cords.

7. Required the number of cords of wood in a pile 100 feet long, 4 feet wide, and 6 feet 11 inches high.

Ans. 2152.

#### DIVISION OF DUODECIMALS.

**601.** To divide one duodecimal by another.

Ex. 1. A board in the form of a rectangle, whose area is 27 sq. ft. 8' 6", is 1ft. 7in. wide; what is its length?

Ans. 17ft. 6in.

We find how many times 27

square feet contains the divisor. and obtain 17 feet for the quo-

tient, we multiply the entire divisor by the 17ft., and subtract the product, 26ft. 11', from the idend, and obtain 9', to which remainder we bring down the 6", and, dividing, obtain 6' for the quotient. Multiplying the entire divisor by 6", we obtain 0ft. 9' 6", which, subtracted as before, leaves

no remainder. Therefore 17 feet 6 inches is the length required.

In the second operation we reduce the feet of the given multiplicand and multiplier to the duodecimal scale, and thus obtain We then conduct 23.86 and 1.7. the division with reference to the radix 12, as is ordinarily done with respect to 10 (Art. 594). The result obtained is 15.6 in the duodecimal, which, on changing the

figures to the left of the point to the decimal scale, and giving the proper notation to the figure on the right of the point, becomes transformed to 17ft. 6' = 17ft. 6in., the answer, as before.

RULE. - Find how many times the highest term of the dividend will contain the divisor. By this quotient multiply the entire divisor, and subtract the product from the corresponding terms of the dividend. To the remainder annex the next denomination of the dividend, and divide as before, and so continue till the division is complete.

- 2. It required 834 sq. ft. 3' of board to cover the side of a certain building. The height was 17ft. 9in.; what was the Ans. 47 feet. length of the side?
- 3. How many feet wide is a plank of uniform width, whose length is 18ft. 9in., thickness 3 inches, and solid contents 84ft. 4' 6"?
- 4. An alley has an area of 792ft. 6' 9" 2"". Its width is Ans. 62ft. 8' 6". 12ft. 7' 8". Required its length.

# MISCELLANEOUS EXAMPLES.

- 1. A merchant engages a clerk at the rate of \$20 for the first month, \$25 for the second, \$30 for the third, &c., thus increasing his salary by \$5 per month. How long must the clerk retain his situation, so as to receive on the whole as much as he would have received had his salary been fixed at \$52.50 per month?

  Ans. 14 months.
- 2. A mason has plastered 3 rooms; the ceiling of each is 20 feet by 16 feet 6 inches, the walls of each are 9 feet 6 inches high, and 90 yards are to be deducted for doors, windows, &c. For how many yards must be be paid? Ans. 251yd. 1ft. 6'.
- 3. A man of wealth, dying, left his property to his ten sons, and the executor of his will, as follows: to his executor, \$1024; to his youngest son, as much and half as much more; and increasing the share of each next elder in the ratio of 1½. What was the share of the eldest?
- 4. A butcher, wishing to buy some sheep, asked the owner how much he must give him for 20; on hearing his price, he said it was too much; the owner replied, that he should have 10, provided he would give him a cent for each different choice of 10 in 20, to which he agreed. How much did he pay for the 10 sheep, according to the bargain?

  Ans. \$ 1847.56.
- 5. If 340 square feet of carpeting are required to cover the floor of a room, how many yards will be required, provided the width of the carpeting is 3 feet 9 inches? Ans. 30yd. 8in.
- 6. If a clergyman's salary of \$700 per annum is 6 years in arrears, how much is due him, allowing compound interest at 6 per cent.?

  Ans. \$4882.72.
- 7. Suppose a clock to have an hour-hand, a minute-hand, and a second-hand, all turning on the same centre. At 12 o'clock all the hands are together and point at 12.
- (1.) How long will it be before the second-hand will be between the other two hands, and at equal distances from each?

  Ans.  $60\frac{789}{7837}$  seconds.
- (2.) Also before the minute-hand will be equally distant between the other two hands?

  Ans. 61 § § 3 seconds.
- (3.) Also before the hour-hand will be equally distant between the other two hands?

  Ans. 59‡3 seconds.

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## MENSURATION.

#### DEFINITIONS.

- 602. A point is that which has neither length, breadth, nor thickness, but position only.
  - 603. A line is length, without breadth or thickness.

A straight line is one which has the same direction in its whole extent; as the line A B.

A curved line is one which continually changes its direction; as the line C D.



**604.** An angle is the inclination or opening of two lines, which meet in a point.

A right angle is an angle formed by a straight line and a perpendicular to it; as the angle ABC.

a, A B

An acute angle is one less than a right angle; as the angle E B C.

An obtuse angle is one greater than a right angle; as the angle F B C.



605. A surface is that which has length and breadth, without thickness.

A plane surface, or simply a plane, is that in which, if any two points whatever be taken, the straight line that joins them will lie wholly in it.

Every surface, which is not a plane, or composed of planes, is a curved surface.

- **606.** The area of a figure is its quantity of surface; and is estimated in the square of some unit of measure, as a square inch, a square foot, &c.
- 607. A solid, or body, is that which has length, breadth, and thickness.
- 608. The solidity, or volume of a solid, is estimated in the cube of some unit of measure; as a cubic inch, a cubic foot, &c.
- 609. MENSURATION is the process of determining the areas of surfaces, and the solidity or volume of solids.

#### MENSURATION OF SURFACES.

**610.** A plane figure is an enclosed plane surface; if bounded by straight lines only, it is called a rectilineal figure, or polygon. The perimeter of a figure is its boundary, or contour.

611. Three-sided polygons are called triangles; those of four sides, quadrilaterals; those of five sides, pentagons, and so on.

### TRIANGLES.

612. An equilateral triangle is one whose sides are all equal; as CAD.

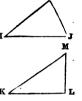
Note. — The line AB, drawn from the angle A perpendicular to the base CD, is the altitude of the triangle CAD.



An isosceles triangle is one which has two of its sides equal; as EFG.



A scalene triangle is one which has its three sides unequal; as HIJ.



A right-angled triangle is one which has a right angle; as KLM.

613. To find the area of a triangle.

Multiply the base by half the altitude, and the product will be the area. Or,

Add the three sides together, take half that sum, and from this subtract each side separately; then multiply the half of the sum and these remainders together, and the square root of this product will be the area.

Ex. 1. What are the contents of a triangle whose perpendicular height is 12 feet, and whose base is 18 feet?

Ans. 108 feet.

2. There is a triangle, the longest side of which is 15.6 feet, the shortest side 9.2 feet, and the other side 10.4 feet. What are the contents?

Ans. 46.139+ feet.

3. The triangular gable of a certain building has a base of 40 feet and an altitude of 15 feet; how many square feet of boards will cover it?

Ans. 300 sq. ft.

4. The perimeter of a certain field in the form of an equilateral triangle is 336 rods; what is the area of the field?

Ans. 33 acres 152 sq. rd.

### QUADRILATERALS.

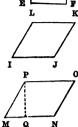
- 614. A parallelogram is any quadrilateral whose opposite sides are parallel.
- **615.** A rectangle is any right-angled parallelogram; as ABCD.



616. A square is a parallelogram whose sides are equal, and whose angles are right angles; as EFGH.



617. A rhombus is a parallelogram whose sides are equal, and whose angles are not right angles; as IJKL.



618. A rhomboid is a parallelogram whose angles are not right angles; as M N O P.

NOTE. — The altitude of a parallelogram is the perpendicular distance between any two of its parallel sides taken as bases, as the line P Q, drawn between two sides of the rhomboid M N O P, and perpendicular to the sides M N and O P.

619. A trapezoid is a quadrilateral which has only two of its sides parallel; as RSTU.



**620.** A trapezium is a quadrilateral which has no two sides parallel; as W X Y Z.



NOTE. — A diagonal of a quadrilateral, or of any polygon of more than four sides, is a straight line which joins the vertices of two opposite angles, or of two angles not adjacent; as the line X Z joining vertices of opposite angles of the trapezium W X Y Z.

621. To find the area of a parallelogram.

Multiply the base by the altitude, and the product will be the area.

Ex. 1. What are the contents of a board 15 feet long and 2 feet wide?

Ans. 30 feet.

2. A rectangular state is 128 miles long and 48 miles wide. How many square miles does it contain?

Ans. 6144 miles.

3. The base of a rhomboid being 12 feet, and its height 8 feet, required the area.

Ans. 96 feet.

4. Required the area of a rhombus of which one of the equal sides is 358 feet, and the perpendicular distance between it and the opposite side is 194 feet.

Ans. 69452 sq. ft.

5. The largest of the Egyptian pyramids is square at its base, and measures 693 feet on a side. How much ground does it cover?

Ans. 11 acres 4 poles.

6. What is the difference between the area of a floor 40 feet square, and that of two others, each 20 feet square? Ans. 800 feet.

7. There is a square whose area is 3600 yards; what is the side

of a square, and the breadth of a walk along each side and each end of the square, which shall take up just one half of the whole?

Ans. {42.42+ yards, side of the square.
8.78+ yards, breadth of the walk.

To find the area of a trapezoid.

Multiply half of the sum of the parallel sides by the altitude, and the product is the area.

If the parallel sides of a trapezoid are 75 and 33 feet, and the perpendicular breadth 20 feet, what is the area?

Ans. 1080 sq. ft. 2. Required the area of a meadow in the form of a trapezoid, whose parallel sides are 786 and 473 links, and whose altitude is 986 Ans. 6 acres 33 rods 3 yards. links.

623. To find the area of a trapezium.

Divide the trapezium into two triangles by a diagonal, and then find the areas of these triangles; their sum will be the area of the trapezium.

Ex. 1. Required the area of a garden in the form of a trapezium, of which the four sides are 328, 456, 572, and 298 feet, and the diagonal, drawn from the angle between the first and second sides, 598 Ans. 8 acres 1 rood 31 rods 29 yards 3.85 feet. feet.

2. Given one of the diagonals of a field, in the form of a trapezium, equal 17 chains 56 links, to compute the area, the perpendiculars to that diagonal from the opposite angles being 8 chains 82 links, and 7 chains 73 links. Ans. 14 acres 2 roods 5 rods.

# Pentagons, Hexagons, &c.

- 624. A pentagon is a polygon of five sides; a hexagon, one of six sides; a heptagon, one of seven sides; an octagon, one of eight sides; a nonagon, one of nine sides; and so on for a decagon, undecagon, dodecagon, &c.
- A regular polygon is one whose sides and angles are equal; as the pentagon ABCDE.

**626.** To find the area of a regular polygon.

Multiply the perimeter by half the perpendicular let fall from the centre upon one of the sides. Or,

Multiply the square of one of the sides by the number against the polygon in the following

#### TABLE.

Pentagon,	1.720477	Nonagon,	6.181824
Hexagon,	2.598076	Decagon,	7.694209
Heptagon,	3.633913	Undecagon,	9.365641
Octagon,	4.828427	Dodecagon,	11.196152

Ex. 1. What is the area of a regular pentagon, of which the side is 250 feet, and the perpendicular from the centre to one side 172.05 feet?

Ans. 107531.25 sq. ft.

2. What is the area of a regular hexagon whose side is 356 yards, and whose perpendicular is 308.305 yards?

Ans. 329269.74yd.

3. The side of a regular octagonal enclosure is 60 yards; how many acres are included? Ans. 3 acres 2 roods 14 rods 19 yards.

4. The side of a field, whose shape is that of a regular decagon, is 243 feet; what is its area?

Ans. 10 acres 1 rood 28 rods 24 yards 6.347 feet.

#### CIRCLES.

627. A circle is a plane figure bounded by a line, every part of which is equally distant from a point within called the centre; as AEFGBD.



The circumference or periphery of a circle is the line that bounds it.

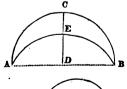
A radius is a line drawn from the centre to the circumference; as C A, or C D.

A diameter is a line which passes through the centre, and is terminated by the circumference; as AB.

An arc is any portion of the circumference; as AD, AE, or EGF.

The chord of an arc is the straight line joining its extremities; as
EF, which is the chord of the arc EGF.

- **628.** The segment of a circle is the portion included by an arc and its chord; as the space included by the arc EGF and the chord EF.
- **629.** The sector of a circle is the portion included by two radii and the intercepted arc; as the space A C D A.
- **630.** A zone is the space between two parallel chords of a circle; as the space AEFBA.
- **631.** A lune, or crescent, is the space included between the intersecting arcs of two eccentric circles; as ACBEA.



632. A circular ring is the space included between the circumference of two concentric a circles; as the space between the rings AB and CD.



633. To find the circumference of a circle, the diameter being given.

Multiply the diameter by 3.141592.

Ex. 1. If the diameter of a circle is 144 feet, what is the circumference?

Ans. 452.389248 feet.

2. If the diameter of the earth is 7964 miles, what is its circumference?

Ans. 25019.638688-|- miles.

- 8. Required the circumference of a circle, whose radius is 512 feet.

  Ans. 4 furlongs 34 rods 5 yards 1 foot.
- 634. To find the diameter of a circle, the circumference being given.

Multiply the circumference by .318309.

- Ex. 1. Required the diameter of a circle, whose circumference is 1043 feet.

  Ans. 331.997+ feet.
- 2. If the circumference of a circle is 25000 miles, what is its diameter?

  Ans. 7957.74+ miles.
- 3. If the circumference of a round stick of timber is 50 inches, what is its diameter?

  Ans. 15.91549+ inches.
- 635. To find the area of a circle, the diameter, or the circumference, or both, being given.

Multiply the square of the diameter by .785898. Or, Multiply the square of the circumference by .079577. Or, Multiply half the diameter by half the circumference.

- Ex. 1. If the diameter of a circle is 761 feet, what is the area?

  Ans. 454840.475158 feet.
- 2. There is a circular island, three miles in diameter; how many acres does it contain?

  Ans. 4523.89+ acres.

3. Required the area of a circle, of which the circumference is 1284 yards.

Ans. 27 acres 17 rods 0.8— yards.

- 4. Required the area of a circle, of which the diameter is 169, and the circumference 532 inches. Ans. 17 yards 3 feet 13 inches.
  - 636. To find the area of a sector of a circle.

Multiply the length of the arc by half the radius of the circle. Or, As 360° are to the degrees in the arc of the sector, so is the area of the circle to the area of the sector.

Ex. 1. Required the area of a sector, of which the arc is 79 and the radius of the circle 47 inches.

Ans. 1856.5 inches.

2. Required the area of a sector, of which the arc is 26°, and the radius of the circle 25 feet.

Ans. 141.8 square feet.

637. To find the area of the segment of a circle.

Find the area of the sector which has the same arc with the segment; and also the area of the triangle formed by the chord and the radii drawn to its extremities. The difference of these areas, when the segment is less, and their sum, when the segment is greater, than the semicircle, will be the area of the segment. Or.

To two thirds of the product of the height of the segment by the chord add the cube of the height, divided by twice the chord.

Ex. 1. Required the area of the segment ABCA, of which the arc ABC is 49.25°, the chord AC 10 feet, and the radii EA, EB, and EC, each 12 feet. Ans. 7.35 sq. ft.



- 2. Required the area of a segment whose height is 15 rods and whose chord is 24 rods. Ans. 1 acre 3 roods 30 rods 9.4 yards.
  - To find the area of a zone of a circle.

From the area of the whole circle subtract the areas of the segments on the sides of the zone.

Ex. 1. Required the area of a zone whose parallel sides are 23.25 and 20.8 feet, in a circle whose radius is 12 feet. Ans. 206+ sq. ft.

2. Required the area of a zone included between two chords of 16 feet each, the diameter of the circle being 20 feet. Ans. 224.7 sq. ft.

To find the area of a lune or crescent.

Find the difference of the areas of the two segments formed by the arcs of the lune and its chord.

- Ex. 1. If the chord of two intersecting arcs is 72 feet, and the height of one of the segments is 30, and of the other 20 feet, what is the area of the crescent? Ans. 612 sq. ft.
  - 640. To find the area of a circular ring.

Multiply the sum of the diameters of the two circles by the difference of the diameters, and that product by .7854.

Ex. 1. What is the area of the ring formed by two circles whose Ans. 235.62 sq. vd.

diameters are 10 and 20 yards?

- 2. In the centre of a circular pond there is an island 128 yards in diameter; what is the area of the pond, provided the exact distance from any part of the outer side of the pond to the centre of the island Ans. 1 acre 1 rood 14 rods 17 yards 7.4 feet. is 781 yards?
- 641. To find the side of a square that shall equal the area of a circle of a given diameter or circumference.

Multiply the diameter of the circle by .886227. Or, Multiply the circumference of the circle by .282094.

Ex. 1. I have a round field, 50 rods in diameter; what is the side of a square field that shall contain the same area?

Ans. 44.31135+ rods.

2. I have a circular field 360 rods in circumference; what must be the side of a square field that shall contain the same area?

Ans. 101.55+ rods.

3. John Smith had a farm which was 10,000 rods in circumference, which he sold at \$71.75 per acre. He purchased another farm containing the same quantity of land in the form of a square; required the length of one of its sides.

Ans. 2820.94+ rods.

642. To find the diameter of a circle that shall contain the area of a given square.

Multiply the side of the given square by 1.12838.

- Ex. 1. The side of a square is 44.31135 rods; required the diameter of a circular field containing the same area.
- 643. To find the side of the largest equilateral triangle that can be inscribed in a circle of a given diameter or circumference.

Multiply the given diameter by .866025. Or, Multiply the given circumference by .275664.

Ex. 1. There is a certain piece of round timber 30 inches in diameter; required the side of an equilateral triangular beam that may be hewn from it.

Ans. 25.98—inches.

2. How large an equilateral triangle may be inscribed in a circle whose circumference is 5000 feet?

Ans. 1378.320 feet.

3. Required the side of an equilateral triangular beam, that may be hewn from a round piece of timber 80 inches in circumference.

Ans. 22.05 + inches.

644. To find the side of the largest square that can be inscribed in a circle of a given diameter or circumference.

Multiply the given diameter by .707106. Or, Multiply the given circumference by .225079.

NOTE. — To find the circumference of a circle required to exactly admit a square of a given side, divide the given side by 225079.

Ex. 1. I have a piece of timber 30 inches in diameter; how large a square stick can be hewn from it?

Ans. 21.21+ in. square.

2. Required the side of a square that may be inscribed in a circle 80 feet in diameter.

Ans. 56.56848+ feet.

3. I have a circular field whose circumference is 5000 rods; what is the side of the largest square field that can be made in it?

Ans. 1125.395+ rods.

4. How large a square stick may be hewn from a piece of round timber 100 inches in circumference?

Ans. 22.5+ inches square.

5. What must be the circumference of a tree that, when hewn, shall be 18 inches square?

Ans. 79.97—inches.

6. I have a garden which is 20 rods square; required, in feet, the circumference of a circle that will enclose this garden.

Ans. 1466.15+ feet.

645. To find the diameter of the three largest equal circles that can be inscribed in a circle of a given diameter.

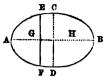
Divide the given diameter by 2.155.

Ex. 1. Required the diameter of each of the largest three circles that can be inscribed in a circle 86.2 inches in diameter.

Ans. 40 inches.

### ELLIPSE.

646. An ellipse is a plane figure bounded by a curve, from any point of which the sum of the distances to two fixed points is equal to a given distance. The two fixed points are called the foci, as G,H in the ellipse AECBDFA. The major or transverse axis of an ellipse is its longest diameter, as A.B. The



minor or conjugate axis of an ellipse is its shortest diameter, as CD. The segment of an ellipse is a portion cut off from the ellipse, as FAEF.

647. To find the area of an ellipse, the two diameters being given.

Multiply the two diameters together, and that product by .785398.

What is the area of an ellipse, whose two diameters are 24 and 18 inches? Ans. 339.2919 inches.

2. What is the area of an elliptical pond, whose longest diameter is 33 feet 5 inches, and whose shortest diameter 20 feet 3 inches? Ans. 59 sq. yd. 67 sq. in.

#### MENSURATION OF SOLIDS.

### Prisms and Cylinders.

648. A prism is a figure whose ends or bases are any plane figures which are equal and similar, and parallel to each other, and whose sides are parallelograms.

A triangular prism is one whose base is a triangle; as the

figure A B.

A square prism is one whose base is a square; a pentagonal prism, one whose base is a pentagon; and so on, according to the figure of the ends or bases.

A parallelopiped is a prism whose ends or bases, as well as

its sides, are parallelograms.

A cylinder is a round body of uniform diameter. and which has circular bases parallel to each other; as the figure CD.

The perimeter of a prism or cylinder is the line that bounds its end or base; and the altitude or height is the distance

between the ends or bases.

N

The convex surface of a prism or cylinder is the entire surface, exclusive of the two ends or bases.

To find the surface of a prism, or of a cylinder.

Multiply the perimeter of the given prism or cylinder by the height, and to the product add the area of the two ends.

Ex. 1. Required the surface of a triangular prism, of which the dis-37





tance between the ends is 13 feet, and the sides of the base 23, 34, and 19 inches.

Ans. 85.22+ square feet.

2. Required the surface of a pentagonal prism, whose length is 14 feet, and each side of whose base is 33 inches. Ans. 218.52 sq. ft.

- 3. Required the surface of a cylinder 13 feet long, the circumference of whose base is 57 inches.

  Ans. 65.34 square feet.
- 4. How often must a cylinder, 5 feet 3 inches long, whose diameter is 21 inches, revolve, to roll an acre?

  Ans. 1509.18 times.
- 5. Required the wall-surface of a square room, whose sides are each 16 feet long and 10 feet high.

  Ans. 71½ square yards.
  - 651. To find the contents or volume of a prism or cylinder.

Multiply the area of the base of the given prism or cylinder by the height.

Ex. 1. What are the contents of a triangular prism, whose length is 12 feet, and each side of whose base is 2½ feet? Ans. 32.47—cu. ft.

- 2. Required the volume of a triangular prism, whose length is 10 feet, and the three sides of whose triangular end or base are 5, 4, and 3 feet.

  Ans. 60 cu. ft.
- 3. How many cubic feet in a block of marble, whose length is 3 feet 2 inches, breadth 2 feet 8 inches, and depth 2 feet 6 inches?

  Ans. 21½ cu. ft.
- 4. What is the volume of a cylinder, whose length is 9 feet, and the circumference of whose base is 6 feet?

  Ans. 25.78+cu. ft.

### PYRAMIDS AND CONES.

652. A pyramid is a solid having for its base some rectilineal figure, and for its sides triangles meeting in a common point called the vertex; as the figure AB.

The slant height of a pyramid is a line drawn from the vertex to the middle of one of the sides of the base.

vertex to the initiale of one of the sides of the base.

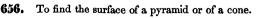


653. A cone is a solid having a circle for its base, and tapering uniformly to a point called the vertex.

The slant height of a cone is a line drawn from the vertex to the

circumference of the base.

- 654. The altitude or height of a pyramid or of a cone is a line drawn from the vertex perpendicular to the plane of the base.
- 655. The frustum of a solid is the part that remains after cutting off the top by a plane parallel to the base; as the frustum of a cone C D.



Multiply the perimeter or the circumference of the base by half of the slant height, and to the product add the area of the base.

- Ex. 1. Required the area of the surface of a square pyramid, whose base is 2 feet 8 inches square, and whose slant height is 3 feet 9 inches.
- 2. What is the convex surface of a cone, whose slant height is 20 feet, and the circumference of whose base is 9 feet? Ans. 90 feet.
  - 657. To find the volume of a pyramid, or of a cone.

Multiply the area of the base by one third of the altitude.

Ex. 1. What is the solidity of a cone, whose height is 12½ feet, and the diameter of whose base is 2½ feet?

Ans. 20.45+ feet.

2. What are the contents of a triangular pyramid, whose height is 14 feet 6 inches, and the sides of whose base are 5, 6, and 7 feet?

Ans. 71.035+ feet.

658. To find the surface of a frustum of a pyramid, or of a cone.

Multiply the sum of the perimeters or of the circumferences of the two ends by half of the slant height; and to the product add the areas of the two ends.

- Ex. 1. Required the surface of a frustum of a pentagonal pyramid, whose slant height is 10 inches, and the sides of whose base are 3 and 5 inches.
- 2. What is the surface of the frustum of a cone, the diameters of the bases being 43 and 23 inches, and the slant height 9 feet?

  Ans. 90.72+ sq. ft.

659. To find the volume of a frustum of a pyramid, or of a cone.

Multiply the areas of the two ends together, and extract the square root of the product. To this root add the two areas, and multiply their sum by one third of the altitude.

Ex. 1. If the length of a frustum of a square pyramid be 18 feet 8 inches, the side of its greater base 27 inches, and that of its less 16 inches, what is the volume?

Ans. 61.228+ cu. ft.

2. What are the contents of a stick of timber, whose length is 40 feet, the diameter of the larger end being 24 inches, and of the smaller end 12 inches?

Ans. 73½ cu. ft., nearly.

### SPHERES, SPHEROIDS, &C.

660. A sphere is a solid, bounded by a curved surface, every part of which is equally distant from a point within, called the centre.

The axis or diameter of a sphere is a line passing through the centre, and terminated by the surface; as the line A B.



The radius of a sphere is a line drawn from the centre to any part of the surface.

**661.** A segment of a sphere is a part of it cut off by any plane; as the figure A C B D. The plane is the base of the segment; the perpendicular distance from the centre of the base to the convex surface is the height of the segment; as C D.



- **662.** A spherical zone is a part of the surface of a sphere included between two parallel planes, which form its bases; and the height of a spherical zone is the perpendicular distance between the planes forming its bases.
- **663.** A cylindrical ring is a figure formed by bending a cylinder uniformly till the two ends meet; as A C D B.



**664.** A spheroid is a figure resembling a sphere, and which may be formed by the revolution of an ellipse about one of its axes; as A E C B D F A.

· If the ellipse revolves about its longer or transverse axis or diameter, the spheroid is *prolate*, or oblong; if about its shorter or conjugate diameter, the spheroid is *oblate*, or flattened.



- **665.** A segment of a spheroid is a part cut off by any plane, as F A E F.
  - 666. To find the surface of a sphere.

Multiply the diameter by the circumference.

Ex. 1. Required the convex surface of a globe, whose diameter is 24 inches.

Ans. 1809.55+ inches.

2. Required the surface of the earth, its diameter being 7957<sup>3</sup>/<sub>2</sub> miles, and its circumference 25,000 miles. Ans. 198943750 square miles.

667. To find the solidity of a sphere.

Multiply the cube of the diameter by .523598.

Ex. 1. What is the solidity of a sphere, whose diameter is 12 inches?

Ans. 904.78+ inches.

2. Required the solidity of the earth, supposing its circumference to be 25,000 miles.

Ans. 263858149120.06886875 miles.

**668.** To find the convex surface of a segment or of a zone of a sphere.

Multiply the height of the segment or zone by the circumference of the sphere of which it is a part.

- Ex. 1. If the diameter of a sphere is  $12\frac{1}{2}$  feet, what is the convex surface of a segment cut off from it, whose height is 2 feet?
- Ans. 78.54 sq. ft.

  2. If the diameter of the earth, considered as a perfect sphere, is
  7970 miles, and the height of each temperate zone is taken at
  2143.623553 miles, what is the surface of each temperate zone?

  Ans. 53673229.81+sq. m.
  - 669. To find the solidity of a segment of a sphere.

Multiply the square of the height plus three times the square of the radius of the base, by the height, and this product by .5236.

Ex. 1. Required the solidity of a spherical segment, whose height is 3 feet, and the radius of whose base is 4½ feet. Ans. 109.56 cu. ft.

2. Required the solidity of the segment of a sphere, whose height is 9 feet, and the diameter of whose base is 20 feet.

670. To find the surface of a cylindrical ring.

Multiply the sum of the thickness and the inner diameter by the thickness, and that product by 9.8696.

Ex. 1. Required the surface of a cylindrical ring, whose inner diameter is 21 inches, and whose thickness is 4 inches.

Ans. 986.96 sq. in.

Ans. 1795.42 cu. ft.

671. To find the solidity of a cylindrical ring.

Multiply the sum of the thickness and the inner diameter by the square of the thickness, and that product by 2.4674.

Ex. 1. Required the solidity of a cylindrical ring, whose inner diameter is 25 inches, and whose thickness is 5 inches.

Ans. 1850.55 cu. in.

672. To find the solidity of a spheroid.

Multiply the square of the revolving axis by the fixed axis, and that product by .523598.

Ex. 1. If the fixed axis of a spheroid is 32 inches, and the revolving axis 20 inches, what is the solidity?

Ans. 6702.08 cu. in.

2. Required the contents of a balloon in the form of a prolate spheroid, having its longer diameter 48 feet, and its shorter 38 feet.

Ans. 36291.76 cu. ft.

#### MENSURATION OF LUMBER.

673. Boards are usually measured by the square foot.

Planks, joists, beams, &c. are usually measured by board measure, the board being considered to be 1 inch in thickness.

Round timber is sometimes measured by the ton, and sometimes by

board measure.

674. To find the number of square feet in a board.

Multiply the length of the board, taken in feet, by its width, taken in inches; and the product divided by 12 will give the contents in square feet. Or,

Take both the length and width in feet, and their product will be the

contents in feet.

NOTE. — If the board is tapering, take half the sum of the width of its ends for the width.

Ex. 1. What are the contents of a board 24 feet long, and 8 inches wide?

Ans. 16 feet.

2. What are the contents of a board 30 feet long, and 16 inches wide?

Ans. 40 feet.

- 3. What are the contents of a tapering board, 30 feet long, whose ends are, the one 26 inches, and the other 14 inches wide?
- 675. To find the number of feet, board measure, in a plank, joist, beam, &c.

Multiply the width taken in inches by the thickness in inches, and this product by the length, in feet; and the last product divided by 12 will give the contents in feet, board measure.

Note. — If the plank, joist, &c. is tapering in width, take half the sum of the width of the ends for the width; and if the taper be both of the width and the thickness, the common rule of obtaining the contents in cubic feet is, to multiply half the sum of the areas of the two ends by the length, and divide the product by 144.

Ex. 1. How many feet are there in 3 joists, which are 15 feet long, 5 inches wide, and 3 inches thick?

Ans. 56½ feet.

2. How many feet in 20 joists, 10 feet long, 6 inches wide, and 2 inches thick?

Ans. 200 feet.

3. How many feet in a beam 20 feet long, 10 inches thick, whose width tapers from 18 to 16 inches?

Ans. 283\frac{1}{3} feet.

**676.** To find the contents of round timber.

Multiply the length, taken in feet, by the square of one fourth of the mean girth, taken in inches; and this product divided by 144 will give the contents in cubic feet.

Note. — The girth of tapering timber is usually taken about 1 the distance

from the larger to the smaller end.

The rule is that in common use, though very far from giving the actual number of cubic feet in round lumber measured by it. 40 cubic feet, as given by the rule, are in fact equal to  $50^{\circ}_{100}^{\circ}$  true cubic feet. The following rule gives results more nearly accurate, requiring to be diminished by only one foot in 190, to give exact contents. Multiply the square of one fifth of the mean girth, taken in inches, by twice the length, in feet; and divide by 144.

Ex. 1. How many cubic feet in a stick of timber which is 30 feet long, and whose girth is 40 inches?

Ans. 20½ feet.

2. If a stick of timber is 50 feet long, and its girth is 56 inches, what number of cubic feet does it contain?

Ans. 68 18 feet.

3. What are the contents of a log 90 feet long, and whose circumference is 120 inches?

Ans. 562½ feet.

### GAUGING OF CASKS.

677. Gauging is the process of finding the capacities of casks or other vessels.

Casks are generally considered to be of four varieties: 1. Having the staves nearly straight; 2. Having the staves very little curved; 3 Having the staves of a medium curve; 4. Having the staves considerably curved.

NOTE. — Casks of the first variety approach very nearly the form of a cylinder; those of the third variety are of the shape of a molasses hogshead; those of the second variety have a curvature of stave between that of the first and third; and the fourth have a greater curvature than that of the third.

- 678. In gauging casks, it is necessary first to find the mean di-This is found by taking the end and middle diameters, and the length in inches; and then adding to the end diameter the product of the difference between the end and middle diameters by .55, .60, .65, or .70, as the cask may be of the first, second, third, or fourth variety.
  - To find the capacity of a cask in gallons.

Multiply the square of the mean diameter, in inches, by the length, in inches; and the product multiplied by 0034 will give the capacity in . liquid or wine gallons.

NOTE 1. — If the capacity is required in ale or beer gallons, use for a multiplier .0028 instead of .0034. If imperial gallons are required, multiply the liquid or wine gallon, as found by the rule, by .833.

NOTE 2. — The contents of any vessel being known in cubic inches, its capacity in liquid gallons may be found by dividing by 231; in ale or beer gallons, by dividing by 282; and in bushels, by dividing by 2150.42.

- Required the capacity in gallons of a cask of the fourth variety, whose middle diameter is 35 inches, head diameter 27 inches, and length 45 inches. Ans. 162.6.
- 2. What is the capacity in gallons of a cask of the third variety, whose middle diameter is 38 inches, head diameter 30 inches, and length 42 inches?
- 3. What are the contents in liquid measure of a tub 40 inches in diameter at the top, 30 inches at the bottom, and whose height is 50 Ans. 209.66gal. inches?
- 4. How many wine gallons will a cubical box contain, that is 10 Ans. 1496 77 gal. feet long, 5 feet wide, and 4 feet high?
- 5. How many ale gallons will a trough contain, that is 12 feet long, 6 feet wide, and 2 feet high? Ans. 88218 gal.
- 6. How many bushels of grain will a box contain that is 15 feet long, 5 feet wide, and 7 feet high? Ans. 421.8bu.

### TONNAGE OF VESSELS.

680. The tonnage of a ship is the number of tons burden it will carry, with safety, under the ordinary circumstances of navigation.

The light-loaded water-line of a vessel is the line made by the water upon the outside of the hull as it floats without load; and the deep-loaded water-line is that made in like manner when it is fully laden.

The number of cubic feet of the hull between these two waterlines, divided by 35, the number of cubic feet of sea-water which must be taken to weigh a ton, represents the weight of water displaced in sinking the vessel from the light to the deep-loaded waterline, and therefore its true tonnage.

681. Government has by law established a rule by which the custom-house officers are to be guided in collecting tonnage duties. But as it does not always give the actual tonnage, builders, and others, usually make their estimates by some other rule.

#### GOVERNMENT RULE.

FOR SINGLE-DECKED VESSELS. Take the length on deck from the forward side of the main stem to the after side of the stern post, and the breadth at the broadest part above the main vales; take the depth from the under side of the deck plank to the ceiling of the hold; and deduct from the length three fifths of the breadth; multiply the remainder by the breadth, and the product by the depth; and divide the last product by 95.

FOR DOUBLE-DECKED VESSELS. Proceed as with single-decked

vessels, except for the depth take half the breadth.

NOTE. — The government rule is differently construed. The length is usually taken in a line with the deck; the depth at the main hatch. But with regard to the breadth, there is a great want of uniformity among measurers; most take the breadth about 45 inches below the plank-sheer at the broadest part; some consider the upper wales, and others the lower, at the main wales, thus making a considerable difference in their results.

The government rule for single-decked vessels operates very well, but the rule for double-decked vessels, which is also intended to include all vessels of more than one deck, often fails to give the true tonnage. A more accurate method would, for DOUBLE-DECKED VESSELS, take the breadth 5 feet below the upper deck, at the broadest part, and for THREE-DECKED VESSELS 7 feet below the upper deck; and in each case for depth of hold three fifths of the breadth.

Ex. 1. A. & G. T. Sampson, of East Boston, have contracted to build a clipper ship 191 \( \frac{1}{2} \) feet long, 36 \( \frac{1}{2} \) feet wide, 22 \( \frac{1}{2} \) feet deep; what is the government tonnage of the ship? Ans. 1184 \( \frac{1}{2} \) \( \frac{1}{2} \) tons.

2. What is the government tonnage of the ship Meridian, whose

length is  $184\frac{6}{13}$ , width  $38\frac{1}{12}$ , and depth 28 feet?

Ans.  $1284\frac{296047}{547260}$  tons.

3. The ship Mattakeeset is  $195_{12}^{2}$  feet long,  $39_{12}^{4}$  wide, and  $27_{12}^{6}$  deep; what is the government tonnage of the same?

5. What is the government tonnage of a double-decked vessel, whose length is 159 feet, and width 30 feet?

Ans. 667<sup>1</sup>/<sub>1</sub> tons.

6. What is the government tonnage of Noah's ark, admitting its length to have been 479 feet, its breadth 80 feet, and its depth 48 feet?

Ans. 14517 17 tons.

### MISCELLANEOUS EXAMPLES.

1. What number is that to which, if \(\frac{2}{3}\) of \(\frac{1}{3}\) be added, the sum will

2. A certain gentleman, at the time of his marriage, agreed to give his wife 2 of his estate, if, at the time of his death, he left only a daughter, and if he left only a son, she should have 1 of his property; but as it happened, he left a son and a daughter, in consequence of which the widow received in equity \$ 2400 less than she would have received if there had been only a daughter. What would have been his wife's dowry if he had left only a son? Ans. \$ 2100.

3. A gentleman being asked what o'clock it was, said that it was between 5 and 6; but, to be more particular, he said that the minutehand had passed as far beyond the 6 as the hour-hand wanted of having reached the 6; that is, that the hour and minute-hands made equal acute angles with a line passing from the 12 through the 6. Required the time of day. Ans. 32m.  $18_{7}^{6}$ s. past 5.

4. Divide 97deg. 55m. 7fur. 35rd. 4ft. 6in. by 6.

5. A, B, and C are to share \$100,000 in the proportion of \(\frac{1}{2}\), \(\frac{1}{2}\), and 1, respectively; but C's part being lost by his death, it is required to divide the whole sum properly between the other two.

Ans. A's part is \$57,142\$, and B's \$42,857\$.

6. A father devised 7 of his estate to one of his sons, and the residue to the other, and the remainder to his wife. The difference of his sons' legacies was found to be 257£. 3s. 4d. What money did he leave for his widow? Ans. 635£. 0s. 10\feat{9}\d.

7. In the walls of Balbec, in Turkey, the ancient Heliopolis, there are three stones laid end to end, now in sight, that measure 61 yards in length, one of which is 63 feet long, 12 feet thick, and 12 feet broad; what is its weight, supposing its specific gravity to be 3 times that of water? Ans. 850d tons.

8. A burden of 200lb., suspended on a pole 4ft. in length, the point of suspension being 6in. from the middle, is carried by two men, the ends of the pole resting on their shoulders; how much of this load

is borne by each man?

Ans. 125lb. and 75lb. 9. The court-house in Boston has eight pillars of granite, each 25ft. 4in. in length, 4ft. 5in. in diameter at one end, and 3ft. 5in. in diameter at the other end. How many cubic feet do they contain, and what is their weight, allowing a cubic foot to weigh 3000 Ans. 2455.03 cub. ft.; 230.15+tons. ounces?

10. A father, dying, left his son a legacy, 1 of which he spent in 8 months; \$ of the remainder lasted him 12 months longer, after which he had only \$410 left. What amount did his father bequeath him? Ans. \$ 956.66%.

11. A merchant sold goods to a certain amount, on a commission of 4 per cent., and, having remitted the net proceeds to the owner, received 1 per cent. for prompt payment, which amounted to \$15.60. Ans. \$ 260. mount of his commission?

12. A, of Boston, remits to B, of New York, a bill of exchange on London, the avails of which he wishes to be invested in goods on his account. B, having disposed of the bill at 7½ per cent. advance, received \$9675; and having reserved for himself ½ per cent. on the sale of the bill, and 2 per cent. for commission, he invests the remainder. What is the amount invested, and for how much was the bill drawn?

Ans. Investment, \$9461.58\frac{3}{4}; the bill was £2025.

13. Bunker Hill Monument is 30ft. square at its base, and 15ft. square at its top; its height is 220 feet. From the bottom to the top, through its centre, is a conical avenue 15ft. in diameter at the bottom, and 11ft. at the top. How many cubic feet are there in the monument?

Ans. 86.068.518++ ft.

14. A hired a house for one year for \$300; at the end of 4 months he takes in M as a partner, and at the end of 8 months he takes in P.

At the end of the year, what rent must each pay?

Ans. A pays \$  $183\frac{1}{3}$ ; M pays \$  $83\frac{1}{3}$ ; P pays \$  $33\frac{1}{3}$ .

15. A merchant receives on commission three kinds of flour; from A he receives 20 barrels, from B 25 barrels, and from C 40 barrels. He finds that A's flour is 10 per cent better than B's, and that B's is 20 per cent. better than C's. He sells the whole at \$6 per barrel. What in justice should each man receive?

Ans. A receives \$  $139\frac{14}{24}$ ; B, \$  $158\frac{17}{24}$ ; C,  $211\frac{14}{24}$ .

16. Bought 100 barrels of flour, at \$5 per barrel, and immediately sold it on a credit of six months. The note which I received for pay I got discounted at the Suffolk Bank, and, on examining my money, I found that I had gained 20 per cent. on my purchase. What did I receive per barrel for the flour?

Ans. \$6.18\frac{1}{6.88}.

17. Required the greatest possible number of hills of corn that can be planted on a square acre, the hills to occupy only a mathematical point, and no two hills to be nearer than three and a half feet. Ans. 4165.

18. Lent a friend \$ 700, which he kept 20 months. Some years after I borrowed of him \$ 300; how long should I keep it to balance the favor?

Ans. 46% months.

19. John Lee gave ½ of his estate to his wife, ½ of the remainder to his oldest son, and ½ of the residue to his oldest daughter, and ½ of what then remained, which was \$1500, was to be equally distributed among his other children, who received \$150 each; required the number of his children, and the value of his estate.

20. A and B set out to travel round a certain island, which is 80 miles in circumference. A travels 5 miles a day, and B 7 miles a day. How far must B travel to overtake A?

Ans. 280 miles.

21. If 24.4 cubic inches of lead weigh 16 pounds, required the number of feet of lead pipe that can be made from 80 pounds of lead, the caliber of the pipe to be 1 inch, and the thickness of it \(\frac{1}{4}\) of an inch.

Ans. 10.35\(-\frac{1}{4}\) feet.

22. How long a tube can be made from a cylinder of lead 8 inches long and 2 inches in diameter, and through the centre of which is a hole \(\frac{3}{4}\) of an inch in diameter; the tube or pipe to be \(\frac{3}{4}\) of an inch in caliber, and \(\frac{3}{8}\) of an inch in thickness?

Ans. 16.29—in.

23. Four men, A, B, C, and D, bought a stack of hay, containing

tons, for \$ 100. A is to have 12 per cent. more of the hay than B, is to have 10 per cent more than C, and C is to have 8 per ent. more than D. Each man is to pay in proportion to the quanty he receives. The stack is 20 feet high, and 12 feet square at its ase, it being an exact pyramid; and it is agreed that A shall take is share first from the top of the stack, B is to take his share the ext, and then C and D. How many feet of the perpendicular neight of the stack shall each take, and what sum shall each pay?

Ans. A takes 13.22+ft, and pays \$  $28.93\frac{12287}{1884}$ ; B takes 3.14+ft. and pays \$ 25.83 $\frac{1}{1}$  \$ 27; C takes 2.06+ft., and pays \$ 23.48 $\frac{1}{1}$  \$ 27;

**D** takes 1.58+ft., and pays \$  $21.74\frac{17066}{28741}$ .

24. A. B. and C bought a grindstone, for which they paid \$10.60. B paid 20 per cent. more than A, and 10 per cent. less than C. The diameter of the stone was 65 inches, and the diameter of the place for What sum did each pay, and how much must the shaft 3 inches. each grind off from the semidiameter to obtain his proper share of the stone?

Ans. A paid \$3, B \$3.60, and C \$4. A grinds off 5 inches; B

71 inches, and C 181 inches.

25. A servant draws off a gallon on each day, for 20 days, from a cask containing 10 gallons of wine, each time supplying the deficiency by the addition of a gallon of water; and then, to escape detection, he again draws off 20 gallons, supplying the deficiency each time by a gallon of wine. How much water still remains in the cask?

Ans. 1.0679577 gallons, or more than a gallon and half a pint.

26. The dimensions of a bushel measure are 181 inches wide, and 8 inches deep; what should be the dimensions of a similar measure that Ans. 37in. wide, 16in. deep. would contain 8 bushels?

27. What is the weight of a hollow spherical iron shell 5 inches in diameter, the thickness of the metal being 1 inch, and a cubic inch of Ans. 13.2387lb.

iron weighing 123 of a pound?

28. At a certain time between 2 and 3 o'clock, the minute-hand Within an hour after, the hour-hand and was between 3 and 4. minute-hand had exactly changed places with each other. was the precise time when the hands were in the first position?

Ans. 2h. 15m. 56 143s.

29. Required the contents of the largest cube that can be inscribed Ans. 1539.58+ cu. in.

in a sphere 20 inches in diameter.

- 30. If in a pair of scales a body weigh 90 pounds in one scale, and only 40 pounds in the other, required the true weight, and the proportion of the lengths of the two arms of the balance-beam on each side of the point of suspension.
- Ans. Weight 60lb., and the proportions 3 to 2. 31. In turning a one-horse chaise within a ring of a certain diameter, it was observed that the outer wheel made two turns, while the inner wheel made but one; the wheels were each 4 feet high; and supposing them fixed at the distance of 5 feet asunder on the axletree, what was the circumference of the track described by the outer wheel? Ans. 62.83 + feet.

32. The ball on the top of St. Paul's Church is 6 feet in diameter.

What did the gilding of it cost, at 31d. per square inch?

33. There is a conical glass, 6 inches high, 5 inches wide at the top, and which is 1 part filled with water. What must be the diameter of a ball, let fall into the water, that shall be immersed by it?

Ans. 2.445+ inches.

34. A certain lady, the mother of three daughters, had a farm of 500 acres, in a circular form, with her dwelling-house in the centre. Being desirous of having her daughters settled near her, she gave to them three equal parcels, as large as could be made in three equal circles included within the periphery of her farm, one to each, with a dwelling-house in the centre of each; that is, there were to be three equal circles, as large as could be made within a circle that contained 500 acres How many acres did the farm of each daughter contain, how many acres did the mother retain, how far apart were the dwelling-houses of the daughters, and how far was the dwelling-house of each daughter from that of the mother?

Ans. Each daughter's farm contained 107 acres 2 roods 31.22+ rods. The mother retained 176 acres 3 roods 26.34+ rods. The distance from one daughter's house to the other was 148.119817+ The mother's dwelling-house was distant from her daugh-

ters' 85.51+ rods.

35. James Page has a circular garden, 10 rods in diameter; how many trees can be set in it, so that no two shall be within ten feet of each other, and no tree within two and a half feet of the fence enclosing the garden?

36. A and B engaged to reap a field for 90 shillings; and as A could reap it in 9 days, they promised to complete it in 5 days. They found, however, that they were obliged to call in C, an inferior workman, to assist them for the last two days, in consequence of which B received 3s. 9d. less than he otherwise would have done. In what time could B and C each reap the field?

Ans. B could reap it in 15 days, and C in 18 days.

37. A merchant tailor bought 40 yards of broadcloth, 21 yards wide; but on sponging it, it shrunk in length upon every 4 yards half a quarter, and in width, one nail and a half upon every 11 yards. To line this cloth, he bought flannel 5 quarters wide, which, being wet, shrunk the whole width on every 20 yards in length, and in width it shrunk half a nail. Required the number of yards of flannel used in Ans.  $71\frac{7}{18}$  yards. lining the cloth.

38. I have a garden in the form of an equilateral triangle, whose sides are 200 feet. At each corner stands a tower; the height of the first is 30 feet, the second 40, and the third 50. At what distance from the base of each tower must a ladder be placed, that it may just reach the top of each? And what is the length of the ladder, the

garden being a horizontal plane?

Ans. The foot of the ladder from the base of the first tower 118.811+ feet; second tower, 115.827+ feet; third tower, 111.875+ feet.

Length of the ladder, 122.535 + feet.

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